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ABSTRACT

This document contains reports and papers from a conference focusing on the political dimension of mathematics education. The program was organized around four large themes: (1) "Mathematics Education and Culture" based on 13 papers; (2) "Society and Institutionalized Mathematics Education" based on 41 papers; (3) "Educational Institutions and the Individual Learner" based on 32 papers dealing with classroom learning practices; and (4) "Mathematics Education in the Global Village," which summarized five hearings that were focused on the accountability of the international mathematics education community. (YP)

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Mathematics, Education, and Society

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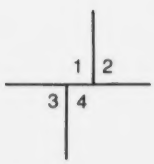

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Document Series No. 35

Mathematics, Education, and Society

Reports and papers presented in the
Fifth Day Special Programme
on "Mathematics, Education, and Society"
at the 6th International Congress on Mathematical Education
Budapest, 27 July - 3 August, 1988

edited by

Christine Keitel
(Chief editor)
and
Peter Damerow, Alan Bishop, Paulus Gerdes
(Chief organizers)

**Division of Science
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Preface

This is the second title which Unesco has brought out based upon the Sixth International Congress on Mathematical Education (ICME 6) which brought some 2,500 mathematics educators from seventy-two countries together in Budapest, Hungary, in August 1988. The Congress devoted its fifth day entirely to examining the political dimension of mathematics education. This was an extension of the previous Congress' theme 'Mathematics for All' (see Mathematics for All, Science and Technology Education Document Series, 20). Another of the theme groups was devoted to evaluation and assessment and Unesco has printed a selection from the forty-seven papers presented in the thirteen sessions, Evaluation and Assessment in Mathematics Education (Science and Technology Education Document Series, 32).

The Mathematics, Education, and Society programme was organized around four fields: Mathematics Education and Culture; Society and Institutionalized Mathematics Education; Education Institutions and the Individual Learner; and Mathematics Education in the Global Village. It is only natural that Unesco, within its domain of education, science, social science, culture and communication, decided to publish these proceedings.

Unesco wishes to express its appreciation to the chief editor, Christine Keitel, to the chief organizers of the Mathematics, Education and Society Special Programme, Peter Damerow, Alan Bishop and Paulus Gerdes, to the ninety contributors from over forty countries, to the Max Planck Institute for Human Development and Education of Berlin (West) for preparing the manuscript and to ICME 6 for permission to publish these proceedings.

The views expressed in this report are those of the editor or the individual authors and not necessarily those of Unesco.

We welcome comments on the contents of this document, which should be sent to: Mathematics Education Programme Specialist, Division of Science, Technical and Environmental Education, Unesco, Place de Fontenoy, 75700 Paris, France. The two documents mentioned above are also available free of charge from this same address.

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Mathematics, Education, and Society (MES)

The main task of scientific community concerned with mathematics education is to support the teaching and learning going on in the schools. However, increasingly the interrelation between mathematics education and educational policies has become a matter of worldwide consciousness, and it is evident now that mathematics education has a serious dimension. It was novel in the ICME Congress to make this political dimension the issue of a whole day, i.e., the 5th day, with its special program called "Mathematics, Education, and Society."

The high number of contributors offered to the organizers indicates the increasing awareness of the relations between education in general as a universal human right and mathematics education in particular. On this day almost 90 contributors from over 40 countries were working together addressing issues of mathematics and mathematics education as human activities, which, on the one hand, belong universally to the core of the school curriculum all over the world, yet, on the other hand, are always embedded in particular cultural settings. Both the historical dimension of the past, the present, and the future of mathematical education, as well as the scale from cultural and socio-political conditions in general to the social conditions of classroom activities and individual learning processes were fundamental dimensions of the program schedule.

However, within this general framework of the 5th-day program the speakers as well as the contributors were concerned with key problems of the social conditions and the outcome of mathematical activities.

For example, one important focus of the day was on the restricted opportunities to learn for children of certain minority groups in industrialized countries, for girls almost everywhere and for the majority of the youth growing up in Third World countries. There is a growing awareness of the importance of ethnomathematical activities as a means to overcome eurocentrism and cultural oppression in mathematical learning.

A second focus was on the consequences of uncontrolled technological development which promotes specialization and suppresses pedagogical aims of education, like general competence and individual autonomy. Creative uses of mathematics, meaningful learning related to social experience and social needs, and critical and multicultural perspectives in the designing of curricula and learning environments were vividly recommended by most of the contributors.

The outcome of the 5th-day special program is more an agenda for future activity than a balanced account of achievements and limitations of mathematics education under the present social and political conditions. This is the message which will be disseminated by means of a UNESCO publication containing the main arguments of all contributions to the 5th-day program of ICME 6.

The day was organized around four large themes. Timeslot 1 dealt with *Mathematics Education and Culture*. General presentations on the central themes helped to set the scene for the day. Timeslot 2 concerned *Society and Institutionalized Mathematics Education*. Timeslot 3 focused on *Educational Institutions and the Individual Learner*. Within both timeslots several parallel sessions took place with panel presentations and discussions on specific topics. In timeslot 4 at the end of the 5th-day program five hearings on *Mathematics Education in the Global Village* raised main questions society asks mathematics educators.

Mathematics Education and Culture

Mathematics education has a cultural history, a diverse present, and a cultural future. How can we represent it to ourselves? This was the leading question of the three opening sessions dealing with historical, actual, and future perspectives.

Social History of Mathematics Education

This theme concerns some significant aspects of the history of mathematics education, which has contributed to the determination of today's situation. The Hungarian mathematics historian Arpàd Szabó shows in his presentation "Mathematics and Dialectics" that fundamental principles like mathematical definitions, reasoning, and proving were raised by the philosophy of the Eleatics, and that the system of mathematics as it is formulated in Euclid's Elements is partly stimulated by the dialectics of Eleaticism and is partly a further developed construction based on these dialectics. Ahmed Djebbar from Algeria compares the contents of mathematics education of North Africa in the Middle Ages with its role in actual teaching. John Fauvel from the Open University, United Kingdom, sets up the question: "Should we bring back the mathematical practitioner?" referring to some important aspects of the social history of mathematics education in the Renaissance. In a case study of Britain he reports how British mathematics educators coped with didactic problems, how they had to struggle for acceptance of mathematics as a practical and academic subject, and in which debates about teaching practices they were involved. Gert Schubring, Federal Republic of Germany, discusses theoretical categories for investigations in the social history of mathematics education and some characteristic patterns. He tries to offer a more comprehensive framework for historical studies as well as for an understanding of the role of mathematics in both liberal education and vocational training.

Mathematics and Dialectics

Arpàd Szabó

This talk argues for the thesis that *Euclid*'s system of mathematics follows the same patterns of dialectics which has been evolved on the ground of Eleatic philosophy.

"Dialectics" is an expression of Greek origin and meant the art of debating or arguing. *Aristotle* in the 4th century B.C. was of the opinion that the philosopher *Zeno of Elea* had been, one, or one-and-a-half centuries earlier, the initiator, so to speak the very first representative of dialectics. This remark reminds of the famous, even baffling arguments of *Zeno*, by which he tried to defend the theory of his master and teacher, *Parmenides*. This latter taught e.g., that "there is no movement." This surprising idea, which obviously contradicts all sound experience, has been defended by *Zeno* in the following argument: "The thing that moves must move either in the place where it is, or the place where it is not, and both are impossible." To be in a place implies position, rest, i.e., the opposite of motion; on the other hand, nothing can happen to a thing in a place where it is not; therefore motion is impossible in this second case, too.

Before going further, let us state here in definite form: both *Parmenides* and *Zeno* had been normal human beings. They knew of course that motion (or movement) is an undeniable fact of everyday experience. Have they denied, all the same, the possibility of every motion, so must this curious position taken up by them have some sense, which we have to try to understand somehow or other.

Indeed, what does it mean to be in movement?—It means to be in some place, and not to be there at the same time. Movement (motion), being a change of place by the thing moved, is in this way a contradictory concept: to be there and not to be there at the same time. Is this, however, possible at all?

If we are of the opinion that every thing can only be the thing itself, and it cannot be another thing at the same time, which is not the said thing itself, then the self-contradiction is the sure sign of impossibility. In other words: It is the sign of a false statement. That is why movement, the undeniable fact of everyday sensory experience, must be—according to Eleatic philosophy—only a misleading illusion, or in other terms: Our everyday experiences, obtained through our senses, cannot catch the truth, which is reserved only for our thinking.

The stated example, one of *Zeno*'s arguments against the movement, illustrates the most important features of Greek dialectics, as evolved in Eleatic philosophy. Let us now recapitulate these features in *four* points:

(1) First of all, the dialectician does not rely upon his sensory experiences. Everyday experience bears witness to the movement through all our senses. However, the testimonies of our sense organs are, at least in many cases, misleading. Therefore, we have to rely more upon our thinking than upon our senses.

(2) The truthfulness of a statement is not warranted by its adequacy to material facts that cannot be acknowledged, but by sensory perception, the latter being unreliable in itself.

(3) The most important intellectual criterion for the truth follows from the recognition that: A statement which contradicts itself cannot be true, it must be false.

(4) But how can it be demonstrated that a statement does not lead to any contradiction? There is a simple method of demonstrating this, which has become generally accepted in the Eleatic philosophy. If a statement *A* is not true, it means that: its opposite, *Non-A*, must evidently be true. Instead of proving any statement, we can refute its opposite. That is the so-called *indirect proof*, *"reductio ad absurdum"* in Latin.

Parmenides, *Zeno*, and in general the philosophers of the Eleatic school have demonstrated that such fundamental concepts as *motion*, *space*, *time*, *pluralism*, etc. are contradictory, and therefore, all these things of material reality are somehow only *illusions of our senses*. If, however, truthfulness cannot be grasped but by statements without contradiction, how can one get such statements, *most of our concepts being contradictory?*

It was a decisive step in the evolution of Greek dialectics the recognition that every demonstration must begin with commonly accepted statements, i.e., the truthfulness of which is accepted without any proof. Such statements—accepted without any proof—are in mathematics the *definitions*, *postulates*, and *axioms*. Having accepted, e.g., in mathematics the definitions of *even* and *odd* numbers, it can be proved by *reductio ad absurdum* that the diagonal of a square must be incommensurable to its side.

First, we formulate the opposite of the statement to be proved: "Diagonal and side of the square *are* commensurable." This opposite statement will be refuted by pointing out that in such a case the same number should be *even and odd* at the same time. This is, however, impossible. Therefore, the statement initially formulated must be false, its opposite is true: "Diagonal and side of the square are incommensurable."

A similar "indirect proof" is applied to demonstrate the Euclidean proposition Elements III 2: "If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.

This statement being completely evident, why prove it? Because the mathematician, as well as the dialectician, does not rely upon their sensory experiences. Although geometry originally used to be an *empirical science*, pure geometry is the science of things that can only be taught. (The point is that which has no part.)

The Content of Mathematics Teaching in North Africa in the Middle Ages and its Role in Present Day Teaching

Ahmed Djebbar

Over the different periods of African history, depending on the particularities of its peoples and their social organization, mathematical activities on this continent have gone through some or all of the following phases:

- longstanding calculation procedures for utilitarian, magic, or artistic purposes (or all three), and which linked essentially with the domains of arithmetic, geometry, and combinatorics, and which were accompanied by the development of a terminology;
- the start of mathematization of certain of these procedures through establishing definitions, inventing, perfecting, and justifying algorithms, and obtaining results;
- the participation in the development of topics and sometimes of entire disciplines;
- contributing to the transmission of a part of this mathematical knowledge to medieval Europe;
- lastly, and in parallel with the previous stage, the slowing down and then the extinction of creative activities, with mathematics becoming gradually the most closed subject, and so the least studied, of a heritage which will be completely lost from the collective memory of certain peoples in North Africa.

In this paper we propose to demonstrate, using as examples the Arabic-speaking regions of Africa (that is to say, Egypt, Sudan, Somalia, and the countries of the Maghreb), the possibilities which are offered by the mathematical heritage of these regions, from a perspective of enriching their pedagogy and of examining the role which this heritage can play as a vehicle of development for pupils, for teachers, and also for researchers.

But, before discussing the possible roles of this heritage, it is necessary to say something of the content and history of its development in order to appreciate its possible place in the scientific and cultural reality of North Africa today.

On the basis of our present knowledge we can affirm that as early as the 9th century, that is to say, before the end of the period of translation into Arabic of certain Greek and Indian mathematical works, the teaching of calculation had started in mosques or in private schools in some villages in Egypt and the Maghreb. This teaching was a response to the needs of a new society, in particular for the determination of the time for the five daily prayers, for the distribution of estates, and above all, for meeting the daily needs provoked by the enormous development of the merchant economy which was to become one of the essential characteristics of the Arabic-Islamic civilization as a whole.

This activity stimulated the preparation of written materials both for teaching and for application, and then more specialized works, the fruits of new research in arithmetic, algebra, geometry, and in the theory of numbers. It was the case in Egypt with Abū Kāmil († 930), whose treatise on algebra was to be studied until the 14th century in both East and West, and with Ibn al-Haytham († 1039), one of the greatest Arabic physicists and mathematicians whose works on optical geometry were authoritative in both Islamic countries and in medieval Europe.

In the 12th century one can equally mention the Maghreb mathematicians like al-Hassār whose books contained, for the first time in our knowledge, the symbolism of fractions as it is taught today, or like Ibn al-Yāsamīn († 1204) who was one of the first mathematicians to have used algebraic symbols for writing equations. This symbolism, moreover, was to be largely spread throughout the Maghreb, as is clearly shown by a number of works by mathematicians in this region of North Africa, and, in particular, those of al-Muwwāhidī and Ibn Qunfudh in the 14th century, and of al-Qalasādī and Ibn Ghāzī in the 15th century.

It is necessary, finally, to mention, along with arithmetic and algebra, the development of the field of combinatorial analysis whose first propositions were established, as far as the Arabic mathematical tradition is concerned, by two mathematicians from the Maghreb: Ibn Munᶜim († 1228) and Ibn al-Bannā († 1321).

Thus, in the light of manuscripts which have reached us and of which only a few have been analyzed, one is quickly convinced that one is in the presence of an important corpus of mathematical knowledge which has been an essential component of the Arabic scientific tradition and, occasionally, has been one of its driving forces. It is therefore natural to ask what happened to this corpus at the time when the different countries of North Africa were establishing their educational policies, that is to say, shortly after they had gained their independence.

Bearing in mind the policy of acculturation or of "cultural substitution" which was knowingly practized through the centuries by the colonial powers who were administering these countries, and equally taking account of the scientific power struggle which quickly turned to the benefit of these powers, the following paradoxical situation was reached in the end: The pupils of North African countries have had access, in the primary and secondary classrooms, to a non-negligible part of a mathematical heritage developed or taught by their ancestors, but these same pupils have been left totally ignorant of the origin of this heritage and of those who developed it. Thus, with the exception of some professors in the Islamic universities of Cairo, of Tunis, or of Fez, it is undoubtedly the case that the other citizens taught in these countries have been initiated into "mathématiques occidentales" (Western mathematics).

Has the situation changed since then? My experience of teaching mathematics and the history of mathematics both in France and in certain countries of the Maghreb (Algeria, Morocco, Tunisia) allows me

to make three points: in the first place, an increasing awareness of the existence of the mathematical heritage of these countries; in the second place, an enormous ignorance of the precise content of this heritage with, at the same time, a very great receptivity for this knowledge and a real desire to spread it; in the third place, finally, and after 25 years of hardly any change, the setting up of cautious experiments of teaching the history of the mathematics produced in North African countries and elsewhere, and the introduction of some elements of this history into secondary school textbooks.

Based on these points, my paper represents both a defense of the argument in favor of an extension of these experiments to other levels of teaching, as well as a detailed exposition of different aspects of mathematics produced in North African countries and the way to insert them in teaching programs and teacher training courses, either as a part of research and practice in the didactics of mathematics or as a set of tools for researching the pedagogy, or, finally, as new material likely to enrich the cultural frame of teacher trainers and their students, and to transform beliefs about the content of mathematics, its role, and its status.

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Should We Bring Back the Mathematical Practitioner? Learning From the Social History of Mathematics Education in the British Renaissance

John Fauvel

I want to ask the naive question: What is the use of the social history of mathematics education? Can it be of any value to mathematics teachers or to those involved in mathematical education? When the organizers chose to insert the theme of "the social history of mathematics education" into a congress on mathematical education, was this anything more than a fashionable, but ultimately distracting, gesture?

My feeling is that the past does indeed have lessons for us, if rightly interpreted. It is a laboratory of practices, decisions, problems, and solutions which have been tried, and recovering what has happened in the past and reassessing what solutions people found to their social and educational problems can help us today. Our problems are different, of course; but learning about the past helps to broaden the base of possible solutions for us, and liberates us from thinking along too rigid lines. Of the many possibilities of place and period, I will concentrate on one in order to make the argument as concrete as possible.

Britain in the 16th and 17th centuries was in many respects a set of developing nations. Literacy and learning in general, and mathematical skills in particular, were becoming more widespread. To think of England as a developing nation is quite a helpful image, both for understanding mathematics in the British Renaissance, and for giving cues towards what we can learn from it for today's situations and challenges.

Perhaps I can guard against misunderstanding: This is *not at all* a kind of deterministic evolutionary model in which today's underdeveloped countries are seen as lagging some 400 years behind Western Europe! My point is actually quite the converse: that modern Europe has lost some of the institutional richness of mathematics education it once had, but underdeveloped countries may still have the opportunity to foster a wider and more fruitful range of mathematical educational practices.

For most people in the Renaissance, mathematics was new and unfamiliar. Much rhetorical effort was expended by textbook writers, therefore, in explaining why mathematics was a good thing and what it would achieve. Let me give an example. William Kempe, a schoolteacher who translated an arithmetic text by Peter Ramus, wrote[1]

> Take away arithmetic, you take away the merchant's eye, whereby he seeth his direction in buying and selling; you take away the goldsmith's discretion, whereby he mixes his metals in due quantities; you take away the captain's dexterity, whereby he embattels his army in convenient order; finally you take from all sorts of men, the faculty of executing their functions aright. Arithmetic then teaches unto us matters in divinity, judges civil causes uprightly, cures diseases, searches out the nature of things created, singeth sweetly, buyeth, selleth, makes accounts, weighs metals and works them, skirmishes with the enemy, goes on warfare, and sets her hand almost to every good work, so profitable is she to mankind.

This is but one example from many dozens. So our attention is drawn to the people who were trying to promote the subject. What was in it for them? Their motives ranged from the narrowly commercial to the spiritual-idealistic. Some were clearly in the business of mathematics promotion because it was a profitable growth industry, but others saw mathematics as the key to personal and social development, whose benefits they wished to spread quite altruistically. This va-

[1] Fauvel, J. & Gray, J. (Eds.) (1987). *The history of mathematics: A reader* (p. 289). Macmillan.

riety in turn reflects the range of reasons for which people wanted to learn mathematics.

If you wanted to learn mathematics, how would you set about it? There were several possibilities. You could study a textbook (if you could read); you could pay for tuition in various skills from a mathematical practitioner; you could attend a ciphering school to gain basic knowledge of numeracy and accounting (even simple ciphering was not on the grammar school curriculum, as a rule); in some places you might have the opportunity of attending lectures.

Thus mathematics had a different place within the structure of the educational world from nowadays. It was more of a resource which you tapped as the need arose for you, within the limits of what was locationally and financially feasible—for instance, in 1662 the 29-year-old Samuel Pepys[2] hired a seaman to teach him how to multiply. Pepys was already a competent civil servant, besides being a graduate of the University of Cambridge, so this episode tells us something of the low spread of elementary mathematical skills in 17th century England.

In trying to understand the role of mathematical practitioners, I make a distinction between two ways of purveying skills and knowledge. A scribe, let us say, in some cultures can *either* write letters on behalf of people (an "expert" role) *or* can teach them to write letters for themselves (a teaching-practitioner role). Similarly the mathematical practitioner can be an expert, if you want to treat him or her as such, but is more valuably a teacher of skills for later independent use.

What problems are there for the practitioner? This model brings the risk of losing one's monopoly: The only writer in a village is a very powerful person; and as Robert P. Hunting has observed,[3] "Possession of mathematics knowledge means power, because such knowledge can be traded in the market-place of problems and ideas." So there is perhaps a temptation for practitioners to stay in the "expert" mode. But in practice, fears of losing status or power are groundless, not least because of the variety of mathematical skills and the length of time mathematical training takes. There will always be plenty for the mathematical practitioner to do.

A more serious difficulty is this: How are people to know what kind of mathematical skills they want or need? Presently there is a widespread system of mathematics education whose content is defined by examination syllabi, with allied top-down teaching structures. A strength of this system is that everyone who gets through it emerges with a fair range of skills that are—in theory—adaptable to a wide range of needs. A strength of the mathematical practitioner system, on the other hand, is that people can learn skills to handle their particular needs, whether these be (in the Renaissance case) navigation or gunnery, astrology or accounting. To know sufficient about your needs to ask for specific mathematical advice does demand more sophistication, however, and so a corollary of taking a mathematical practitioner, access-to-skills, approach more seriously is a revival of the old Renaissance practice of putting widespread effort into explaining what mathematics is good for, what it will do for you, and how it will meet different needs. An education system might emerge with a dual focus, on two levels of skills: both the mathematical skills themselves, for particular problems, and, equally important, the higher-level skills of judging what mathematical tools are appropriate and needed for different problems.

This multivalent valuation of mathematical learning is not new, of course. Indeed, there is evidence from Babylon onwards of the close connection between mathematical training and making one's way in the world. As the poet wrote on a clay tablet[4] over 4,000 years ago

> The scribal art is the bond of all.
> Work hard for it and it will give you its beautiful prosperity.
> To write a stele, to draw a field, to settle accounts.
> Strive to master the scribal art and it will enrich you.

How mathematics has at various times been seen as crucial, or otherwise, in training the next generation is at the heart of the social history of mathematics education. And in turn, its study points us towards an awareness that in order to do our best by the young mathematics learners of today, we need to understand what they are being fitted for and why.

Again, the mathematical practitioner was sometimes selling a particular kind of skill: training in the use of mathematical instruments—for computation, geometry, surveying, astronomy, etc. The debates surrounding such activities have no less bearing today. There were debates about particular teaching practices as well as about what kind of mathematics to teach. In the teaching of techniques such as the use of the slide rule—or, today, the computer—how important is it that students understand the underlying principles? What are you teaching people mathematics *for?* This was a heated issue in 1630s London.

So, studying the mathematical practitioner broadens our conception of the possibilities of mathematics education. In a recent newspaper letter,[5] Professor Griffiths spoke dismissively of the view that *teachers can pass mathematics to pupils like shop assistants handing loaves to customers.* One understands his point, of course, but the imagery is thought-provoking. Perhaps the antithesis could be taken more seriously, to the extent of training people to recognize what kind of loaf will satisfy their needs any particular morning. Then learners could gain the strength of being responsible for their own purchases and not be recipients of bread-line charity.

I conclude: No historical situation repeats itself exactly. What we learn from history is never a simple

[2]Fauvel and Gray, pp. 317–318.

[3]Hunting R. P. (1987). Mathematics and Australian Aboriginal culture. *For the Learning of Mathematics, 7,* 5.

[4]Fauvel and Gray, pp. 40–41 (slightly readjusted for readability).

[5]Griffiths, H. B. (1988). *The Guardian,* 22 March 1988, p. 27.

message or easy moral. But to see something of the social situation and didactic problems faced by mathematics educators of the 16th and 17th centuries, and learn how they coped with them, helps to broaden the basis on which we form judgments and plan strategies for mathematics education today. It also alerts us and enables us to evaluate today's situation more critically, in the sense of presenting, as problems to be analyzed and assessed, procedures which we may have grown up thinking are in the nature of things. In studying possibilities for mathematics education in developing countries, there is obviously much to be learned from the experience of more developed countries—and vice versa; that is why we are here. But it is helpful too to compare with the problems and solutions of other developing countries in the past—Renaissance Britain, for example. A truth increasingly apparent as the century wears on is that the practices of developed countries today do not contain all the answers, and indeed their historical development has been one of restricting certain avenues of possibility, for the sake of other values which seemed like a good idea at the time. Recapturing all our roots, and fertilizing them afresh, is essential for all our futures.

Theoretical Categories for Investigations in the Social History of Mathematics Education and Some Characteristic Patterns

Gert Schubring

The number of activities invested in changing the mathematics curriculum and the subsequent failure of the curriculum reform projects has enhanced interest in and reflection of history of mathematics education. The failure showed that the protagonists' good will for reform is not sufficient to effect changes and that there are deep and more persistent structures, due to the social functions of knowledge. The history of mathematics education can contribute to revealing these underlying structures.

It must be said, however, that historiography of mathematics education in its present state is largely not yet prepared to provide answers to such questions.

I presume that the reasons for the insufficient methodology can be ascribed to two levels: to a factual one and to a theoretical one. As regards the factual level, I suppose that the traditional concentration on administrative history is due to the relatively easy accessibility of these data—whereas systematic access to more refined primary historical sources like archives or analyses of the textbook production in a given period are far beyond the state of professionalization of those working on history of mathematics education. As regards the second and deeper level, one has to consider that the enormous amount of raw data obtainable from primary sources can only be appropriately structured and will "speak" to us if theoretical categories guiding the primary analysis and permitting an interpretation of the findings have been elaborated. However, this domain of theoretical structurization still shows the largest deficits. In fact, the general underlying methodology consists of assuming that the task of history of mathematics education is just a descriptive one. Today, I should like to emphasize particularly the importance of elaborating theoretical categories in order to be able to analyze historical data effectively and to attain structural insights. I shall just summarize some results of research on such categories.

The most basic conception is constituted by the assertion that there is no "neutral" knowledge—no knowledge follows exclusively an "internal" logic. There will always be a relation between intellectual factors and social factors in the development and transmission of knowledge. History of mathematics education must therefore be conceived of as a part of the social history of knowledge.

A second theoretical conception is given by the particular epistemological nature of mathematics which I should like to call its "double-face-nature": Mathematics belongs at the same time to the humanities and to the natural sciences. It follows immediately from this double character that mathematics education has to be analyzed historically at the crossroads of liberal education and of vocational training. Hence also the primary importance of institutional studies on the function of mathematics within the educational system.

Eventually, a third theoretical category is provided by the notion of *development*. Usually, it is taken for granted that education should apply a developmental approach. This common-sense conviction is often expressed as adherence to a so-called genetic principle: developing the mind of the adolescent by following a developmental exposition of, say, mathematics. Yet, the most fervent plea for a developmental approach might prove to imply a rather static view of a child's abilities or even a belief in innate qualities. The category of development, therefore, is a highly complex one and has its impact both on the notion of a scientific concept and its change in history, and on organization and function of the school system and the syllabus.

I shall now outline a number of research topics which have proved to be key issues for applying theoretical categories to a deeper understanding of the complex historical reality:

State of mathematics within general education for all. There is a remarkable contradiction: Although mathematics constituted one of the basic branches of instruction since the earliest times of institutionalized instruction in ancient Egypt and Mesopotamia, mathematics was assigned a marginal role in medieval and pre-modern schooling. And even in modern times, when mathematics, in the early 19th century, attained the position of a major subject in the systems of public education of most of the developed countries,

this discipline, in the eyes of the greater public, retained the character of something accessible only to specially gifted people or of a craft restricted to vocational purposes. We are still far from understanding the involved complex system of relations and its concrete impact on the position of mathematics in society and particularly in schools.

Delimitations between secondary and higher education and the regulative influence of the transition from secondary to higher education in the school syllabus.
Upon passing from the general level just outlined to the more concrete institutional system of schooling, it is seen that there is no institutional unity, but rather a hierarchy of institutions. Each type of institution incorporated a set of values of its own, according to the social functions of the knowledge it taught. In agreement with these values, each institution used to mold a particular corpus of school knowledge, accompanied by particular methodologies and epistemologies. Particularly relevant is the delimitation between the knowledge domains for secondary schools and for universities. The static and elementary character of the secondary school mathematics throughout the 19th century proves to have been largely effected by university mathematicians in order to preserve a reasonable number of students. And, given the fact that primary school mathematics was able to maintain over centuries an almost independent and hermetical character with regard to the higher-ranking institutions, the "new math"-movement can be understood to be the abolition of this independence and the complete integration of primary mathematics into the hierarchy dominated from above (cf. Schubring, 1988a).

The professional role of the mathematics teacher.
In order to counterbalance the traditional history of administrative decisions and in order to better approach school reality, I propose to make the professional life of mathematics teachers the focus of historical research. Starting from the well-documentable evidence that the professional life of mathematics teachers is shaped by their personal view of and their relation to mathematics, investigations into history of the mathematics teacher profession provide a privileged key to the reality of mathematics instruction. Investigating the modes of the teachers' formation, of their examinations, of their publications and epistemological convictions will vivify the historical insight.

The function of textbooks within the instructional process and their relation to the role of the teacher.
Within the traditional paradigm of concentrating on published texts, textbooks for school instruction have played a prominent role. However, neither a systematic analysis of the production of textbooks in a given period has been undertaken, nor was there an analysis how textbook and teacher—the two main factors influencing the instructional process—interact with each other historically. In fact, historical case studies for Germany and France have shown that there is a dichotomy between improvement of teacher education and the elaboration of good textbooks: The two factors cannot be optimized simultaneously. There is not only a series of intervening factors—human and social factors, like competition between authors, market strategies of publishers—but also the type of organization of the respective state, which defines the support for the educational institutions and by the state's objectives with regard to education and science. Depending on these objectives, a given state tends either to improve the teachers' qualifications and their methodological autonomy or to prescribe just one textbook for all schools in order to achieve control and uniformity in teaching (cf. Schubring, 1988b).

The contribution of textbooks to a social history of ideas (sensu W. B. Brockliss, 1987).
Historical case studies are also very important as they can contribute to denounce the one-sided "genius-view" in the history of the sciences: According to this view, the history of ideas can be studied sufficiently by concentrating on the works of the few geniuses or giants in the sciences. In recent decades, however, historiography of science has proved that extraordinary achievements are not explainable without the contributions of the larger community of scientists. One has to include textbook authors into this larger community, since in particular their striving for clarity in the fundamental notions and concepts contributed, most evidently in the 19th century, to the advances of science.

Relation between scientific knowledge and school knowledge.
There is not just *one* source for new knowledge: the framework of scholarly work in universities and academies; rather, there are additional and probably independent sources for the production of knowledge. It depends therefore on a process of social deliberation and on a society's value system as to which source becomes privileged and dominant. The "didactical transposition" from scholarly knowledge to school knowledge is therefore, in principle, not a one-sided top-down process as it is often understood, but rather entails complex interactions on all levels. The *elementarization* of *new* knowledge to *teachable* knowledge is therefore in itself a social process which is determined by factors particular to each culture and state.

Cultural determination of school knowledge.
School knowledge is not only shaped by institutional hierarchies, but affected by cultural and social factors and constraints. These constraints used to be translated into epistemological notions. School syllabi of different countries are therefore not directly comparable; one has to take into account the context factors, for instance, dominating or rivalling epistemologies.

Transmission between different cultures.
The methodological diversity between, for example, France, Greece, and Italy in the 19th century which can be said to belong to a common cultural universe, demonstrates how deeper even the differences between

really different cultures will be, and how difficult it will be to elaborate underlying structural patterns. Regarding the highly attractive and controversial topic of *transmission*, it is necessary to distinguish transmission between independent *metropoles* (say: China and India) from transmission between metropoles and a politically or socially dependent periphery (cf. Pyenson, 1985; Schubring, 1987).

In order to conclude, I should like to summarize that with the outlined research program and its dimensions one will be enabled to accede to a meaningful history of mathematics education which transcends the superficial collection of descriptive data.

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Cultural Diversity and Conflicts in Mathematics Education

Cultural diversity is commonplace in education around the world, and mathematics educators have a particular interest in certain issues which will be addressed in this theme. Terezinha Nunes Carraher, Brazil, describes material embodiments of mathematical models found in use in everyday life in Brazil. Lloyd Dawe, Australia, discusses typical problems of mathematics teaching and learning in village schools of the South Pacific. Murad Jurdak, Lebanon, focuses on the roles of religion and language of instruction in mathematics in as far as the two factors operate as cultural carriers or cultural barriers in mathematics education. By contrasting two main and provoking theses, illustrated by the counterexamples of Saudi Arabia (rich) and Lebanon (poor), he explains the dilemma of balancing the needs of socio-economic development and the preservation of the ecology of the culture. Claudia Zaslavsky, USA, emphasizes the necessity for the expansion of the curriculum at any level to include culturally specific mathematical practices as social practices in daily life ("ethnomathematics") and presents possibilities for doing so in the elementary and middle grades.

Material Embodiments of Mathematical Models in Everyday Life

Terezinha Nunes Carraher

Problem-solving strategies may be used to identify a subject's conception of arithmetic operations through an analysis of how objects are manipulated, when concrete materials are available, and of the situations to which the same strategies are applied. To clarify, let us take an example from the field of addition and subtraction. Carraher and Bryant (1987) suggested that addition/subtraction are conceived by young children only as transformations in which amounts increase or decrease. Their strategies in object manipulation during problem solving involve actions that increase or decrease amounts. When a problem is presented, which involves an increase in amount but has as the unknown one of the addends, young children represent the known added, then add new elements (objects or fingers) until the total is reached, then go back and count the elements which had to be added to get to the total. Their problem-solving strategy can be represented as $a + x = b$; they act this out and then count the number of elements in the set x, which was unknown. This use of the transformation schema maintains the meaning of the situation—an addition situation—and results in accurate problem solving. However, it represents a mismatch to the schooled conception of addition/subtraction, which defines missing addend problems as a "subtraction" problem, calling for the representation $b - a = x$.

The conflict between the concepts used by children and those currently used in elementary school mathematics is likely to be a source of motivation for change; as children go to school for longer periods of time, they end up adopting schooled ways of solving addition/subtraction problems. But what happens to adults who solve arithmetic problems in their everyday lives but do not go to school? Do they learn the schooled ways informally or do they retain their own ways? These questions are of significance to mathematical education in two respects. First, it is important that mathematical educators be able to distinguish what is learned under the pressure of school work from what can be spontaneously learned outside school. Second, mathematics education designed for adults can find new directions on the basis of an anal-

ysis of their problem-solving strategies. Mathematics education designed for adults, even more so that that designed for children, cannot afford to ignore what previous conceptions the students bring into the classroom. If the adult-students' conceptions already match the schooled ways, instruction does not have to be designed for the teaching of these concepts. On the other hand, if adult-students use other conceptions of arithmetic operations, instruction must take this into account—especially because their social needs cannot be ignored. In fact, regular calculators embody the schooled concept of arithmetic operations. Although problems can be solved in more than one way and through the use of different representations, when a calculator is employed, the choice of operation is determined by the model it embodies; missing addend problems, for example, are solved through subtraction, and not through addition. The now widespread use of calculators in many different job situations requires that the schooled concept be readily available to people in these jobs, or else they will be classified as "ignorant" or "lacking in reasoning skills" because they will be unable to "choose the proper operation" and solve problems quickly despite the availability of a calculator. Once the school model of arithmetic operation is built into calculators and becomes a part of an officially recognized way of thinking in the culture, the use of alternative conceptions is hardly acceptable even outside school.

A study was conducted with 90 adults who were attending evening school at three grade levels: first grade, in which mostly reading instruction is offered, third and fifth grade. Students were interviewed in their schools and asked to solve a total of 12 word problems, which could be classified into two types: (1) problems in which the operation to be entered into the calculator matched the problem situation (such as problems in which an increase in amount is said to happen and an addition is required); and (2) problems in which the operation to be entered into the calculator did not match the problem situation (such as problems in which amounts increase, but there is a missing addend and, thus, a subtraction is required). Numbers in the problems were two- and three-digit numbers, so that calculation was not quickly carried out mentally, and the interviewer could exert pressure on the subjects to use the calculator. Using the calculator under pressure, subjects may make more errors than they would otherwise if their conception of arithmetic operations does not match the schooled concept. It must be noted, though, that despite the interviewer's attempts to lead subjects to use calculators, they did not always do so.

Significant differences were found across types of problems when percentages of correct responses and percentages of choices of operation according to the schooled model were compared (see Table 1). Significant differences were observed also between first and third graders' performance, but no difference was observed when third and fifth graders were compared. These results indicate that adults solving problems outside school probably do not encounter enough pressure to change their spontaneous conceptions to our schooled conceptions of addition/subtraction. They can, as other studies have shown (e.g., Schliemann, 1984; Carraher, 1986; Acioly & Schliemann, 1987), solve problems accurately in everyday life, yet do not succeed in situations in which there is pressure to solve problems according to the schooled conception of operations.

Table 1
Percentage of Correct Responses (and Choice of Operation Following the Schooled-Model) by Problem Type and by Level of Schooling

Problem type	Level of schooling					
	First		Third		Fifth	
Amount increases; addition required	80	(93)	93	(87)	93	(93)
Amount decreases; addition required	50	(68)	40	(43)	67	(70)
Amount decreases; subtraction required	70	(81)	93	(97)	83	(93)
Amount increases; subtraction required	27	(16)	53	(47)	60	(31)

Multiplication and division problems were also included in this study. The basic situational meaning of multiplication was treated as addition of equal amounts, which will be referred to here as problems of rate. The basic situational meaning of division was sharing out equal amounts. With multiplication and division, the findings obtained for addition/subtraction were replicated: Significant differences were obtained across problem types and between first and third graders but not between third and fifth graders. Results are summarized in Table 2. In multiplication problems, only multiplication was considered as "the schooled choice of operation," but a fair amount of first grade subjects solved these problems accurately through repeated additions. In division problems, only division was treated as "the schooled choice of operation" but problems were solved correctly through repeated subtractions by some first graders and through multiplication by both third and fifth grade students.

Conclusions

Regular calculators embody an arithmetic model of problem solving. Their current widespread use introduces in the culture the belief that there is only one way to solve arithmetic problems. Other representations are neither recognized as appropriate in elementary school (although they may be considered acceptable later, when algebra is introduced) nor outside, once the calculator is used in problem solving. However, young children and unschooled adults appear to

use other valid problem representations when solving problems.

Table 2
Percentage of Correct Responses (and Choice of Operation Following the Schooled-Model) by Problem Type and by Level of Schooling

Problem type	Level of schooling First	Third	Fifth
Rate situation; multiplication required	50 (33)	100 (100)	80 (70)
Sharing situation; multiplication required	40 (22)	89 (90)	70 (70)
Sharing situation; division required	30 (45)	100 (100)	90 (90)
Rate situation; division required	10 (12)	30 (44)	20 (20)

In view of these findings, schools are faced with a dilemma. On the one hand, there is a need to teach the arithmetic model embodied in calculators, which are more and more prevalent in everyday activities outside school. On the other hand, schools need to reconsider the exclusive use of arithmetic representation in elementary school, which may suppress the understanding of a to-be-later-introduced algebraic representation of problems. Adult education in particular must face this dilemma once it is known that adults come into school with ways of representing and solving problems which are alternative to the arithmetic model.

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Mathematics Teaching and Learning in Village Schools of the South Pacific

Lloyd Dawe

In recent times, the Island states of the South Pacific have attracted a great deal of attention in the world media. Political and cultural conflicts have been witnessed in New Caledonia, Vanuatu, and Fiji. These and economic needs resulting in conflict with regards to tuna fishing rights and the sensitive issue of nuclear testing, are just a few factors which have changed the context in which education policy and development is now being made.

Despite tremendous advances in the general development of education in the Pacific Island states, economic restrictions continue to be a barrier to growth, and these restrictions have immediate consequences for young people leaving school. Unemployment is high and many face a return to village life or leaving the country for further education in countries like Australia, Fiji, or New Zealand. Unfortunately, this is simply not possible for the vast majority. Further, the impact of technology has had a profound effect on their culture. The traditional ways of family life are in conflict with the lifestyles young people aspire to. The issue at stake for these countries is how to retain their cultural identity while adapting to inevitable technological change. Teachers have a fundamental role to play in the process.

Since the second world war, Australia and New Zealand have given a great deal of support to the development of education in various Pacific Island states. In particular, Australia has given special support to Papua New Guinea and New Zealand special support to Western Samoa. We now turn to three specific cases to illustrate how cultural diversity influences the teaching and learning of mathematics in village schools.

Western Samoa

I am thinking of a village school on the eastern side of the main island Upolu. The school consists of six or more buildings built in the traditional Polynesian way, roughly circular, supported by solid poles and thatched with palms. The buildings are open to the fresh sea air. Inside one of the classrooms, the children are having a mathematics lesson. There are a few roughly constructed desks and benches at the back of the room where the children write. However, most of their mathematics is learnt on the grass mats in the center of the room. There is a heavy emphasis on learning arithmetic by rote in Samoan, although English is the preferred medium in later years. The class responds to questions together in a chorus with frequent repetition. There is virtually no "hands on" activity. Problem solving, geometry, and measurement run a poor second to number work. Speed and accuracy are rewarded at the expense of reflective thought. The

curriculum is imported from Australia and New Zealand and supported by old American textbooks acquired from American Samoa.

The Samoan tradition of passing on knowledge orally from the Matai (or chief) to the people is paralleled in the mathematics classroom. The teacher passes on knowledge to the children who listen respectfully. Authority and responsibility are invested in the teacher in the same way that it is in the Matai. Western methods of acquiring knowledge through investigational means, of hypothesis, experiment, and analysis, conflict with traditional ways. Thus, although there is a wealth of mathematical knowledge built into their culture, Polynesians do not see its relevance to the curriculum. Indeed, drawing attention to their splendid architecture, or intricate weaving techniques as possible material for teaching mathematics is often seen as patronizing.

Outback Australia

In the remote outback areas of central Australia aboriginal children are generally taught by white teachers often with aboriginal support teachers. Bilingual schools have been set up with the aim of establishing literacy in the child's first language before learning to read and write English. Thus for many outback aboriginal children their first experiences in mathematics are in their native tongue.

There are several cultural factors which directly impinge on how mathematics is taught and learned in such schools. Firstly, the children appear to have an extraordinary capacity for rote learning. Secondly, they work far better in groups with the elimination of individual competitiveness. Thirdly, they have a very strong visuo/spatial sense accompanied by strong visual memories. In terms of learning, active trial and error followed by consolidation and practice is the norm of family life. Book learning is seen as peculiar to whites. Knowledge comes from experience and certain types of knowledge are not considered the right of all. Knowing does not mean sharing. It depends on the knowledge itself and the relative status of the questioner to that person. Thus, when an aboriginal child is asked a question in a mathematics class and does not offer a reply, it could be because

(1) the child believes he/she is being asked something the teacher already knows and therefore there is no point in replying;
(2) the child is reflecting on his/her reply. Immediate answers are not considered wise. Reflection is often needed and valued.

It is of interest that many urban aboriginal children share these same cultural characteristics, even though they have never been to the outback. What is clear is that such factors must be taken into account in the mathematics education of aboriginal children.

Papua New Guinea

The third case is a village school on the outskirts of Lae in northern Papua New Guinea. In this school the children come with a variety of first languages, but all speak Pidgin and there is a gradual move to English. Thus the children are likely to be fluent in at least three languages as schooling increases. (It is of interest to note that the government of this country is coping with the task of administering education in the context of over 700 distinct native languages.)

At the school pupils enter at a variety of ages due to a waiting list for places. Many begin school at ages ranging from 8 to 11. The mathematics course for grade 1 must therefore be very flexible. Pre-number and pre-measurement activities are an integral part of the program. The children enter school with a knowledge, often incomplete, of their own traditional means of counting and measuring and with a means of classifying built into their language. Such factors are not often taken into account by teachers. Indeed the nexus of strong communal relationships of their families as part of traditional society, and the competitive individualistic approach to education in the school is a source of growing tension as the child grows older.

In summary, PNG children as shown by this school appear to exhibit many of the characteristics shown by Polynesian children in Western Samoa and aboriginal children in northern Australia. They learn in a context which does not take the ethnomathematics of their culture into account, and experience the tension of cooperative group learning versus individual competitiveness.

Conclusion

Throughout the world cultural diversity is commonplace in mathematics education. It has a profound influence on learning and teaching. Such a rich diversity between cultures, and therefore within mathematics as a whole, can considerably enrich the quality of mathematical activities in different classrooms around the world. The issue at stake for developing nations of the South Pacific, however, is how to retain their distinctive identity while facing the pressures of economic, political, and technological change. These countries have a wealth of ethnomathematics in their cultures which can contribute towards this goal. Countries like Australia and New Zealand can do much more to encourage such development.

Religion and Language as Cultural Carriers and Barriers in Mathematics Education

Murad Jurdak

Distant as they are, mathematics education and religion are not as strange bedfellows as they seem. This stems from the fact that each may be regarded as a subculture of the "home" culture. Anthropologists such as White (1959) identified four components for any culture: technological (manufacture and use of tools), ideological (beliefs and values), sentimental (attitudes and feelings), and sociological (customs and institutions). It is basically on the ideological plane that religion and mathematics education may meet or conflict. In some societies, depending on the nature of both religion and mathematics education, the planes of incidence may extend to the sentimental and sociological components.

Although mathematics education and religion are not necessarily discrepant, they may conflict under certain circumstances. One would expect little conflict if each keeps to its own territory. However, a potential conflict between the two becomes a possibility if the nature of the religion is so pervasive as to encompass, beside the ideological framework, significant aspects of the individual's private and social life which is a natural field for applying mathematics. Again, the conflict between the two depends, to a large extent, upon the degree to which mathematics education is perceived as an alien culture to the mother culture. The possibility of conflict between religion and mathematics education becomes an actuality if the values of the two are not compatible.

The incompatibility of the values of mathematics education and the mother culture is not a mere hypothetical possibility, but rather a historical actuality. The last few decades witnessed the emergence of the so-called developing countries as independent states. Already disadvantaged by their low level of socio-economic development, such countries have aspired for accelerating their development through education, science, and technology. Very frequently, such countries looked up to the former colonial powers, with which they were linked by ties of educational and cultural interactions, as models and sources of science and technology. Rightly or wrongly, mathematics was perceived, first, as the basic tool of science and technology, and, second, as a heaven-sent, universal, and culture-free discipline. As a result, a massive movement to adopt and/or adapt mathematics, Western curricula was started with the support of international and regional organizations. In doing so, the developing countries were not importing a culture-free technology, but rather a generalized "Western" culture described by Bishop (1985) as a mathematico-technological (MT) culture. In many cases, the values of the MT culture came in direct conflict with the ideological values, and in particular, the religious values of the mother cultures.

The Arab States are not an exception in this respect. The dominant culture in the Arab States is, to a varying degree, derived from two sources: Islam and Arab nationalism. The concerns of the Arab States were reflected in regional conferences of the Ministers of Education (Arab League, 1953; 1968; 1965) and conferences of Ministers of Education and those Responsible for Economic Planning in the Arab States (UNESCO, 1966; 1970; 1977). These concerns focussed on four issues: (1) unification of curricula within the framework of the Arab-Islamic culture, (2) provision for the quantitative expansion of education, (3) improving the quality and productivity of education, and (4) orientation of education toward social and economic development. The 1966 Conference of Ministers of Education and Ministers Responsible for Economic Planning in the Arab States requested that emphasis be placed primarily on mathematics and science, thus in 1967, the UNESCO Mathematics Project for the Arab States (UMPAS) was launched (Jurdak & Jacobsen, 1981). The project was modelled in content and development after similar projects in the West. It was led by international Western mathematics educators and Arab mathematics educators, who by virtue of their education, reflected the MT culture. Similar national projects, in some of which the author has been involved, were similarly modelled. The impact of all such projects was rather limited. My claim is that one major reason for the apparent shallow adaptation of such projects is the incompatibility of the Islamic-Arab culture and the MT culture carried by the methods and models of these projects.

Against this background, I present my first thesis, which says that if the values of mathematics education are incompatible with the value system of the mother culture, then mathematics will be "appended" to the culture as a "technology," rather than assimilated as a "mode of thinking." The thesis is to be discussed with reference to the interaction of the mathematico-technological (MT) culture and the Islamic-Arab (IA) culture.

The incompatibility of the values of IA and MT cultures are apparent on the ideological and sentimental levels. On the ideological level, Bishop (1985) identified logic as the important criterion in the MT culture rather than traditions, experience, or personal status. In the IA culture, although logic is used extensively, it is not the ultimate criterion. The latter is reserved to the authority of the Holy Koran (the words of God). It is not an unusual practice to provide an evidence by citing a relevant quotation from the Koran. On the sentimental level, the MT culture assumes that values can be questioned and statements can be subjected to empirical test. In any religious culture, and certainly in the IA culture, this assumption is not tenable because in such cultures there are some values which should not be questioned. In the MT culture, knowledge is not a possession of certain members of the culture, whereas, in the IA culture, religious authorities have to possess certain knowledge not available to others (such authorities are called in Arabic All-ullama, the literal English translation is scientists). One would say that the above applies, in varying degrees, to any religious culture. I do not

contest that, but I wish to note that Islam is special in at least two ways. First, Islam is not only a religion, but an ideology which provides a complete system for civil laws and social and political institutions. The domain of Islam is not only the private spiritual life of the individual, but also the ideological, sentimental, and sociological values of the individual and the society. Second, the Holy Koran is not simply a revelation, but the exact words of God, embodied in an immutable text, which is not subject to change and which allows little for interpretation.

The conflict between the IA and MT cultures neither reveals itself in the kind of mathematics taught nor in the specialized mathematical techniques and procedures used in the classroom. The mathematical content and procedures are not essentially different than what you find in classrooms in other countries. When it comes to mathematics (number, space, operation, ...) religion apparently plays a minor role.

However, the conflict between the IA and MT cultures reveals itself in at least two areas: production of mathematical knowledge and use of this knowledge to cope with reality. Mellin-Olsen (1985) emphasizes the link between the production of knowledge (mathematical knowledge in this case) and the production of culture. In societies where religion plays such a central role in the society, the enculturation process has to be started and incorporated in formal education very early. In some Arab countries induction into the religious culture starts as early as the first grade with more than 25% of periods allotted to religious studies. By its nature, induction into the religious culture has to be done, particularly at this young age, by imitative and reproductive methods. On the other hand, the MT culture relies on productive methods (constructing, discovering, and testing hypotheses). By exposing the learner to two conflicting modes of producing knowledge, the cultural conflict is reinforced by the culturation process itself.

Using mathematical knowledge to cope with reality is a second area where cultural conflict occurs with more far-reaching implications. As explained before, Islam dominates the existence of the individual and the society outside the classroom. In using the mathematics of the MT culture to cope with reality, the learners find themselves in cultural conflict situations. Thus, the so-called "mathematical mode of thinking" is easily dominated by the authoritative mode of thinking. What actually is transferred to real life is the technological aspect of mathematics. This is probably one reason why such societies have "gone back to basics in their curricular work" (Bishop, 1985, p. 63). Mathematics thus survives as a collection of tools. "Old techniques" are easily replaced with "new techniques" with the value system rarely affected by these innovations.

A closely related issue which has cultural implications is the use of a foreign language as a language of instruction in mathematics. Much has been said about psychological and pedagogical implications of using a foreign language as a medium of instruction (Berry, 1985), but not enough about the cultural and social implications. It seems to me that, at the basis of this, is an implicit assumption to the effect that the language of mathematics is culture-fair because it is culture-free. As a consequence, the language of instruction, whether native or foreign, does not have cultural implications. This assumption may be challenged in the same way that one challenges the universality of mathematical concepts. Although this assumption may be true for mathematics at the scientific level (even though this is questionable if one considers the different roles ascribed to language in different philosophies of mathematics), it is not necessarily true on the educational level. In mathematics education, the language of instruction and learning is far from being a pure formal language. It is rather more of the ordinary language accentuated with symbols and technical terms. As soon as mathematics is applied in problems and situations, the language of instruction and learning becomes a cultural carrier in terms of behaviors, social relations, habits, and values. There is even some evidence that formal deductive reasoning is language-specific (Jurdak, 1977).

The social implications of the use of foreign language as a medium of learning and instruction are obvious. Mathematics is often described as a "critical filter" to university education in general and to scientific occupations in particular. If foreign language is introduced as an additional factor, then instead of one we end up with a double-critical-filter. It is often the case that the socially and economically disadvantaged have less opportunities to make it through the double filter than the more advantaged. Consequently, mathematics taught in a foreign language tends to discriminate among some social groups, thus, reinforcing social disharmony. The situation becomes more serious if the lines of cultural conflict coincide with those of the social conflict.

Lebanon is a good example of a country where the issues of social and cultural conflicts are much alive. Lebanon has been, and still is, a crossroad of different cultures. In its recent history, Lebanon has become a bridge between the Western culture and the Arab-Islamic culture. In the 19th century, Christian missionaries started to establish schools which reflected Western values and practices through several means, the most important of which was using French or English, not only as a foreign language, but as a medium of instruction. The foreign languages took such a strong hold that mathematics continued to be taught in a foreign language even after independence in 1943 and despite laws and regulations to limit such practices. On the other hand, Arabic, the native language, is looked at by the majority not only as a symbol of national identity, but also as the carrier of the Arab-Islamic culture. If one accepts the thesis that the language of instruction, even in mathematics, is a cultural carrier, then it is not difficult to anticipate the negative consequences of such a situation. The historical accumulation has resulted in cultural divergence, which goes under the name of "cultural diversity" or "colonialism," depending on which side one is.

In the many panels which often take place to discuss the Arabization of mathematics teaching, the issue soon becomes an epitome of the larger issue of cultural conflict. The pedagogical arguments in favor of Arabization are self-evident and need not be elaborated. That Arabic cannot accommodate modern mathematical terms and structures is easily refuted by the fact that Arabic is a sophisticated, powerful language which, at one time, preserved and extended Greek mathematics. What is it then that stands in the way of Arabizing the teaching of mathematics? The claim is that the obstacle is an accumulation of two discrepant culturation processes. The resolution of this seemingly pedagogical issue becomes hopelessly entangled with the conflicting political and social issues.

In conclusion, I ask the following question: To what extent can the technological aspects of mathematics education be adapted without significantly changing the cultural values? This is a good question for which we do not have a good answer. Some Arab countries, which have material resources to buy not only the technology but the means of its production, are as determined to adapt technology as to preserve the ecology of their culture. This is a historical encounter, whose outcome only the future can tell.

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Integrating Mathematics With the Study of Cultural Traditions

Claudia Zaslavsky

All societies have developed mathematical practices appropriate to their daily lives and their cultures, an area of mathematics now known as "ethnomathematics." Yet, very little information about these practices has entered the curriculum at any level. Children tend to view mathematics as a cut-and-dried, esoteric subject that arose full-blown from the minds of a few great men in the past. Small wonder that many students seem to leave behind them all knowledge of the real world when they enter the mathematics classroom. It should be no surprise that many students find mathematics irrelevant, develop fear and anxiety about the subject, and drop it as soon as possible.

Mathematics educators in Third World countries now recognize the need to "multi-culturalize" the curriculum (D'Ambrosio, 1985; Gerdes, 1985). The educational failure of ethnic minority children in the industrialized countries has persuaded some educators to incorporate multicultural perspectives into their mathematics programs (Bishop, 1988).

Indeed, all children profit from such an expansion of the curriculum. Children learn that mathematical practices arose out of the real needs and desires of all societies. Mathematics comes alive when children study the measurement and numeration systems, the patterns in art and architecture, the games of skill and games of chance of various cultures. Students have the opportunity to learn about mathematical contribution of Third World societies, a generally neglected area of mathematics. They can take pride in their own heritage, and at the same time become familiar with and learn to respect the cultures of other societies.

For the past 15 years I have been conducting seminars and workshops for teachers on the theme: "Bring the world into the mathematics class" (Zaslavsky, 1973; 1985; 1987). The participants are encouraged to explore the mathematical practices of their own students' cultures, and wherever feasible, to integrate mathematics with other subjects. I will relate a few typical outcomes.

(1) Language-minority children often feel inferior because of their inability to speak the language of the country. However, given the opportunity to count in their own language, to teach the number words to their classmates and even to the teacher, and perhaps to explain the structure of the numeration system, their self-esteem grows immeasurably.

(2) In a session on the topic of mathematical probability, the participants discussed the tossing of coins, dice, cowrie shells, half-shells of nuts, and other objects appropriate to various societies. After they had carried out experiments with some of these devices, a first grade teacher in the South Bronx, the poorest and most neglected area of New York City, decided to introduce the subject to her students. According to her

plan, she would explain "head" and "tail" of a coin, then toss one coin, and so on. To her surprise, the children, with their streetwise experience, knew all about tossing coins, and entered enthusiastically into the activity.

(3) A project dealing with the shape of a house (Zaslavsky, in press) involved finding the areas of several different shapes, all having the same perimeter, by sketching the shapes on grid paper and counting the squares enclosed by each shape. As an aftermath to this activity, one class, working in small groups, designed, constructed and decorated several African-style compounds of round houses with conical roofs. In the course of this activity they learned that a cylinder is a rectangle whose two opposite edges have been joined, and that a cone is a circle with a sector removed. Another class used rulers, tape measures, and lengths of string to find areas and perimeters of many objects in the classroom.

(4) In response to a unit on African sand drawings, an aspect of graph theory, one ninth grade student commented: "I loved the fact that I got a chance to learn some of the math from a completely different country." Another said: "It showed that we shouldn't think our way of doing math is the only way there is." Ironically, the most talented mathematics student in that class wrote: "The traceable networks are nice for recreation, but they aren't *real* mathematics." To him, real mathematics meant the standard academic curriculum.

(5) A teacher of black adults in a drug rehabilitation center devised a teaching unit that required participants to find the value of a basket of food, using the Igbo (Nigeria) base-twenty numeration system. These adults were spared the shame of repeating the lessons they had not learned in the early grades, and at the same time they took pride in their African heritage. Non-decimal numeration systems are not unusual, and people who are not familiar with them can learn to work with bases other than ten when they are presented in a meaningful context.

(6) An Afro-American teacher wrote a paper about black children's exposure to number concepts through the illegal lottery known as the "numbers game." She described the use of probability, the skill of semi-literate people in recalling and recording numbers, and the system of hand gestures used when the police were on the scene. In conclusion she wrote: "Certainly if teachers could in some way use some of this number logic with the children, or at least recognize their familiarity with numbers when they meet them, perhaps number games based on Playing the Numbers might be substituted for the boring activities now presented to the children" (quoted in Zaslavsky, 1975).

Many obstacles block the implementation of multicultural mathematics education, among them a dearth of materials and inadequate teacher training. The most daunting impediment is the attitude of educators and their conception of what constitutes proper mathematics education. As an example, one referee, in an evaluation of my article, *Symmetry in American folk art* (Zaslavsky, to appear), crossed out all my references to the affective aspects of these activities, without so much as a comment. Apparently this educator believes that students' attitudes towards mathematics are of no consequence!

A further deterrent to enriching the mathematics curriculum came with the announcement of the results of the 1986 National Assessment of Educational Progress in mathematics. Commenting on the improvement in scores on basic skills, the president of the Educational Testing Service commented: "Thanks to the back-to-basics thrust, we've brought up the students who were at the bottom" (New York Times, 8 June 1988: A1). In other words, children have improved in their ability to carry out computations, a task that can be performed more quickly and accurately by a calculator. At the same time, the test results showed little or no progress on higher-order reasoning abilities. It is time to ask what mathematics education is for!

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The Cultural Role of Mathematics Education in the Future

World-wide cultural domination and unequal distribution of opportunities to take part in education characterize the present situation. In theme 3, therefore, future perspectives of mathematics education under these conditions were explored. Leone Burton, United Kingdom, going beyond already discussed features on gender and mathematics education, focuses on another area which still requires investigation: This is the degree to which the discipline of mathematics together with its predominant pedagogy reflects an inaccurate, socially stereotyped and misguided view of the nature of mathematical activities. This she clarifies by putting the question "Mathematics as a cultural experience—whose culture?" Desmond Broomes, Barbados, describes a strategy which seems most promising for developing, implementing, and evaluating an appropriate mathematics curriculum for rural communities: Community Involvement Strategy, in order to bring the curriculum closer to the activities of community life, to integrate educational institutions into the community, to broaden the curriculum in a meaningful way, so that it includes socio-economic, technical, and practical knowledge and skills. Kathryn Crawford, Australia, argues that—as school mathematics is not socially neutral—students from minority groups need experiences which provide concept development in qualitative aspects of the contexts in which mathematics is used in the dominant technical culture, that is, they need to know what to reason about, instead of overemphasis on instrumentalism offered by most teachers to know how. Philip J. Davis, USA, takes the position that mathematics education has to redefine its goals so as to create a citizenry with sufficient knowledge to provide social backpressure on the relevance and development of future mathematizations and to view applied mathematics as a "social contract." The Belgian anthropologist Rik Pinxten focuses on the relevance of differences in "cultural knowledge" for mathematics education. As cultural knowledge he describes kinds of ethno-ontology, cultural premises, and institutions to view the world, which are partly implicit and only partly explicit, but pervade and direct the explicit knowledge system and thinking within a culture.

Mathematics as a Cultural Experience: Whose Experience?

Leone Burton

In reading literature in the fields of mathematics education and mathematics, one is struck by the contrast between those who present results, outcomes, "product," and those, for whom the process is integral to what is being processed (see, for example, Poincaré). This so-called product/process debate has, to some extent, obscured the distinction between mathematical activity which brings, in its own right, satisfaction and pleasure, but might ultimately accomplish particular outcomes.

> From the viewpoint of a user, it is possible, and sometimes even convenient, to identify mathematics itself with its axiomatic presentation in textbooks. From the viewpoint of the producer, the axiomatic presentation is secondary. It is only a refinement that is provided after the primary work, the process of mathematical discovery, has been carried out. (Davis & Hersh, 1983, p. 343)

However, the apparent close links between mathematical, scientific and technological advances (i.e., products) and economic development, has helped to reinforce a reified view of these products in connecting them to the social/cultural experience which they help to promote. Despite rebuttals from those mathematicians who describe their world in terms of its aesthetics, its personalness, its culture-boundedness, mathematics continues to be experienced by the majority, especially those still in formal education, as "dry as dust, as exciting as a telephone book, as remote as the laws of infangthief of fifteenth century Scotland" (Davis & Hersh, 1983, p. 169). Nonetheless, or perhaps because of this removal of personal relatedness, mathematical power is acknowledged as touching most human concerns, but is seen as doing so in a depersonalized, i.e., objective, abstract, and rational manner.

I wish to raise a number of questions about the relationship between mathematics, the way in which it is described, and the manner in which it is experienced particularly by those who are still in the early days of their mathematical journey. My perspective is one of equal opportunities—the challenge, to reflect on the current accessibility of mathematics education.

First question: *How substantive is the view of mathematics as objective, rigorous, and convergent?* Three shifts can be detected over time in the understanding of mathematics. One is a shift from completeness to incompleteness, another from certainty to conjecture, and a third from absolutism to relativity. The Platonist position in ancient Greece was that mathematical statements represented eternal truths which were complete descriptions in both absolute terms and in terms of certainty. For example, Euclid offered his system of thought as a complete description of physical space after which no other was deemed necessary or required. The axioms on which the system was constructed were not seen as problematic but as self-evident foundations on which the system could rest with confidence. After the development of non-Euclidean geometries post 1800, a system of geometrical thought was seen to be the result of a particular set of conjectures. Changing the perspective leads to a new set of conjectures, which results in a different mathematical model being developed. In the present century, absolutism has succumbed to Einsteinian relativity and Godel dislodged completeness. Proofs are now seen to be demonstrations which take place within fixed systems of propositions and are, consequently, relative to those systems. Objec-

tivity is "at best" seen to be relative, and "at worst" abandoned altogether. In 1972, Kline wrote:

> Mathematics is not a structure of steel resting on the bedrock of objective reality but gossamer floating with other speculations in the partially explored regions of the human mind. (p. 481)

The mathematics which is finally codified and appears in journals or texts in the axiomatic presentation referred to above, is the product which results from the discovery process which Davis and Hersh have described so vividly. This process is personal, intuitive, and culturally dependent, it is informal and creative. Although the outcome of the process is rigorously and logically argued, the journey to this end has usually involved struggle, blockages, and failures, evidence of which, in its public presentation, has been carefully removed. While not wishing in any way to reduce the value of an elegant and rigorously argued piece of mathematics, why should such a high value be assigned to the product in comparison with the mathematical thinking which has produced it? Undervaluing the processes of mathematical thought also undervalues the mathematician who has engaged in that thought, leaving us with a body of thought which is disembodied. This depersonalization changes the character of the process, but simultaneously reinforces a social view of mathematics as "belonging" only to a very particular set of people, a closed club who subscribe to the same rules and speak the same language. Arguing that mathematics is abstract, objective, and independent of social, cultural, and political conditions has left the members of the club in an elitist and privileged position, from which they can attempt to control both the product and those who are admitted to their circle.

> We could look at these developments in mathematics simply as the onward and upward march of truth in the service of intellectual progress. But to do so hides the social imagery within which numbers and other mathematical notions have been conceptualized, and the very interesting processes of social negotiation through which one cultural image for thinking about mathematical concepts comes to replace another. (Harding, 1986, pp. 50–51)

Clearly, there are mathematicians whose image of mathematics is not dominated by notions of objectivity, rigor, and convergence. However, the existence of such descriptors of mathematics represents one means by which the cultural experience of mathematics can be constrained. For the mathematics educator, therefore, how to broaden not only physical access to mathematics, but more crucially psychological access, becomes a central issue. The social image of mathematics, which those within the discipline have permitted to be created, is not only destructive of potential mathematical students. It also has a ripple effect on attitudes and performance of those outside the discipline. In these circumstances, how should we engage with the responsibility for encouraging a social reperception of our discipline?

Second question. *What is the relationship between the view of mathematics as objective, rigorous, and convergent and a pedagogy based on a transmission model of learning?* In the famous dialog between Socrates and the slave, so often cited as an example of good teaching, Socrates was at pains to demonstrate that there was no such thing as teaching, only recollection by the learner of truths already known. Socrates' role was to help in the uncovering process by engaging the learner in dialog through which the mathematics was elicited. If we are honest, does this not continue as the practice used in many classrooms today? A good teacher helps a pupil by judicious questioning and encouragement, so that the required mathematical knowledge is recognized and reclaimed. The output of this dialog is the formal mathematics of the school curriculum, the content of which is validated through the teaching style. The model of teaching reflects one or both of two images—the filling of the empty vessel, that is the transfer of knowledge from teacher to pupil, or the peeling of the onion, the uncovering process already described. Some teachers use a combination, the first to transfer knowledge and skills and the second to help the unsuccessful pupil recapture the taught knowledge. The two are linked by the assumption that transfer of knowledge can and does take place. Consistent with the model is a heavy emphasis on rightness, both of solution and of method. Also consistent is a defined curriculum, which is assessed by examination of its contents. The formal teaching environment highlights single methods and single solutions. It stresses correctness and product. In his address to ICME 2, Rene Thom said:

> In practice, a mathematician's thought is never a formalised one ... The only real processes in mathematics are those of numerical and algebraic computation (but) even in a situation entirely concerned with calculation, the steps must be chosen from a large number of possibilities. One's choice is guided by the intuitive interpretation of the quantities involved. Thus the emphasis on axiomatics is not only a pedagogical aberration but also a truly mathematical one ... The real problem which confronts mathematics teaching is not that of rigour but the problem of the development of "meaning." (1972)

The conception of mathematics transmitted through the formal educational curriculum appears, therefore, to be misguided on two counts, that of the discipline itself, as well as that of the pedagogy. If we shift our pedagogical stance, what effects will be produced on the subject and the learners? What are the most effective means of encouraging teachers to make these shifts, intellectually in their reperceptions of mathematics, and pedagogically in their classroom practice?

Third question: *What pedagogical stance is most appropriate to a view of mathematics based on incompleteness, conjecture, and relativity?* In recent years there has been growing evidence of the inadvisability for teachers of perceiving their students as empty vessels.It has, for example, been conjectured that one reason why the performance of females is not sustained through the secondary schooling years is because of their compliance, and consequent dependency, on the authority of the teacher, since an empty vessel relies upon someone else for its management! Where teachers of mathematics themselves are not strong, as in many educational systems, this gets very close to the phenomenon of the blind leading the blind, or the partially sighted dictating to the blind! A different metaphor, that of construction and explo-

ration, has been proposed to challenge the assumptions of transfer and uncovering. In constructing and exploring, the emphasis shifts from product to process, from single solution to alternatives. Students are acknowledged as lively and profound thinkers, engaged upon trying to make sense of the new in the context of what is already understood. The teacher's role has to change to one of resource provider, rather than explainer. The students are placed in a context of cognitive conflict, through which they work together towards a resolution which makes personal sense.

For teachers this is problematic—how much simpler to be in control of a classroom, in which the pace, content, and outcomes are within your jurisdiction, than to observe and interact with small groups or individuals, while being aware that you are only accessible to a small number. The time scale of the enterprise has to change. The participants must assume different roles and responsibilities. The teacher has to accommodate to students pursuing their own lines of enquiry, and managing their learning independently. A new professional stance places much greater emphasis on judgments in the cognitive domain, than on clarity of transmission. At least for those of us with an interest in research in mathematics education, this style holds out more possibility of the use of our work than the former model, since a product, content-dependent model of teaching, has less obvious need of learning theories! However, the outcomes are also different as students demonstrate a wide range of understandings and misunderstandings, often exceeding the expectations of the teacher in one area, while astonishing with their failure to grasp in another. Outstanding questions to be addressed in respect of this pedagogic shift remain. For example, of what does a curriculum consist? What happens to the distribution of power both in relation to the discipline and to the learning? Does the learning model consist of different components and, if so, what are these? For example, when and what does a teacher "tell," does skill learning always sit comfortably within skill using, what professional judgments is it reasonable to expect of a teacher, and how do they acquire the knowledge to make these?

Fourth question: *What impact does the shift in mathematical pedagogy have on accessibility to mathematics?* I have claimed that the pedagogical processes, which are most common in the teaching of mathematics, deny the influence of the individual or the social context and present a pretend world of certainty, exactitude, and objectivity. I believe that this world is associated with power and control and that, given our social history, in each society power and control are vested in particular races and particular sexes. By validating a model of teaching which rests upon knowing, and consequently upon "expertness," we reinforce this hierarchical view. By validating a depersonalised model of mathematics, we ensure that the subject remains aloof from the concerns and interests of most members of society.

> First, there is a general disinclination to locate mathematics in a materialistic base and thus link its development with economic, political, and cultural changes. Second, there is a tendency to perceive mathematical pursuits as confined to an elite, a select few who possess the requisite qualities or gifts denied to the vast majority of humanity. This is a view prevalent even today in the classroom and thus determines what is taught and who benefits from learning mathematics. Third, there is a widespread acceptance of the view that mathematical discovery can only follow from a rigorous application of a form of deductive axiomatic logic which is perceived as a unique product of Greek mathematics ... finally, the presentation of mathematical results must conform to the formal and didactic style following the pattern set by the Greeks over 2,000 years ago ... as a corollary, the validation of new additions to mathematical knowledge can only be undertaken by a small, self-selecting coterie whose control over the acquisition and dissemination of such knowledge through journals has a highly Eurocentric character today. (Joseph, 1987).

I believe that only by moving mathematical pedagogy into the same world of conjecture, relativism, and incompleteness, that many mathematicians describe, will we succeed in breaking the image of mathematics as European and male and in creating the kind of classroom in which all students can find excitement, interest, and success. It seems to me to be a supreme irony that students disengage from the subject known as school mathematics, which itself is rejected as an adequate representation by many professional mathematicians. Were mathematics to be experienced as a searching, hesitant, intuitive area of study, open to interpretation and challenge, I feel confident that there would be a much greater identification with its style and ideas by pupils from both sexes, different classes, and different races. Accumulating evidence reinforces the view that a competitive style of learning is destructive for a lot of students who change their view of mathematics, and of their potential to learn the subject, when placed in a climate which encourages them to work together, listen and learn from each other, explore and respect different perspectives (see, for example, Gerdes, 1985). Equally, the growing body of work on ethnomathematics highlights the importance of respecting all traditions and acknowledging relevance and relatedness, not only to enhance the discipline itself, but also to affect the potentiality for learning it.

> For effective educational action not only an intense experience in curriculum development is required, but also investigative and research methods that can absorb and understand ethnomathematics ... as the underlying ground upon which we can develop curriculum in a relevant way. (D'Ambrosio, 1985, p. 47)

The child's growing interest and skill in pursuing objective mathematical enquiry depends upon sustaining her interest in the world around her, such that she can formulate questions about it and develop her capacity to attend to, focus on, and investigate aspects of that world that are distinct from herself. This does not require mastery or domination although those are the terms used so often by male mathematicians when describing their activities. Far from being a process of closure, as experienced by so many students, it is cyclical where choices dictate new ideas, future questions, further investigations. Such enquiries are not conditioned by power and domination, but thrive in an atmosphere of collaboration and connectedness. An environment which combines intellectual questioning with social support might prove to be the most effec-

tive for providing a cultural context which is meaningful to the majority of mathematical learners.

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The Mathematical Demands of a Rural Economy

Desmond Broomes

Mathematics in the School Curriculum

Any analysis of the curriculum that actually exists in primary and secondary schools of almost every country, developed or developing, shows that mathematics occupies a central place in school and in the activities of pupils and their teachers. Consider, for example, the African and Asian experiences (Khartoum Final Report, 1978), the American experiences (Suydam & Osbourne, 1977), the European experiences (Christiansen, 1975), and the British experiences (Cockcroft Report, 1982).

And although more careful analyses might reveal that levels of economic development within countries seem to relate intimately to the mathematical expertise of the citizens, it would be difficult to identify or trace the actual impact of mathematics on the economic well-being of a country or the impact of the economic and social needs of the country on mathematics eduction.

To attempt that exercise would require heavy investment of time, effort, and wit beyond the resources of the researcher even in the 1980s.

It is therefore prudent, within the confines of this discussion, to state three reasonable postulates as points of departure:

(1) Mathematics is a major activity in the curriculum of primary schools and the lower grades of secondary schools in almost all countries.
(2) The mathematics taught in primary schools is very similar in content, but differs somewhat in teaching methods and in ways the teaching sequences are organized from grade to grade.
(3) The mathematics taught in secondary schools differ markedly in content, in teaching methods, and in ways of assessing performance.

A close look at these three postulates should emphasize the need to clarify first, what is meant *by school mathematics*, and second, in what sense school mathematics is linked to the mathematics of the work place, the market place, and the playing fields.

Once again, it is prudent not to confront the issues head on. The shape of school mathematics, its content, its methods, its manifestations in the performance of the pupils are constructs of the goals of school mathematics.

Goals of Mathematics Education for Rural Development

In this presentation, the *rural world* is defined to include all those developing countries where 70 to 95% of all people live and work in rural areas, and those segments of developed countries where rural activities are prominent.

And *rural development* could be thought of as the consequence of finding solutions to the problems of health, nutrition, community welfare, public works, agriculture, irrelevant school curricula, and sterile educational philosophies.

Broomes (1981), having described the problems faced by rural communities and the tasks all societies undertake in preparing their young for full membership in the adult community, and bearing in mind that all cultures include education as the principle means of perpetuating themselves, pleaded for a concept of a school that is integrated with the community and operates so that other institutions (economic, social, political) contribute to the education of the young.

Within this context of development and redefinition of a school, five broad goals of mathematics education for rural development were generated, namely,

(1) Preparation of citizens as users of mathematics (Social Goal);
(2) Contribution of mathematics to cultural synthesis (Cultural Goal);
(3) Mathematics as an educative force in every person's life (Personal Goal);
(4) Training of mathematicians and other mathematical scientists (Technical Goal);
(5) Mathematics as a science with its own unique characteristics and beauty (Aesthetic Goal).

Quadling (1982), in attempting to probe the importance of learning mathematics within a British context, identified three categories of mathematics.

First, there is *survival* mathematics, that is, the mathematics that we need in order to go about our daily business and make good use of our leisure time ... It is a reflection of a person's life-style. It requires an immediate response, it is rarely done with paper and pencil, and it is not consciously called mathematics.

Essentially, survival mathematics is what Broomes called "preparation of citizens as users of mathematics" (Social Goal).

Next, [Quadling states that] much of the mathematics in the school curriculum is *mathematics for use*. This extends from simple skills, such as decimal arithmetic to the use of differential calculus. It is the mathematics some people needed to do their work successfully; it is job specific, trigonometry for navigators, statistics for economists, and it changes with time [logarithms]; it is also an essential tool for the scientist.

"Mathematics for use" corresponds to the goals Broomes defined as "Training of mathematicians and other mathematical scientists" (Technical Goal) and also "Mathematics as an educative force in every person's life" (Personal Goal). "Mathematics for use" cannot, as Quadling rightly agreed, "be divorced from the context which could give it meaning for the pupils."

And lastly, Quadling identified the "mathematician's mathematics," which is "about definitions, proof and abstract structures." "It is about the pleasure which many people get from solving mathematical puzzles, or the sense of personal achievement from investigating number patterns."

The "mathematicians' mathematics" corresponds to what Broomes called "Mathematics as a science with its own unique characteristics and Beauty" (Aesthetic Goal) and "Contribution of mathematics to cultural synthesis" (Cultural Goal).

Quadling's analysis of the nature of categories of mathematics led him to pose this final question:

If arithmetic (or mathematics) has as central a place in education as is often claimed, should it not—like its companions reading and writing amongst the three Rs—eventually become absorbed into the total curriculum?" (Quadling, 1982, p. 419)

Similar viewpoints were expressed by the participants at an International Conference held in Khartoum (1978) that was concerned with the linkages between mathematics and development. The participants were of the opinion that mathematics must be considered with three general objectives in mind.

First, the development of a country involves related cultural and material activities. And mathematics firmly belongs to the class of intellectual activities which together determine the cultural character of a country. Second, self-determination in scientific and technological matters is becoming increasingly important in the development process of any country. And again this can only be achieved through the existence of a wide and strong mathematical community. Third, mathematical skills can play an important role in the solution of concrete and immediate problems in the national development. Applications in mathematics have proved useful solutions to problems in health, economic planning, population growth, and agricultural development (Elton, 1979).

Using Goals to Construct Useful Forms of School Mathematics

Goals then are the forms of usefulness by which school mathematics are linked, with varying strengths, to economic, social, and political conditions of the society. For economic development, useful mathematics needs to acquire *special forms*.

... In stressing the interplay between the needs of the community and the needs and interests of individuals, it is not intended to give mathematics a narrowly utilitarian purpose, or to make mathematics a simple counting exercise—the popular conception of what is "useful mathematics." The interplay that is chosen for study should ensure that the learners experience high motivation, that teachers use a pedagogy that is consonant with the dominant cultural values of learner and teacher, that the content relates to the interests of the learner, and the needs of the society, and that a psychological and philosophical closeness is encouraged between schools and society. This interaction between mathematics and other subjects and between mathematics and the external world can be effectively sustained *through project work* in depth or by the study of real-world, practical problems. Such project work provides the learners with insight into 'model-building,' insight into the bridge between the abstract world of mathematics as something they can use as members of society. (Broomes, 1981, p. 57)

Here and there, the literature in the developed and developing world includes examples of clever ways of probing the interface between mathematics and employment and mathematics and economic activities (Knox, 1977; Pollak, 1979; Sträßer, 1987).

In its simplest form the teaching of this special mathematics requires the teacher to *pose problems* to the learner, and these problems should

- be replete with the cultural experiences of the learner,
- be, for the most part but not solely, practical and utilitarian,
- illustrate mathematics in use,
- be capable of being tackled profitably using mathematics,
- not make excessive mathematical demands (in terms of the learner's level of mathematical sophistication),
- provide ample scope for cooperative activity among persons at different levels of mathematics competence.

Avital and Parness (1978), working from a different perspective, presented other criteria for identifying problems which might be posed by teachers. This strategy—posing problems—has profound implications for

(a) Teacher Training;
(b) Curriculum Reform in primary and secondary schools;
(c) Cooperative activity within schools, and between schools and other institutions.

It would take this presentation far off course to outline, even briefly, the scope of these implications (for sound ideas and useful insights see Fonancier, 1984; Shuard, 1984; Stenmark, 1987; Sträßer, 1987). It is sufficient to remark on why problem posing is not

used by teachers as a major strategy to produce useful mathematics. About five points rush to mind:

(1) Lack of confidence among teachers who have not studied mathematics in sufficient depth.
(2) Attitude of teachers who studied mathematics as a fixed body of knowledge and who conceive of mathematics as a convergent discipline.
(3) Dearth of suitable textbook materials that make use of cultural experiences of the learner.
(4) Constraints of examination syllabuses that have been prepared for regional examinations.
(5) Constraints of timetabling and school organization that cut across the need for flexible time slots.

Summary and Conclusion

From an analysis of how mathematics education has grappled with the problems of rural development across different countries, Broomes and Kuperus (1983) identified *three major strategies* which seemed most promising for developing, implementing, and evaluating an appropriate mathematics curriculum for rural communities. One of these strategies—*Community Involvement Strategy*—sought to ensure that knowledge is acquired by persons as a living force, not as dogma; that mathematics is treated as an effort to present the quantitative aspect of reality and to analyze (and sometimes create) reality in quantitative terms rather than as an ability to do "sums" in artificial, abstract contexts.

This strategy requires that the activities which traditionally were organized piecemeal as responses to particular needs within the community must now be conceived as part of an educational philosophy which should undergird any community effort.

The major argument in this presentation is a further explication of the *Community Involvement Strategy*. Mathematics education for economic development in rural communities can be usefully analyzed and understood only in terms of a redefinition of schools and education.

To conclude, and at the same time to tease out the key issues for discussion, debate, and reflection, the nature and direction for *educational regeneration* within developing countries suggest activities that

(a) bring the curriculum of schools closer to the activities of community life and to the needs and aspirations of individuals;
(b) integrate educational institutions, vertically and horizontally, into the community so that outputs of such institutions are better adapted to the life and work in the community;
(c) redistribute teaching in space, time, and form and so include in the education process certain living experiences that are found in the community and in the lives of persons;
(d) broaden the curriculum of schools so that it includes, in a meaningful way, socioeconomic, technical and practical knowledge and skills, that is, activities that allow persons to combine mental and manual skills to create and maintain and promote self-sufficiency as members of their community. (Broomes & Kuperus, 1983, p. 710)

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Knowing What Versus Knowing How: The Need for a Change in Emphasis for Minority Group Education in Mathematics

Kathryn Crawford

Introduction

There is a growing consensus among cognitive scientists and mathematics educators (Resnick, 1988; Saxe, 1984) that abstract mathematical concepts are constructed by learners through reflection and conceptual activity. A greater awareness of the effect of experience of both the socio-cultural and physical environments on human development is evident in changing views of intelligence (Sternberg, 1985; Wertsch, 1985). Individual differences in intellectual ability are increasingly seen as the result of both genetic variation and the cumulative effect of differences in experience.

In contrast, many teachers, parents, and educational policy makers believe that mathematics is a socially neutral body of objective knowledge that is taught by teachers to students. Intelligent students are able to remember and apply what they have learnt. The less able find mathematics difficult. Mathematical tasks are presented in a largely instrumental fashion in a curriculum in which little reference is made to the socio-cultural contexts in which mathematical ideas are commonly used. Differences in aptitude are widely believed to be of genetic origin and thus "natural" and unchangeable. School structures, policy and teaching practice largely ignore differences in the socio-cultural experience of students.

In bicultural educational situations, it cannot be argued that students have a congruent socio-cultural experience outside school. The notion that socio-cultural experience influences conceptual development raises important questions for mathematics education in such contexts. To what extent do qualitative differences in social experience outside school influence the conceptual activity (and conceptual development) of students, from minority cultural groups, as they seek to make sense of the learning environment. Can one assume that such students have the knowledge and values that are implicit in an elementary mathematics curriculum?

Some Theoretical Perspectives

The relationship between cognitive and social acts have been described in detail by Vygotskii (1978) and Leont'ev (1981). Vygotskii asserts that individual conscious awareness is an internalized outcome of activity in a social context and not a necessary outcome of experience of an absolute reality. According to Vygotskii, this is particularly so in the case of higher order concepts such as those in mathematics and science. Luria (1973) describes the concepts and the relationships between them that are the learned consciousness derived from cultural activity as "matrices of community relationships." The notion of "Activity" is used by Leont'ev (1981) to describe the purposeful and creative activity (both physical and cognitive) which occurs when individuals or groups are stimulated by needs to *consciously* seek goals. The subjective nature of individual needs and goals is a key aspect of Leont'ev's theoretical perspective. Activity involves volition and arises from an individual's subjective interpretation of the context and the need for action. He makes a clear distinction between two types of cognitive function that occur during Activity. Conscious, reflective conceptual activity is described as "activity." A second type of cognitive function is described as an "operation." The second term is used to describe the automated and unreflective implementation of well-established and automated routines and procedures. Operations are described as tools used, usually with little conscious attention, during "Activity."

In the context of mathematics education, Mellin-Olsen (1987) stresses the importance of student experiences of mathematical "Activity" rather than merely instruction in instrumental rules and procedures or "operations." He argues that these experiences are particularly important for students who lack experience of mathematical "Activity" outside school. He also emphasizes the extent that the needs and goals of school students (and hence the quality of their conceptual activity) are influenced by their past cultural experience and the present social context of school tasks. Research by Crawford (1986) suggests that cognitive functions associated with mathematical "activity" but not those associated with "operations" are influenced by prolonged differences in socio-cultural experience. The research suggests that for some students conceptual "activity" is not an option in relation to many mathematical tasks. These students, and their teachers, are forced to define learning goals in terms of the development of task specific "operations" in cases where the curriculum content is defined by age and grade rather than the conceptual development of the child. Such a situation is dysfunctional in terms of mathematical development, but may have a short-term function in cases where educational opportunities depend on performance in standardized tests of skills.

Mathematics Education for Traditionally Oriented Aboriginal Children

It is clear from the above discussion that the development of mathematical knowledge needs to be considered in a cultural context. Decisions about school curricula in mathematics need to take into account the contexts in which mathematics is used in the wider society, students' experience of mathematical "Activity" in their daily lives, and the social context in which the mathematical experiences are provided in schools. In Australia the primary school curriculum in mathematics traditionally reflects the priorities, values, and activities of the dominant culture. Aboriginal students from traditionally oriented communities enter

school from a very different cultural experience. They begin school with different values, knowledge of different cultural activities, and speaking a different language.

Many children (particularly boys) from high socioeconomic status urban settings acquire a substantial knowledge of and interest in contexts for mathematical reasoning through observation of the activities and interests of their parents and through experiencing for themselves the structure and priorities of a complex, technical, urban culture. The behavioral expectations of teachers and parents are similar. Also, the form and function of language used at school is similar to that used at home. For them, mathematics is relevant and valued as a means to achieve aspirations. Success in competitive forms of assessment in mathematics is viewed as a necessary prerequisite to the gaining of recognition and privilege. For such children, a curriculum with an instrumental emphasis may have apparent relevance as an opportunity to develop desirable skills that are important as a means of gaining status in a highly competitive culture.

In contrast, Aboriginal children from traditionally oriented communities have difficulty interpreting the motivation behind mathematical activity and the intended meaning of mathematical information as it is usually presented in the school curriculum. Mathematical behaviors such as questioning, comparing, speculating, and testing actually conflict with the values associated with "appropriate" behavior in Aboriginal communities.

For example, students from Pitjantjatjara communities live in a culture that has evolved to meet the needs of subsistence in a harsh and relatively stable environment. Property rights are not clearly defined. Material possessions are not the key determinant of status in these communities. Aboriginal people view their physical environment as an extension of themselves rather than as something to be conquered. Traditional cultural activities typically revolve around small numbers of specifically named natural objects. Quantity is not usually considered and this low priority is reflected in a vernacular with few words to denote number. So they ask:
"Why do you always own things?"
"Why do you always compare?"
"Why do you want to know how much?"
"Why do you measure?"

For them, the assumed value of quantification, comparison, and measurement as cultural activities is a mystery. So too is the emphasis on number in the elementary curriculum. Indeed, in Aboriginal social contexts comparison is often viewed with some distaste and for many people questioning is viewed as most inappropriate behavior. As a result, teacher expectations of these behaviors may cause value conflict for many Aboriginal students.

Western notions of time and competition which are implicit in the social organization of schools also cause difficulty for most Aboriginal learners. For them, time is associated with space and seasonal cycles rather than the linear dimension that is measured precisely and used to coordinate activities in complex urban settings. Status and behavioral expectations in everyday activity, in Aboriginal communities are derived fundamentally from a complex set of stable and prescribed interpersonal relationships. Within these constraints, collaboration is highly valued but individual competition, within a community group, is actively censured. It is not surprising that Aboriginal students generally do not value punctuality and carefully avoid competing with other students.

These cultural differences are reflected in Aboriginal perceptions of the need for mathematical Activity and their perceptions of the subjective goal of mathematical tasks. The situation is further complicated by the effect of cultural learning on concept development. Piagetian studies have generally shown a developmental lag (Hunting & Whitley, 1983) with respect to those concepts commonly associated with mathematical reasoning in the dominant culture. Other concepts, commonly associated with mathematical activity, such as direction, are developed at an earlier stage than is usual in urban settings. For example, very young children in some communities are able to point in a Northerly direction consistently. Rudder (1983) reports on evidence of cultural differences in even fundamental qualitative conceptual categories, such as color, among the Yolnu people. He also describes the effect of the Aboriginal emphasis on quality rather than quantity in descriptions of the world. When discussing what English speakers speak of as "length," which signifies a quantitative approach to the attribute, "shortness" (gurriri) may be noted. However, objects are either short or not-short and once an initial assessment has been made, a second qualitative description is offered. That is, a description of the quality of the quality. The idea of short and long as opposite ends of a continuum is not easy for Aboriginal children. They grow up in a culture which emphasizes, in its activities, different categories and different relationships between them. These cultural differences cause particular difficulty when mathematics is presented in "concrete" contexts. Even in cases where English is used in schools, nouns describing the qualitative aspects of contexts for quantitative reasoning (for example short) are often interpreted by Aboriginal people in a sense directly translated from the vernacular and reflecting a conceptual framework originating in community activities.

Conclusions

The values, priorities, and Activities of the dominant culture in Australia are implicit in the content and organization of the mathematics curriculum and the social organization of schools. Because of teacher attitudes and beliefs about the objective and culture-free nature of mathematics and the genetic origins of differences in ability, the cultural context of mathematical development has been largely ignored in schools. The elementary curriculum content is still presented with a strong emphasis on "know-how"

with respect to arithmetic skills and content is not explicitly related to children's everyday lives. For Aboriginal children and other children from minority cultural groups this situation presents particular problems. They are usually perceived by teachers as lacking in ability. They certainly lack knowledge and understanding of the cultural values and the cultural Activities that are associated with mathematical activity in the wider culture. There is a need for these to be made explicit in the school curriculum and experiences of mathematical Activity provided. These children need to know *why* and *where* as well as *how* if they are to share cultural goals in mathematical Activity. The possibility of value conflict needs to be acknowledged particularly in relation to traditional forms of social organization and assessment in schools. Furthermore, children enter school with a culturally distinct view of reality which causes further confusion about exactly *what* is relevant when mathematical contexts are discussed or modelled. There is a need for a shift in emphasis towards clarification of categories and the relationships between them before quantitative reasoning is possible, even in cases where this type of cognitive activity is perceived as appropriate behavior by students.

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Applied Mathematics as Social Contract

Philip J. Davis

I shall emphasize the applications of mathematics to the social or humanistic areas, though one can make a case for applications to scientific areas and indeed to pure mathematics itself (see, e.g., Spalt).

Today's world is full of mathematizations that were not here last year or ten years ago. There are other mathematizations which have been discarded (e.g., Ptolemaic astronomy, numerological interpretations of the cosmos, last years' tax laws). How do these mathematizations come about? How are they implemented, why are they accepted? Some are so new, for example, credit cards, that we can actually document their installation. Some are so ancient, e.g, numbers themselves, that the historical scenarios that have been written are largely speculative. Are mathematizations put in place by divine fiat or revelation? By a convention of Elders? By the insights of a gifted few? By an evolutionary process? By the forces of the market place or of biology? And once they are in place what keeps them there? Law? Compulsion? Inertia? Darwinian advantage? The development of a bureaucracy where sole function it is to maintain the mathematization? The development of businesses whose function it is to create and sell the mathematization? Well, all of the above, at times, and more. But, for all the lavish attention that our historians of mathematics have paid to the evolution of ideas within mathematics itself, only token attention has been paid by scholars and teachers to the interrelationship between mathematics and society. A description of mathematics as a human institution would be complex indeed, and not be easily epitomized by a catch phrase or two.

The employment of mathematics in a social context is the imposition of a certain order, a certain type of organization. Government, as well, is a certain type of organization and order. Philosophers of the 17th and 18th century (Hobbes, Locke, Rousseau, Thomas Paine, etc.) put forward an idea, known as social contract, to explain the origin of government. Social contract is an act by which an agreed upon form of social organization is established. (Here I follow an article by Michael Levin.) Prior to the contract, there

was supposedly a "state of nature." This was far from ideal. The object of the contract, as Rousseau put it, was

> to find a form of association which will defend the person and goods of each member with the collective force of all, and under which each individual, while counting himself with the others, obeys no one but himself, and remains as free as before.

In this way, one may improve on a life which, as Hobbes put it in a famous sentence, was "solitary, poor, nasty, brutish, and short." The contract itself, whether oral or written, was almost thought of as having been entered into at a definite time and place. Old Testament history, with its covenants between God and Noah Abraham, Moses, The Children of Israel, was clearly in the minds of contract theorists. In the United States, political thinking has often been in terms of contracts, as in the Mayflower Compact, the Constitutions of the United States and of the individual states, the Establishment of the United Nations in San Francisco in 1945, and periodic proposals for constitutional amendments and reform.

It was generally assumed by the contract theorists that "Human society and government are the work of man constructed according to human will even if sometimes operating under divine guidance." That "man is a free agent, rather than a being totally determined by external forces," and that society and government are based on mutual agreement rather than on force (see Levin).

The acceptability of social contract as a historical explanation hardly lasted until the 19th century, even if political contracts continued to be entered into as instances of democratic polity. It is an instructive exercise, I believe, in order to get a grasp on the relationship between society and mathematics, to take the outline of social contract just given and replace the words "government and society" by the word "mathematization." Though it is naive to think that most mathematizations came about by formal contracts, the "contract" metaphor is a useful phrase to designate the interplay between people and their mathematics and to make the point that mathematizations are the work of man, constructed according to human will, even if operating under a guidance which may be termed divine or logical or experimental according to one's philosophic predilection.

A number of authors, some writing about theology, and others about political or economic processes, have pointed out that contracts are continuously entered into, broken, and reestablished. I believe the same is the case for mathematizations. Consider, for example, insurance. This is one of the great mathematizations currently in place, and I personally, without adequate coverage, I would consider myself naked to the world. Yet, I am free to throw away my insurance policies. Consider the riders that insurance companies send me, unilaterally abridging their previous agreements. Consider also that in a litigious age, with a populace abetted by eager lawyers and unthinking juries, what appears as the "natural" stability of the averages upon which possibility of insurance is based, emerges, on deeper analysis, to incorporate the willingness of the community to adjust its affairs in such a way that the averages are maintained. The possibilities of insurance can be destroyed by our own actions.

Another example that displays the relationship between mathematics, experience, and law is the highway speed limit in the United States. Before the gas shortage in 1974, the limit was 65 miles per hour. In 1974, the speed limits were reduced to 55 miles per hour, in order to conserve gasoline. As a side effect, it was found that the number of highway accidents was reduced significantly. Now (1987) the gas shortage is over, and there is pressure to raise the legal speed limit. Society must decide what price it is willing to pay for what some see as the convenience or the thrill of higher speeds. Here is mathematical contract at work.

The process of contract maintenance, renewal, or reaffirmation, in all its complexities, is open to study and description. This is a proper part of applied mathematics, and I shall argue that it should be a proper part of mathematical education.

Mathematical Education at a Higher Metalevel

A mathematized and computerized world brings with it many benefits and many dangers. It opens many avenues and closes many others. I do not want to elaborate this point as I and my co-author Reuben Hersh have done so in our book *Descartes' dream*, as have numerous other authors.

The benefits and dangers both derive from the fact that the mathematical/computational way of thinking is different from other ways. Philosopher and historian Sir Isaiah Berlin called attention to this divergence when he wrote "A person who lacks common intelligence can be a physicist of genius, but not even a mediocre historian." For the mathematical way to gain ascendancy over other modes is to create an imbalance in human life.

The benefits and dangers derive also from the fact that mathematics is a kind of language, and this language creates a milieu for thought that is hard to escape. It both sustains us and confines us. As George Steiner has written of natural language (1986):

> The oppressive birthright is the language, the conventions of identification and perception. It is the established but customarily subconscious unargued constraints of awareness that enslave.

One can assert as much for mathematics as a language. The subconscious modalities of mathematics and of its applications must be made clear, must be taught, watched, argued. Since we are all consumers of mathematics, and since we are both beneficiaries as well as victims, all mathematizations ought to be opened up in the public forums where ideas are debated. These debates ought to begin in the secondary school.

Discussions of changes in mathematics curricula generally center around (a) the specific mathematical topics to be taught, e.g., whether to teach the square

root algorithm, or continued fractions or projective geometry or Boolean algebra, and if so, in what grade, and (b) the instructional approaches to the specific topics, e.g., should they be taught with proofs or without; from the concrete to the abstract or vice versa, what emphasis should be placed on formal manipulations and what on intuitive understanding; with computers or without; with open ended problems or with "plug and chug" drilling.

Because of widespread, almost universal computerization, with handheld computers that carry out formal manipulations and computations of lower and higher mathematics rapidly and routinely, because also of the growing number of mathematizations, I should like to argue that mathematics instruction should, over the next generation, be *radically* changed. It should be moved up from subject-oriented instruction to instruction in what the mathematical structures and processes mean in their own terms and what they mean when they form a basis on which civilization conducts its affairs. The emphasis in mathematics instruction ought to be moved from the syntactic-logico component to the semantic component. To use programming jargon, it ought to be "popped up" a metalevel. If, as some computer scientists believe, instruction is to move from being teacher-oriented to knowledge-oriented—and I believe this would be disastrous—the way in which the role of the teacher can be preserved is for the teacher to become an interpreter and a critic of the mathematical processes and of the way these processes interact with knowledge as a database. Instruction in mathematics must enter an altogether new and revolutionary phase.

The Interpretive Component of Mathematical Education

Let me begin by asking the question: To what end do we teach mathematics? Over the millennia, answers have been given and they have differed. Some of them have been: We teach it for its own sake, because it is beautiful; we teach it because it reveals the divine; because it helps us think logically; because it is the language of science and helps us to understand and reveal the world; we teach mathematics because it helps our students to get a job either directly, in those areas of social or physical sciences that require mathematics, or indirectly, insofar as mathematics, through testing, acts as a social filter, admitting to certain professional possibilities those who can master the material. We teach it also to reproduce ourselves by producing future research mathematicians and mathematics teachers.

Ask the inverse question: What is it that we want students to learn? We may answer this by citing specific course contents. For example, we may say that we now want to emphasize discrete mathematics as opposed to continuous mathematics. Or that we want to develop a course in non-standard analysis on tape so that joggers may learn about hyperreal numbers even as they run. Or, we may decide for ourselves what the characteristic, constitutive ingredients of mathematical thought are: space, quantity, deductive structures, algorithms, abstraction, generalization, etc., and simply assure that the student is fed these basic ingredients, like vitamins. All of these questions and answers have some validity, and tradeoffs must occur in laying out a curriculum.

Within an overcrowded mathematical arena with many new ideas competing for inclusion in a curriculum, I am asking for a substantial elevation in the awareness of the applications of mathematics that affect society and of the consequences of these applications. If formal computations and manipulations can be learned rapidly and performed routinely by computer, what purpose would be served by tedious drilling either by hand or by computer? On-the-job training is certainly called for, whether at the supermarket checkout counter or on the blackboards of a hi-tech development company. If mathematics is a language, it is time to put an end to overconcentration on its grammar and to study the "literature" that mathematics has created and to interpret that literature. If mathematics is a logico-mechanism of a sort, then just as only a very few of us learn how to construct an automobile carburetor, but all of us take instruction in driving, so we must teach how to "drive" mathematically and *to interpret what it means when we have been driven mathematically in a certain manner.*

What does it mean when we are asked to create sex-free insurance pools? What are the consequences when people are admitted or excluded from a program on the basis of numerical criteria? How does one assess a statement that procedure A is usually effective in dealing with medical condition B? What does it mean when a mathematical criterion is employed to judge the quality of prose or the comprehensibility of a poem or to create music in a programmatic way? What are the consequences of a computer program whose output is automatic military retaliation? The list of questions that need discussion is endless. Each mathematization-computerization requires explanation and interpretation and assessment. None of these things are now discussed in mathematics courses in the concrete form that confronts the public. If a teacher were inclined to do so, the reaction from his colleagues would probably be "Well, that is not mathematics. That is applied mathematics or that is psychology or economics or social-anthropology or law or whatever." My answer would be: I am trying, little by little, to bring in discussions of this sort into my teaching. It is difficult but important.

If the claim were made, with justice, that these matters cannot be discussed intelligently without deep knowledge both of mathematics and of the particular area of the real world, then I would agree, and point out that this claim forces into the open the conflict between democracy and "expertocracy" (see, e.g., Prewitt). This conflict has received considerable attention in areas such as medicine, defence, and technological pollution, but has hardly been discussed at the

level of an underlying mathematical language. The tension between the two claims, that of democracy and of expertocracy, could be made more socially productive by an education which enables a wide public to arrive at deeper assessments, moving from daily experience toward the details of the particular mathematizations. While we must keep in mind certain basic mathematical material, we must also learn to develop mathematical "street smarts" which enable us to form judgments in the absence of technical expertise (cf. Prewitt).

A philosophy of mathematics which is "public" and not "private," lends support to introducing this kind of material into the curriculum. The discussion of such curriculum changes will be assisted by the perception of the mathematical enterprise as a human experience with contractual elements; and by the realization that every civilized person practices and utilizes mathematics at some level, and thereby enters a certain knowledge and belief community.

Again, following Kenneth Bruffee's article (with additions and modifications), I would like to suggest several lines of inquiry.

(a) Identify and describe the mathematical beliefs, constructions, practices that are now in place. Where and how is mathematics employed in real life?

(b) Describe the mathematical beliefs, constructs, and practices that have been justified by the community. What are justifiable and unjustifiable? what are the modes of justification?

(c) Describe the social dimensions of mathematical practice. What constitutes a knowledge community? What does the community of mathematicians think are the best examples that the past has to offer?

As part both of (a) and (b) one should add: Describe the nature of the various methods of *prediction* and the bases upon which prediction can become prescription (i.e., policy).

This type of inquiry is rarely carried out for mathematics. For example, the concrete question of where such and such a piece of university mathematics is used in practical life and how widespread is its use, is seldom answered. Many textbook claims are made in textbooks, but show me the real bottom line. It is important to know. How, in fact, would you define the bottom line?

The technical term for inquiries such as the above is "hermeneutics." This word is well established in theology, and in the last generation has been commonly employed in literary criticism. It means the principles or the lines along which explanation and interpretation are carried out. It is time that this word be given a mathematical context. Instruction in mathematics must enter a hermeneutic phase. This is the price that must be paid for the sudden, massive, and revolutionary intrusion of mathematizations—computerizations into our daily lives.

Conclusion

Mathematics is a social practice. This practice must be made the object of description and interpretation. It is ill-advised to allow the practice to proceed blindly by "mindless market forces" or as the result of the private decisions of a cadre of experts. Mathematical education must find a proper vocabulary of description and interpretation, so that we are enabled to live in a mathematized world and to contribute to this world with intelligence.

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World View and Mathematics Teaching

Rik Pinxten

The Problem

In previous work (Pinxten et al., 1983; Pinxten et al., 1987) I did research on the spatial concepts and the intuitive or everyday mathematics of another culture (i.e., the Navajo Indians in the USA). In unpublished work, my group concentrated on the same topics in the Turkish immigrant community of Ghent, Belgium. For the purpose of this symposium, a similar problem emerges from both domains: With the growing importance of mathematical competence in a high-tech society we witness a growth of the inequality of opportunities to take part in mathematical education. Our research group found that both in the native groups of North American Indians we studied and in the immigrant community of Turks, the level of education (and schooling) in general and of mathematics education in particular showed a discrepancy in favor of the members of the white dominant culture around them. It looks to me as if mathematics education might thus even become an instrument to create or widen the gap between competence (and ensuing social opportunities) in subjects from the dominant culture and members of the minority groups (cf. also D'Ambrosio's warning in this regard, 1987). In my diagnosis of the actual learning processes in all the communities concerned here, I found that cultural traditions play an important role. I concentrate on one aspect only here, the world view or cognitive setting of a community. This does not imply that religions, interactional, linguistic, or other aspects of a culture do not interfere in mathematical education. But, within the constraints of this paper, my focus is exclusively on the cognitive parameter. My suggestion is that mathematics teaching can be more effective and will yield more equal opportunities, provided it starts from and feeds on the cultural knowledge or cognitive background of the subjects. Thus, mathematics education will differ in contents and strategy depending on the cultural background of the students. It is my conviction that the remarkable successfulness of Asian students in mathematics can be diagnosed in the same sense, working to their benefit in this case. However, this phenomenon again falls outside of the scope of this short paper.

World View and Curriculum

Every cultural tradition develops a particular knowledge system adapted to the ecological circumstances and coached by the ethnohistory of the community. Anthropologists, and cognitive anthropologists in particular, have learned over the generations that each tradition has idiosyncrasies, which do not translate fully into and hence are incommensurable to a degree with other traditions. In cognitive anthropology in general, and in my work on spatial knowledge of Navajo Indians in particular, one finds ample evidence of the partial overlap between diverse knowledge systems: e.g., all human beings live in a three-dimensional spatial representation system, all cultures have some notions of an elementary metric system (with distance, length, and width), and so on, but at the same time, the particular notions can be incomparable to a very high degree. Let me explain this a little bit in the Navajo case, linking it immediately with the mathematics curriculum.

Western children grow up in a world which is framed continuously, at a pre-school and at a school level, in terms of a deep-rooted atomistic view on the world. In other words, the intuitive physics or the intuitive geometry of the westerner starts from the basic metaphor that the world can be seen as a huge construction (a house, a ship) built out of small building units or atoms or solid bodies. We soon learn that knowledge of the whole can be gathered by the clever combination of knowledge about the parts. Inversely, we learn to attack a problem by breaking it up in parts, in analogy with the strategy to gain knowledge about the universe by gathering sufficient information about parts. Not only mathematics teaching (in the ordinary and, of course, in the programmed learning version) testifies to this belief, but the mathematical discipline itself starts from the part-whole dichotomy as a basic given. From it follows the fundamental importance of formal features in description and modeling.

Navajo Indians developed a view on the world, whereby process and movement are the basic stuff of the universe. The language consists nearly entirely out of verb forms, such that the grammatical couple of norm and verb categories do not pertain. A correlate of this way of thinking and speaking about the world is that the verb "to be" does not exist in Navajo, and that a set of verbs for "to go, to move about" in a way holds its place. A second correlate to this cultural approach to the world is that the part-whole distinction is absent in the Navajo knowledge system: Our attempts to investigate bodypart, carpart or holistic notions met with repeated uneasiness and puzzlement. You just do not think in a Navajo way when trying to fit the world in an atomistic framework. Jumping to another level now, this different cognitive outlook led to a very different filling in of spatial and intuitive geometric notions: The notions of horizontality, distance, etc. that Navajos use are constituted in an altogether different and incomparable way from the Western ones (using movement as a constant constituent, for example). For details, I have to refer to the books cited above. The point I want to stress here is that we could also witness that mathematics education (in a Western curriculum in Navajo schools) did not yield a great success and led to serious cognitive problems and eventual drop out in Navajos. On the other hand, the bilingual program (with Navajo as a first and English as a second language) yielded a much better level of performance than any other language curriculum so far (König, 1983). My diagnosis is that

in the mathematical education the foundation for understanding and hence for translation was missing.

The diagnosis on the Turkish immigrant group is different: Linguistically and cognitively the immigrant groups are much more akin to the Western dominant culture they live with than the Navajo Indians. On closer examination their situation reminds one of that of most poor groups: They just get less opportunities in their environment for exploration, they develop on less sophisticated and poorer semantics and are hindered by the poor language and cognition they have to use for the opportunities of the dominant cultural group. It seems advisable to carefully distinguish this form (typical for most or all immigrant groups?) from the cultural difference we witness in, e.g., the Navajos.

Towards Native Curricula

My suggestion for the teaching of mathematics in other cultural communities is simple and straight forward: Let us study and develop the insights that are there in the pre-school knowledge system. The approach is very similar to 'English as a second language': To get to a rich semantics and a fluent use of language, it is important to develop and train the mother tongue first, and—on the basis of the skills and insights with regard to the mother tongue—then to eventually teach a second language. The same strategy should apply with regard to mathematics: First study, train, and develop the semantically rich notions about space, form, number, measure, etc. which are there. Study and train them in the language of the people and focus as much as possible on insightful learning of the knowledge which is there in the pre-school situation. Through a conscious and systematic search for regularities and formalizable characteristics of the Navajo (or any other) tradition one can then by means of insightful comparison come to the instruction of our mathematics as a second approach. The choice for this second part of the curriculum must be left to the local community, of course.

The importance of this approach, according to me, is that it can counterbalance the underdeveloping effect of a lot of present-day curricula, which do not take into account the importance of insight on a culture-specific cognitive basis and see mathematics as an a priori, culturally neutral discipline. We have a clear political impact , I think, while choosing for the one or the other alternative.

A final word should be of a practical order: Cognitive psychology and cognitive anthropology are sufficiently developed now to be of substantial assistance in the "diagnosis" phase of this strategy for education. The development of other (non-Western) curricula can gain from their insights, I guess, but it will most of all depend on the inventiveness and the courage of mathematics educationalists.

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Society and Institutionalized Mathematics Education

Mathematics education and culture, the theme of the timeslot 1, is now examined at two different levels: in timeslot 2 at the level of institutionalized education embedded into the society, and in timeslot 3 culture is seen as a factor in the relationship of the individual process of learning mathematics and the social interactions framed by the institutions of the educational systems.

Mathematics as a Cultural Product

Mathematics is not culturally neutral. The relationships are discussed between the cultural origins of mathematics and its future perspectives. By reflecting implications of the recently developed new and (mathematized) technologies and by combining arguments from the history and sociology of sciences with epistemological reflections, the Danish mathematics historian Jens Høyrup, in "On Mathematics and War," asks the mathematicians (especially in their role as teachers) to take over the responsibility for the social use of their work. Jens Høyrup asks: "In which sense is it possible to understand scientific cognition as a neutral instrument, and in which not? What can mathematicians do in order to avoid the situation of their science and the scientific institutions in general staying or becoming virtual criminals against humanity?" These are questions which do concern all mathematics educators. George Ghevarghese Joseph, United Kingdom, deplores the fact that there is a widespread Eurocentric bias in the production, dissemination, and evaluation of scientific knowledge for centuries, and seeks to destroy the existing Eurocentric paradigmatic norms, not only in history, anthropology, and ethnology, but also in history of mathematics and mathematics education. Sam O. Ale, Nigeria, describes existing rural mathematics in Nigeria, which is oral and unwritten and generally serves as a tool for directly solving practical problems. He argues for taking this kind of rural and very meaningful mathematics as a base to build on mathematics programs for further instruction. Leo Rogers, United Kingdom, pleas for examining the history of mathematics in its social context to identify possible conflicts in the meaning and intention between "intended" and "actual" uses of mathematics, and to create a social epistemology of mathematics.

On Mathematics and War*

Jens Høyrup

The development of new weapon technologies and new military strategies build, on the one hand on highly formalized (whence also strongly mathematized) sciences, on the other directly on the use of mathematical models and techniques. It is claimed by many that the sciences, including mathematics, are thoroughly corrupted because of this involvement in the horrors of mass destruction—and claimed perhaps by just as many that one must distinguish the fire from the incendiary killer.

Both sides can advance valid arguments in favor of their respective position; none of them can thus be totally wrong, and none of them totally right. In order to get behind the two spontaneous moralisms we must transform their question, using an approach combining arguments from the history and the sociology of science with epistemological reflections: *In which sense* is it possible to understand mathematical knowledge as a neutral instrument, *and in which not?* In which sense do mathematicians and other scientists stand like the incendiary in front of the burning house? Only understanding this will we know more about what mathematicians can do (especially in their role as teachers) in order to avoid that their science and the scientific institution in general stay or become virtual criminals against humanity.

A lot of anecdotes are used as documentation that mathematics was always the godchild of war, from Archimedes' supposed use of burning mirrors to Leonardo da Vinci's presentation of himself as equally useful as a military engineer winning wars for the Sforzas and for producing the equestrian statues celebrating their power. Until well after the Renaissance the intercourse of mathematics with military practice has, however, consisted in the application of principles of general validity,—be it Babylonian formulas for the calculation of earth volumes equally valid for siege ramps and for irrigation channels and temple building; Alexander's planning of rations for armies by means of techniques used to plan the supply of food for workers; or Early Modern mathematical navigation shared by the mercantile marine and the navy. Most often, furthermore, the techniques and principles used for the purposes of war were elementary compared to the level of mathematical knowledge attained at the time. In the Early Modern era, true, the improvement of a few techniques was furthered systematically for military reasons by scientific academies and at officers' schools, but on the whole without

* The detailed arguments on which the following compact statements are built are expounded in Bernhelm Booß & Jens Høyrup (1984). *Von Mathematik und Krieg. Über die Bedeutung von Rüstung und militärischen Anforderungen für die Entwicklung der Mathematik in Geschichte und Gegenwart* (Schriftenreihe Wissenschaft und Frieden, Nr. 1). Marburg: Bund demokratischer Wissenschaftler.

perceptible effect on the long-run development of mathematics as a corpus of coherent and applicable knowledge. Mathematics considered in this way as a whole developed in contact with and under the inspiration and pressure of societal practice *in toto*. In as far as war was a dominant societal preoccupation military practices played their role, too; but most often there was nothing specific about those military practices which called for mathematization. War in its destructive specificity thus had no impact upon mathematics, neither as a body of knowledge nor as a style of thought.

Against this it could be claimed that the intent and principle of the application of mathematics is planning of action under unquestioned conditions, and that this is the very principle of military planning, be it strategic or tactical. But, firstly, the latter statement is only really true from Clausewitz onwards. Secondly, war is not the only field where strategic rationality *(Zweckrationalität)* dominates; since the late Middle Ages both the commercial and the technical sphere were increasingly governed by this principle, centuries before Mars left the realm of Honor for that of Comparative Advantage, and both were better qualified for mathematization than was the conduct and planning of war. If any societal concept of sub-optimizing strategic rationality has invaded the spirit of mathematics (which is not incredible) it is that of the Market Economy and the Technical Civilization.

From the French Revolution onwards, and especially since 1939, new trends have occurred. In the post-Clausewitz era of staff planning and of "total wars" between strong, industrialized, and capitalist national states, *war* became not only the supreme social effort but until 1945 also (postal services and similar static sectors apart) the only activity submitted to central planning, and the only time in which governmentally directed and financed applied research of major weight was pursued[1] (in which connection we should remember that the OEEC and its successor organization OECD were created in order to secure the competitive power of capitalism; their promotion of governmental science policy for non-military purposes can thus be understood as another aspect of the Cold War).

In this era we have seen major changes in the constitution of mathematics connected to the impact of war. Paradigmatic examples are the development of electronic computers, operations research, and applied statistics during World War II. These were no longer elementary applications of familiar procedures but opened up new vistas. And yet they share important features with traditional military mathematics. We may look at the case of the computers. Computers based on relays had been created shortly before the war by Stibitz (USA) and Zuse (Germany) for civilian purposes; Stibitz's machine had even been financed and applied by Bell Telephone. Then came the war, and a number of further machines were built, each of them however intended for a specific purpose. Even after the war, development was rather slow because of narrow binding to pre-determined military applications; in Zuse's case, the war was even a main reason that he had to concentrate on the application in greater scale of the principles of his pre-war calculators instead of implementing his more revolutionary ideas. Only when IBM (closely followed by others) began producing general-purpose electronic computers in the early 1950s and brought them into the civilian market did the decisive breakthrough take place. Only the needs of the civilian market were wide and diverse enough to call for and hence make possible the unfolding of the full potentialities of this new mathematical technology. Furthermore, the services rendered by electronic computing to the military were not different in character from those rendered to civilian agencies and commercial firms: Even though the development of COBOL was financed by the Pentagon, the product serves equally well and *in the same way* for civilian and military stock bookkeeping.

Similar stories could be told of applied statistics and even of operations research, in origin as well as etymology the mathematization of military tactics, and also of later developments, e.g., in pattern recognition and other branches of artificial intelligence. As in the 17th century and even in Ancient Babylonia, the character of military and non-military practice are sufficiently similar to call for common principles of mathematization (single problems and solutions will of course differ, but single problems do not determine the global development of mathematics or mathematical disciplines); in as far as the development of mathematics reflects societal practice, it reflects it *as a whole*. Furthermore, of course, the secrecy surrounding innovation in the military domain will make problem solutions arising in that context less liable to call forth scientific (including mathematical) development than solutions occurring in the more open civilian domain.

Mathematics as a field of knowledge is hence not *specifically* involved in the virtual auto-destruction of mankind; but mathematics is *more than a field of knowledge*, and involvement need not be *specific* in order to be *involvement*.

Firstly, mathematicians are members of a social community connected in various ways to the rest of society. Often they take their grants and their jobs where they can get them, and in many cases that happens to be within the military establishment. The theorems or techniques which they develop may be just as valid for civilian as for military purposes, but their working conditions will often determine that communication of technically relevant information only goes to the employer. They are thus no less morally involved in military undertakings than any other member of the military establishment, and in

[1] I disregard the somewhat similar yet different structures of socialist countries.

fact *more* than average when it comes to the dangers of nuclear war: Mathematicians, indeed, are critically needed everywhere planning is necessary for *that which has never been and cannot be tested in practice:* mathematicians are thus indispensable servants wherever strategists are "thinking about the unthinkable."

It was argued that the progress of mathematical knowledge was not geared to any military spirit. That does not necessarily hold for the mathematicians' community (or communities—the plural may be needed). Loyalty will easily go in the direction intimated by grants and job possibilities (one's own or those of one's students), and the colleague who jeopardizes by "disloyalty" research and employment prospects will soon get under pressure. Hence, the moral involvement of mathematics in the virtual nuclear holocaust is not just the affair of the relatively few directly taking part but of much wider circles.

Secondly, we come to the question of "unspecific involvement." As mathematics reflects social practice *as a whole,* it also serves social practices indiscriminately. It can do so without being torn epistemologically and socially to pieces because of a general characteristic of the complex *scientific knowledge:technological practice,* which from a global point of view could be called "organized segmentation" and from each specific field looks as wide-range autonomy. This can be explained in metaphors referring to systems theory.

Since long, indeed, not only the single sciences and the single sectors of material production but also subdisciplines and -specialties have grown so large and complex that most interactions of most elements—be it the references of mathematical theorems or the mutual fitting and physical interaction of parts of a supersonic airplane—are with elements of the same system. For most elements, the direct references to or interactions with the external world are marginal, few, or non-existent. For the system as a whole, the dense network of internal interactions produce the inertia of the system—i.e., its tendency to go on reproducing the same sort of activity. On the other hand, external interactions though hardly visible from the perspective of the single element are necessary as a source of energy, and they influence decisively the *changes* in the way the system works.

This segmentation is a prerequisite for the effectiveness of the complex of technical and scientific knowledge. Because of the inertia produced in subsystems, however, it is also a reason that the effectiveness is increasingly off the rail, each system continuing for decades the course set by the navigation marks of yesterday. Not only has it proven much easier to make the general public ecologically conscious than to reorient the chemical industry; mathematics, too, as the embodiment of average mathematicians experiencing only professional interaction with other mathematicians, tends to see itself as not mixed up in the affairs of this world ("I prove theorems; what happens to them afterwards I do not know," to quote one very honest guy). Since, however, only mathematicians are able to clear up what is done by mathematics and to bring *their* perspective on deadly issues into the public arena, shielding oneself off from the implications of co-responsibility by subjectively experienced innocence amounts to guilt.

Eurocentrism in Mathematics: The Historical Dimensions

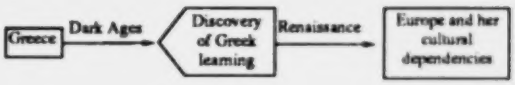
George Ghevarghese Joseph

The standard treatment of the history of non-European mathematics in Western texts exhibits a deep-rooted historiographical bias in the selection and interpretation of facts which, as a consequence, has resulted in ignoring, devaluing, or distorting mathematical activity outside Europe. Most histories of mathematics which were to become standard bearers for later work were written in the late 19th or early 20th century. During that period, two contrasting developments were taking place which would have an impact both on the content and the balance of the books, especially those produced on both sides of the Atlantic. On the one hand, exciting discoveries of ancient mathematics on papyri in Egypt and clay tablets in Mesopotamia pushed back the origins of written mathematical records by at least 1,500 years.

But a far stronger counter-influence was the political climate of the day, when the same period saw the culmination of European domination in the shape of political control of vast tracts of Africa and Asia. As an adjunct to this domination arose the ideology of racism and white superiority which spread over a wide range of social and economic activities, with traces to be found in the writing of histories of science. These histories emphasized the unique role of Europe in providing the soil and spirit for scientific discovery. The contributions of the colonized were ignored or devalued as part of the rationale for subjugation and dominance. And the development of mathematics before the Greeks—notably in Egypt and Mesopotamia—suffered a similar fate being dismissed as of little importance to the future of the subject.

Figure I presents the "classical" Eurocentric view of how mathematics developed over the ages. This development is seen as taking place in two areas separated by a period of stagnation lasting for over a 1,000 years—Greece (from about 600 B.C. to 400 A.D.) and post-Renaissance Europe from the 16th century to the present day. The intervening period of inactivity was the "Dark Ages"—a convenient label that expressed both the post-Renaissance prejudices about its immediate past and the intellectual self-confidence of those who saw themselves as the true inheritors of the "Greek miracle" which was supposed to have occurred 2,000 years earlier.

Figure I: The "Classical" Eurocentric Trajectory



But this comforting rationale for European dominance became increasingly untenable for a number of reasons. First, there was the fulsome acknowledgment by the ancient Greeks themselves of the intellectual debt they owed to the Egyptians. There are scattered references from Herodotus (about 450 B.C.) to Plutarch (about 100 A.D.) of the knowledge acquired from the Egyptians in various fields including astronomy, mathematics, and surveying, with some commentators even considering the priests of Memphis as founders of science. Herodotus had implied that the Greeks were heavily influenced by Egyptian settlers in their own land, and that their civilization owed to no small measure the establishment of Egyptian rule in southern Greece in earlier centuries.

A second reason why the trajectory described in Figure I was found to be untenable arose from the combined efforts of archaeologists, translators, and interpreters, who unearthed evidence of a high level of mathematics practiced in Mesopotamia and in Egypt at the beginning of the second millennium B.C., which provided further confirmation of Greek reports on the nature of such mathematics. In particular, the inhabitants of Mesopotamia had invented a place value number system, knew different methods of solving quadratic equations which would only be improved upon in the 16th century A.D., and understood (but not proved) the relationship between sides of a right-angled triangle which came to be known as the so-called Pythagorean theorem.

The neglect of the Arab contribution to the development of European intellectual life in general and mathematics in particular is another serious draw-back of the Eurocentric trajectory contained in Figure I. The course of European cultural history and the history of European thought are inseparably tied up with the activities of Arab scholars during the Middle Ages on account of their seminal contributions in mathematics, natural sciences, medicine, and philosophy. In particular, we owe to the Arabs in the field of mathematics the bringing together of the technique of measurement, evolved from its Egyptian roots to the final form in the hands of the Alexandrians, with the remarkable instrument of computation (our number system) which originated in India, and finally supplementing these strands with a systematic and consistent language of calculation which came to be known by its Arabic name, *algebra*.

Finally, in talking about the Greek contribution, there is a need to recognize the differences between the "classical" period of Greek civilization (i.e., from about 600 to 300 B.C) and the post-Alexandrian period (i.e., from about 300 B.C. to 400 A.D.). In early Eurocentric scholarship, the Greeks of the ancient world were perceived as an ethnically homogeneous group, originating from areas which were mainly within the geographical boundaries of present day Greece. It was part of the Eurocentric mythology that from the mainland of Europe had emerged a group of people who had created virtually out of nothing the most impressive of all civilizations of ancient times. And from the civilization had sprung not only the cherished institutions of the present day Western culture but also the mainspring of modern science. The reality was, however, different.

Before the appearance of Alexander (356–323 B.C.), the term "Greek" was applied to a number of independent city states, often at war with one another, but exhibiting close ethnic or cultural affinities, and above all sharing a common language. The conquests of Alexander changed the situation dramatically, for at his death his Empire was divided among his generals who established separate dynasties. The two notable dynasties from the point of view of mathematics were the Ptolemaic dynasty of Egypt and the Seleucid dynasty which ruled over lands that included the earlier sites of the Mesopotamian civilization. The most famous center of learning and trade became Alexandria in Egypt, established in 332 B.C. and named after the Conqueror. From its foundation, one of its most striking features was its cosmopolitanism—part Egyptian, part Greek with a liberal sprinkling of Jews, Persians, Phoenicians, and Babylonians, and even attracting scholars and traders from as far away as India. A lively contact was maintained with the Seleucid dynasty. It thus became the meeting place for ideas and different traditions, and over the period the character of Greek mathematics changed slowly, mainly as a result of the continuing cross-fertilization between different mathematical traditions, notably the algebraic and empirical basis of Babylonian and Egyptian mathematics interacting with the geometric and anti-empirical traditions of Greek mathematics. And from this mixture came some of the greatest mathematicians of antiquity, including Archimedes and Diophantus.

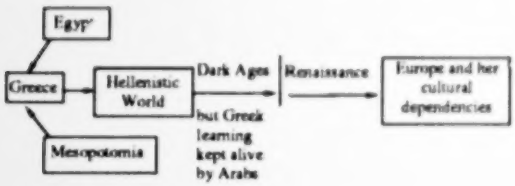
Figure II takes on board some of the objections raised about the "classical" Eurocentric trajectory. There is some awareness of the existence of mathematics before the Greeks and their debt to earlier mathematical traditions, notably those of Mesopotamia and Egypt. But this awareness is often likely to be tempered by a dismissive rejection of their importance compared to Greek mathematics—"(the) scrawling of children just learning to write as opposed to great literature" (Kline, 1962, p. 14).

The differences in character of the Greek contribution before and after Alexander is also recognized to a limited extent in Figure II by a separation of Greece from the Hellenistic world (where the Ptolemaic and Seleucid dynasties were the crucial instruments of mathematical creation for that period). There is also some acknowledgment of the Arabs, but mainly as *custodians* of Greek learning during the so-called Dark Ages in Europe. Their role as transmitters and creators of knowledge is ignored. So are the contributions of other civilizations—notably those of China and In-

dia—which are perceived either as borrowers from Greek sources, or as having made only minor contributions that play an insignificant role in mainstream mathematical development (i.e., development eventually culminating in European mathematics).

Figure II, therefore, remains a flawed representation of how mathematics developed over time. It contains a series of biases and remains relatively impervious to new evidence and arguments. With minor modifications, it presents the model to which many books on the history of mathematics conform. It is interesting to note that similar Europe-centered bias exists in other disciplines as well. For example, diffusion theories in anthropology and social geography indicate that "civilization" spreads from the Center (i.e., "Greater" Europe) to the Periphery (i.e., to the rest of the world). Again, the theories of modernization or evolutionary schemes developed within the framework of certain brands of Marxism are characterized by a similar type of Eurocentrism. In all such conceptual schemes, the development of Europe is seen to serve as a precedent for the way in which the rest of the world will develop in the future—a trajectory whose spirit is not dissimilar to the one suggested in Figures I and II.

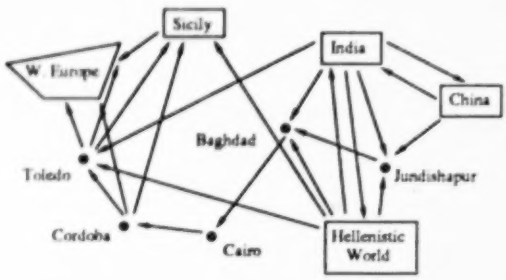
Figure II: The "Modified" Eurocentric Trajectory



In offering alternative trajectories to Figures I and II, a guiding principle should be the recognition that different cultures in different periods of history have contributed to the world stock of mathematical knowledge. Figure III presents such a trajectory of mathematical development but confines itself to the period between the 5th and 15th centuries A.D., i.e., the period represented by the arrow labelled in Figures I and II as the "Dark Ages" in Europe. The choice of this trajectory as an illustration is deliberate. It serves to highlight the sheer variety of mathematical activity and exchange taking place in a number of cultural areas when Europe was in deep slumber. A trajectory constructed after the 15th century would by the same token show that transmissions and mathematical creativity were more or less confined to countries within Europe until the emergence of the truly international character of modern mathematics during the 20th century.

The role of the Arabs is highlighted in Figure III. Scientific knowledge which originated in India, China, and the Hellenistic world was sought out by the Arab scholars and then translated, refined, synthesized, and augmented at different centers of learning, starting at Jundishapur, in Persia around the 6th century (even before the coming of Islam), and then moving to Baghdad, Cairo, and finally to Toledo and Cordoba in Spain from where this knowledge spread into Western Europe. Considerable resources were made available to the scholars through the benevolent patronage of the caliphs of *Abbasid* (i.e., the rulers of the eastern Arab empire with its capital at Baghdad) and of *Ummayad* (i.e., the rulers of the western Arab empire with its capital first at Damascus, then moving briefly to Cairo, and finally to Cordoba).

Figure III: An Alternative Trajectory (from 8th to 14th Century)



The role of the Abbasid Caliphate was particularly important for the future development of mathematics. The Caliphs, notably al-Mansur (754–775 A.D.), Harun al-Rashid (786–806 A.D.), and al-Mamun (813–833 A.D.), were in the forefront of promoting the study of astronomy and mathematics in Baghdad. Indian scientists were invited to Baghdad. At the closure of Plato's Academy in 529 A.D., some of its scholars found refuge in Jundishapur, which a century later became part of the Arab world. Greek manuscripts from the Byzantine Empire, the translations of the Syriac schools of Antioch and Damascus and the remains of the Alexandrian library in the hands of the Nestorian Christians at Edessa were all sought out eagerly by Arab scholars, aided and abetted by the rulers who had control over or access to men and materials from the Byzantine Empire, Persia, Egypt, Mesopotamia, and places as far east as India and China.

Caliph al-Mansur built at Baghdad a *Bait al-Hikma* (House of Wisdom) which contained a large library to stock the manuscripts that had been collected from various sources, an observatory which became a meeting place of Indian, Babylonian, Hellenistic, and Chinese astronomical traditions, and a university where scientific research continued apace. A notable member of the institution, Mohamed ibn-Musa al-Khwarizmi (ca. 825 A.D.) wrote two books which were of crucial importance to the future development of mathematics. One of the books, the Arabic text of which is extant, is entitled *Hisab al-djabr wa'l Muqabalah* which gave the name to an important part of mathematics. Al-Khwarizmi wrote a second book, of which only a Latin translation is extant. *Algorithmi de Numero Indorum* explained the Indian numeral system. While al-Khwarizmi was at pains to point out the Indian origin of the numeration system, subsequent translations of the book attributed not only

the book but the numerals to the author. Hence, in Europe any scheme using these numerals came to be known as algorism or later algorithm (i.e., a corruption of the name al-Khwarizmi), and the numerals became Arabic numerals.

Figure III shows the importance of two areas of Southern Europe in the transmission of mathematical knowledge to Western Europe. Spain and Sicily were the nearest points of contact with Arab science and had been under Arab hegemony, with Cordoba succeeding Cairo as the capital and center of learning of the Ummayid Caliphate during the 9th and 10th centuries. Scholars from different parts of Western Europe congregated in Cordoba and Toledo in search of both ancient and contemporary knowledge. As an illustration of this great thirst for knowledge, it is reported that Gherardo of Cremona (1114–1187) went to Toledo, after its recapture by the Christians, in search of Ptolemy's Almagest, an astronomical work of great importance produced in Alexandria during the 2nd century A.D. He was so taken by the intellectual activity there that he stayed there for 20 years, during which he was reported to have copied or translated 80 manuscripts of Arab science or Greek classics, which were later disseminated across Western Europe. Gherardo was one of a number of European scholars, including Plato of Tivoli, Adelard of Bath, and Robert of Chester, who flocked to Spain in search of knowledge.

There are other features that Figure III serves to highlight. It is not generally recognized that many of the topics taught in schools today are directly derived from the work of mathematicians from outside Western Europe before the 15th century. The failure to do so is both a reflection of the general Euro-centered attitude to the school curriculum and the general neglect of the history of mathematics from a mathematician's education.

But the main message of Figure III is the danger of characterizing mathematical development solely in terms of the Eurocentric scenario. The darkness that was supposed to have descended over Europe for a thousand years before the illumination that came with the Renaissance did not interrupt mathematical activity elsewhere. Indeed, the period marked not only a mathematical renaissance in the Arab world but were high points of Indian and Chinese mathematics. The fallacy in short lies in the tacit equating of global mathematics with European mathematics.

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Mathematics in Rural Societies

Sam O. Ale

Mathematics exists in rural communities, indeed they use it every day. A simple characteristic of the mathematics is that it is oral and not written. One problem however is that no one is investigating and developing the mathematics the rural communities use. We briefly outline here, the mathematics the rural societies use. Considering their societal needs, it is pertinent that an appropriate curriculum be drawn for rural communities. Such a curriculum must be based on the socio-cultural consideration of the communities. Special reference is made to the nomads as a rural society in Nigeria.

Bochner (1966) rightly commented that before 1600 A.D., there was an Egyptian mathematics, a Greek mathematics, an Indian mathematics, a Chinese mathematics. Indeed, one can extend it that every community has one form of mathematics. Even, during the stone age, there was a form of mathematics and before the so-called dark ages, there was some mathematics. There has been an erroneous view that mathematics is an activity of the Western culture. Certainly some form of mathematics exists in every culture. Some cultures have developed their knowledge very fast, some are underdeveloped, and to us today, the later group appears to have a knowledge that is elementary. However, this elementary knowledge can be built upon and made culturally acceptable to their society.

Every community needs mathematics especially if we consider mathematics as a unique art and science that enables man to cope with the complexity of economic, social, and technical problems of our world. We note and as Burghes (1988) rightly said "mathematics is playing an ever increasing role in today's society." The role of mathematics is not limited to the Western culture but to every society in all parts of the world. Indeed this is most evident in the mathematics the rural societies use.

From our own research findings, the present need of many rural societies is a program of numeracy that will enable them not only to translate their oral mathematics to a written one but also more importantly to cope with the real-life problems that they encounter daily. Even in the Western culture today there is the orientation to teach mathematics through real-life problems, so that pupils can cope with practical problems of life (see Ernest, 1988; The Spode Group, 1983). The background to the mathematics that the rural societies use are based on the real-life problems they are often faced with. Therefore any mathematics curriculum to be drawn for rural communities must continue to be based on these problems and their vocation. For example, the mathematics curriculum that will be acceptable to the nomadic Fulani of West Africa must be centered on their

vocation which is cattle-rearing. Such a mathematics program will not only be acceptable to them but will also be relevant.

Rural Mathematics

We have emphasized that there is a form of mathematics already existing in rural societies. This we refer to here as rural mathematics. This rural mathematics is oral and unwritten and as Carraher (1988) described the street mathematics of Brazil as oral both in the sense that it is spoken and in the sense that it is used for communication so also is the case for the rural mathematics that is also used for communication. There are other characteristics of this rural mathematics. It is generally a tool for solving real-life problems. We consider the following practical problem a young nomad often faces.

Problem

A young nomadic man wants to get married, he wants to pay his dowry before having his bride. He could not get cash, so instead he wants to give cows for the dowry. To give three cows is too much and to give two is too little. What does he do?

He solves this problem by giving two cows which he values at 600 naira (naira is the unit of Nigerian money). To pay the remaining 100 naira, he works ten days for the father-in-law estimating his daily labor at 10 naira a day.

Here we see that some simple mathematics has been carried out, though oral, he actually got the perfect solution to solve the real-life problem. This example also brings out another characteristic of rural mathematics, which, using the words of Carraher (1988), is "conducive to the development of problem-solving strategies which reveal a representation of the problem situation." The strategy used above is convenient and perfectly acceptable in the community. Though the mathematics is unwritten it is perfectly correct. The strategy is also an ethical solution. This also leads us to the fact that rural mathematics agrees with the ethics of the community. Certainly ethical consideration should be considered in the solution of mathematical problems. To simplify this notion, we would say that mathematical statements or solutions should be reasonable. For example we say a man walks at 4 km per hour, but not at 80 km per hour. This may be an approximation but the approximation, is reasonable. There are many such approximations in rural mathematics. For example, the notion of distance in some community is equivalent to periods a journey takés. So we hear of the distance of two villages as a day's journey. Also the notion of time is crude and is also an approximation. The sun is used as reference point. Since the rural societies under consideration in this paper live around the equator, the variation in sunrise and sunset throughout the year is not too diverse as, say, in Europe. Hence, the sun could be used for timing.

Further Examples of Rural Mathematics

It is most interesting to note that some rural communities have their symbolic representations, which have not yet being explored. They use these symbols to represent numbers which are primarily used to show their possessions. The symbols were similar to the Roman numbers. For example:

100 is represented by two short sticks placed in the following form V
50 is represented by two sticks placed in the form X
3 is represented by three sticks III.

Hence, to represent 153 it is placed consecutively as VXIII. Other representations are as follows:

6 by IIIIII
15 by – IIIII (this implies 10 is –)
32 by – – – II
54 by XIIII.

Thus, a man can show through earth moulding or placing of sticks in front of his house the number of cows or goats he possesses. For example, the following was found in the house of a rich cattle owner: VVVVVVXII showing that he had 652 cows.

Unfortunately, the symbols used by this community have not been thoroughly investigated by mathematics educators.

In most rural societies, one basic method for performing arithmetic operations is through repeated groupings. We illustrate this with the following simple examples found in rural mathematics:

(1) The head of a family was asked to pay 20 naira each for himself and his three sons. To do this, he took 20 naira into four different places, then he gathered them together to count before knowing that the total required from him was 80 naira.
(2) A man had 120 oranges to be shared among the six members of his family. He did not know immediately that each person would take 20 each if they were to share it equally. What he did was to ask each person to take five each at the first instance, and then come back for another five. When he noticed that the number of oranges were getting smaller, he asked that they should take one at a time. It was when everything had been exhausted that each person counted his or her own before they knew that they had 20 each.
(3) A village market woman was selling 80 oranges for 20 naira; a man wanted to buy 60 of the oranges. She could not tell immediately what the price of the 60 oranges would be. She went about solving the problem in this manner: She separated them into two equal parts. Then she took the half which is 40 and separated them into two halves again. She arrived at the cost in this way. Forty oranges costed 10 naira and 20 oranges costed 5 naira. Therefore, 60 oranges costed 15 naira.

The oral solution is correct and mathematically sound. The repeated grouping gives a solution. However, it often takes some time for rural dwellers to solve easy mathematical problems. In fact, it is often difficult for

them to solve some fairly complicated arithmetical problems. But quite often the need to solve complicated problems does not always arise.

One may ask if the rural mathematics does not go beyond simple arithmetic operations. To a certain extent, this is principally the mathematics they need. However, some of their mathematics goes beyond arithmetic operations although it may not appear obvious. For example, there are elements of statistics, inequality, sets, probability, geometry, and basic algebra in rural mathematics. It is often most interesting to see the geometrical constructions they make; for example, in the construction of huts, the weaving of local clothes, and the making of farm ridges. One may describe their geometry as experimental.

Also one may describe the rural games as being based on the theory of probability. Thus, a good player will be able to solve probability problems better. We discovered that practically all the games they play can be connected to one form of mathematics. And the problem-solving strategies which are oral can be very intriguing, although most calculations often go from hundreds to tens to units.

The Need for Relevant Mathematics Curriculum for Rural Societies

Most of the mathematics learnt in primary school is somehow relevant to rural dwellers; in other words, topics like Basic Numeracy, Percentages, Weighing, Measuring, Money, Odds, etc. However, the way it is being taught in schools will not be appropriate to rural societies. For example, a rural dweller has no need of operation of the form

$282 - 17$

$(15 \times 5) + (38 - 7)$

$115 + 20 - 8 + 27$

$18\ \text{kg} + 5\ \text{kg} - 3\ \text{kg}$.

However, he needs mathematical problems like:

(1) If 282 cows went grazing and 70 got lost, how many are left and how does he trace the ones that are lost?
(2) If in a settlement, 15 bags of fertilizer are to be shared equally, how much does each get?
(3) What path does the herdsman and his herd take to get to their destination faster?
(4) Will it be cheaper to transport a herd of cows by lorry than allowing them to walk the distance?
(5) How much grazing land will be sufficient for a herd of cattle to graze on for a period of one month?

These types of problems are most relevant to the cow Fulanis than the straight mathematics often taught in schools.

The Nigerian government has a program of adult literacy for rural dwellers. However, the case is often that the literacy programs are devoid of numeracy contents. A good literacy program must include numeracy. The latter will enable the rural dwellers function when they come to cities to sell their products.

Societal Needs of Rural Dwellers

The basic problems of rural dwellers surround health, education, and transportation. These may in a glance appear not to be mathematical problems, but a little consideration will show the everyday questions coming from these problems are mathematical. For example, questions like how many, how far, how big, what time, what directions are certainly mathematical questions. A non-numerate society cannot contribute maximally to the economy of the nation. Hence, a sound numeracy program is most necessary for rural societies. The rural numeracy program must contain the following mathematical contents:

Arithmetic. The traditional contents of numbers would be included, e.g., natural numbers, relative integers, etc. However, the contents must be put in the social context of the rural dwellers.

Geometry and Measures. Geometrical shapes, standard measures. This study must be related tot he physical and economic reality of the rural community and, in particular, with the daily pictorial experiences of their environment.

Statistics. Elementary statistical data, especially of figures and diagrams and as found in the dailies. Understanding of economic data as often presented in the dailies.

Logic. Problems of classification, the use of graphs and diagrams, study of sequences, and simple introduction to ideas of sets.

Probability. In the context of the common games available in the locality.

Conclusion

The mathematics existing in rural communities is a basis to which to build upon in any mathematics program to be drawn up for them. They need further instructions in mathematics, but such instructions must be relevant to them. Certainly, there are problems to face in any attempt to promote mathematics education in rural societies. However, with good educational strategies based upon their way of thinking and manipulation, a numeracy program or mathematics education in general will succeed in rural communities.

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The Cultural History of Mathematics as a Basis for Philosophies of Mathematics Education

Leo Rogers

Introduction

Twenty years ago, Davis (1967) put forward the idea of a "teacher-belief system," a rag-bag of rules of procedure, principles of subject matter and pedagogy, inherited prejudices and ideologies, etc. upon which most of an individual teacher's classroom practice was based.

Elsewhere (Rogers, 1976), I have attempted to examine this idea in the context of the training of teachers, from the point of view of how a knowledge of the history of mathematics can help to turn these ideologies into a critical philosophy which may suggest some more reliable methodological approaches to the teaching of mathematics.

This paper, based on the conviction that mathematics can have an empirical foundation, and on the idea of an ethnographic methodology, is a contribution to the current dialectic.[1]

Mathematical Culture

The idea of mathematical culture seems to have entered the literature with a paper by L. A. White (1947) where he talks about the 'locus of mathematical reality' and puts forward two propositions:

(a) Mathematical truths have an existence and validity independent of the 'human mind' (I).
(b) Mathematical truths have no existence or validity apart from the 'human mind' (II).

These apparently contradictory statements are explained by pointing out the two meanings where:

'human mind' (I) refers to an individual person (singular)

and 'human mind' (II) refers to the human species or cultural group (plural).

White suggests that mathematical truths enter the mind of each individual person from 'outside'; they have no existence or meaning apart from the cultural tradition in which they are found, and that mathematics is the cumulative product of ages of endeavor.

Wilder took up these ideas, and in a series of writings elaborated the 'concept of culture' applied to mathematics, its teaching, traditions, current development, and historical evolution. All these ideas were finally brought together in his book *Mathematics as a cultural system* (1981).

The broad framework having been constructed, new studies in "ethnomathematics" appeared. Researchers look at "native" mathematics, or the context of mathematics, its uses, learning, and teaching in other cultures.

The Problem of Identification

One of the significant problems we have to tackle here is that we view other cultures—inevitably—from our own standpoint.

How do we recognize the mathematics of another culture? One answer, provided by Ascher and Ascher (1986), is for the investigators to be "as children"; to be prepared to be instructed by the members of another culture, in a similar way to one of their own children growing up in that culture. But what we see may not be called mathematics nor even remotely recognizable as such.

We have to look in other places and learn to recognize "proto-mathematics" (the activities that may lead to mathematics) in areas outside that which we now admit as "mathematical" (Rogers, 1976, section 4b).

So we may now ask: Are there some aspects of mathematics by which we may recognize it in another culture, perhaps as

(a) an activity: the individual engaging in a creative act ("I am doing mathematics"),
(b) a process: the observer reacting to the activity of the group or individual ("look, here mathematics is being done"),
(c) a product: the artefacts, writings, or learnt skills residing in an individual or group ("this is the result of doing some mathematics").

Another concept widely used to characterize human activity in the anthropological sense is the idea of the symbol. Applied to mathematics, this widens out the usually accepted mathematical meaning into something which is at the same time powerful and confusing. A language, in its broadest sense, is a developable symbol system. So also is mathematics. But the human activities of making, choosing, and using symbols depend on our reactions to external stimuli and to our cultural dispositions.

How do we characterize mathematics? As mathematics develops to deal with even wider generalizations, more complex relations, and more abstract concepts, we have another problem: While there is more likelihood of a broad characterization being open to a variety of interpretations and able to adapt itself to new and unforeseen circumstances, we may at the same time be claiming that *some* aspects of *all* human

[1] Dialectical thinking signifies (normatively) conceptual flexibility and novelty, and (descriptively) processes of conceptual transformation and change. (Macmillan Dictionary of the History of Science, 1981)

activity are "mathematical." What criteria do we have for deciding where, say "basket-weaving" stops, and "mathematics" begins?

White (1947) claims that mathematics is a form of behavior—a response to external stimuli in the culture, and Eisenhart (1988) describing ethnography gives us the idea that all human activity is fundamentally a social and meaning-making experience. Wilder has developed this idea and views mathematics as a culture held by mathematicians and discusses the ways in which identifiable factors develop or stress this culture. But which meaning-making experiences can be identified as mathematics? Can only people with "inside knowledge" do this and then only by "hindsight"?

White's original claims (above) put us in an ambivalent position. The body of knowledge, "mathematics" is a culture created and developed by individuals, existing apart from any individual and at the same time possessed by mathematicians, children, teachers, and others to a greater or lesser degree.

Ethnographic Methodology

The anthropological approach means that the researcher becomes a

> participant-observer ... able to step back from the immediate scenes of activity and reflect on what is occurring from the perspective of someone aware of other systems and of theoretical perspectives on sociocultural systems. (Eisenhart, 1988, p. 105)

Thus, the interpretation of ritual and customs as a kind of "proto-mathematics" and of inscriptions or prehistoric artefacts, take on another status when regarded from the ethnomathematical point of view.

The task of the historian of mathematics is to endeavor, working in the most sensitive way we can, to provide the most accurate account, the most plausible paradigm, or even the 'preferred myth.' Provided, like the ethnographer, we are aware of at least some alternatives, we do the best we can, and we come to realize that some myths may work better than others, for pedagogical purposes.[2]

Interpreting History of Mathematics

Bos (1984) provides a number of examples to show us how the social issues in history can influence mathematics and contribute to curriculum and pedagogy, anticipating and supporting Bishop's views on the social construction of meaning.

The more this kind of approach to historical problems accumulates its evidence, the harder it is to ignore. Each piece of evidence puts another nail in the coffin of Platonism.

Yet, while many teachers are fully aware of the social context of the classroom, with all its interactive learning implications, with regard to mathematics, they still act as if there were a body of immutable truths out there to be transmitted.

Routes to Philosophies of Mathematics Teaching

The fundamental statement of belief expressed in the introduction to *Notes on mathematics in primary schools* (A.T.M., 1967) that mathematics is a social, creative activity, little different in kind, between the research mathematician and the child learning mathematics, already recognized some fundamental problems of epistemology which have been made clearer since then by the 'Lakatosian Revolution.'[3] Accounts of how 'real' science actually develops have been influenced by other writers, and Hersh (1979) gives us some proposals for reviving the philosophy of mathematics, arguing forcefully that we can no longer hold a world view which separates the individual on one hand, and the exterior world on the other.

> The existence of mathematics is enough to show the inadequacy of such a world view. The customs, traditions, and institutions of our society—all our non-mathematical culture—are aspects of the world which are neither in the private 'inner' nor the non-human 'outer' world. Mathematics is also this third kind of reality—a reality that is 'inner' from the viewpoint of society as a whole, yet 'outer' from the viewpoint of each individual member of society. (Hersh, 1979, p. 40)

My own convictions, derived from studies in the history of mathematics, and held for some time, are that all mathematics essentially originates from human 'real-world' problems by long historical-cultural processes. It has been difficult until recently to find enough sufficiently detailed support for this view. Lakatos' original simulation of the classroom dialog also showed us that many of the actual examples used in the historical debate were taken from 'real-world' crystallography and other physical experiments.

Since then, new attitudes to the historiography of mathematics, and detailed expositions of formative periods in the history of mathematics over the last 20 years or so, have given us accounts which clearly feature the human personal and social aspects of the ideas involved.[4]

Kitcher (1984) also provides us with some insightful case studies and an accumulation of evidence that reinforces these convictions and leads to the conclusion that

> ... proto-mathematical knowledge can be obtained by manipulating the world and observing the manipulations. From these

[2]For example, stories for children about the evolution of numbers or measures, and the acting-out of moments of discovery are powerful media for conveying the drama of science and the personal involvement of the people concerned. (The "Molecule Club" is a theater group in England which specializes in dramatizing scientific events and ideas for children.)

[3]Copies of Lakatos' original (1963/64) publication in the British Journal for the Philosophy of Science were circulating amongst teachers long before the book (1976) was published. For a discusssion of the far-reaching effects this has had, see Wolfson (1981).

[4]For example, a considerable amount of material on the evolution of mathematical physics from the 17th to the 19th century is now readily available.

humble beginnings mathematical knowledge develops into the impressive corpus of contemporary theory. (Kitcher, 1984, p. 148)

From this and other evidence we conclude that mathematics is grounded in an empiricist epistemology, because all mathematics has developed from human problems and situations. "Higher" mathematics can be demonstrated to have developed from "rudimentary" mathematics which can be perceptually warranted, relying on the reactions of individuals and groups to stimuli in the real world and to each other. The construction of mathematical knowledge is socially based, and the types of changes in the evolution that led to current practice show us a very different product from the situation that may have prevailed where the evolutionary sequence started.

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The Image of Mathematics in Society

Mathematics is widely considered to be independent of social, institutional, and moral implications and consequences. This image is analyzed from different viewpoints. Gilah C. Leder, Australia, reports a case study, in which she tries to identify, quantify, and operationalize the society's image of mathematics by examining the perceptions held about mathematics by a group of tertiary educators without special expertise in mathematics. Stephen Lerman, United Kingdom, shows that despite the apparent proliferation of philosophies of mathematics (as discussed by mathematicians, too) schools of thought fall into two streams, the absolutist and the fallibillist; i.e., search for certainty of mathematical knowledge versus viewing mathematics as a social invention. Some implications for mathematics education are discussed. Chandler Davis, Canada, renews the discussion of the responsibility of scientists (mathematicians) for the uses to which society puts the results of their research, and asks for "A Hippocratic Oath for Mathematicians!"

The Image of Mathematics in Society: A Case Study

Gilah C. Leder

There are various ways in which society's image of mathematics can be quantified and operationalized. This paper examines the perceptions held about mathematics by a group of competent and highly qualified education faculty members, most of whom had no special expertise in mathematics. However, their familiarity with general learning and teaching principles enabled them to give particularly useful and constructive feedback when they themselves became learners of mathematics. Their views and feelings about mathematics reflect those of a broader cross section of society.

A series of four lectures, specially prepared for this purpose, was organized to ensure commonality of experience and help focus the discussion. Two experienced tertiary educators, one a pure, the other an applied mathematician, were invited to act as instructors. They selected their own material (Pythagoras Theorem and matrix applications to digraphs), which they were encouraged to present in as clear and logical a manner as possible. The learners' task was not only to attend carefully to the material being taught by, e.g., taking notes and asking further questions, but they were also asked to respond in a margin left on each page for this purpose to the two questions: "What is making this subject difficult for me?", "What can I, or the instructor, do to make it come clear?"

All sessions were videotaped and subsequently transcribed. Participants' own words, taken from the transcript, illustrate vividly their views about mathematics. (For a discussion of other findings see Leder, 1987; Leder, Jones, Paget, & Stillwell, 1987.)

Scene: Some 45 minutes into Lecture 1. The instructor has finished showing, to his satisfaction, that $\sqrt{2}$ cannot be rational.

Cast: 21 education faculty members including: Lesley, Shirley, Dianne, Peter, Gordon, Monica, and Chris (non-mathematicians); John and David (who have a degree in mathematics).

Lesley: Could I ask what you feel you've taught us so far? If you walked out into the corridor and a colleague said, "Oh, what did you do in math today?" What would you say? "I told them about what?"

Instructor (I): Irrational and rational members and we discussed it. I tried to get across the idea that $\sqrt{2}$ and Π were examples of these. I tried to prove that $\sqrt{2}$ is an irrational number.

Shirley: Root 2 is an irrational number?

I: Yes.

Lesley: See, I would not have known that.

Shirley: Well I can remember that a rational number is a fraction because that's something new I learnt today—Okay, so now I know because you've actually said it in words.

Dianne: If you do know all this, what use is it to you?

I: Let's be honest—The purpose of this exercise is to have some experience in learning mathematics. Now probably motivation is important in maths, but the first thing is to have the experience, and that's what I hope you're having. You're experiencing difficulties but that's typical because mathematics is difficult. Most of the time mathematicians are trying to solve problems and not succeeding. I don't want to rub in that aspect of it. I'm trying to give you some success, but it is fairly typical to have difficulties learning mathematics.

Peter: If someone is having difficulties, what could they do to overcome them? Do they just drop out and say maths is not for me?

I: They would have to discuss it with their teacher.

Peter: With you?

I: With me in this case. The teacher's responsibility would be to analyze where the difficulties lay and to try and remedy them.

Lesley: Can I ask, did you deliberately use more words rather than symbols in your instruction than you would with your mathematics students over in your department?

I: No, I didn't, but then I use more words than most mathematicians.

Lesley: Because I appreciate that—I want words. The fact that you write words down as well as numbers was a great help to me.

I: No, that's the way I always work because I'm not very good with symbols myself. I prefer words and I prefer geometry. I hope we can do some more geometry next session.

Shirley: Don't use that word. You don't know what visions that brings up!

Lesley: Well, I'm happy with geometry. Geometry is tangible. Geometry are figures that we can touch. I'd be happy with geometry.

Gordon: As a non-mathematician ... From a pedagogical point of view, it would seem to me that breaking the explanation into infinitely smaller steps all the time might be a way of making the link from one step to another when someone gets lost. Isn't the knack to try to break it down by the insertion of a sort of substep or whatever?

I: Yes. To find out what they've missed and put it in.

Gordon: The one thing I wrote down was that there are certain things being presupposed all the time. Definitions. The point where you introduced m/n to talk about $\sqrt{2}$. Whole numbers. Now you're presupposing an understanding of some concept of whole numbers.

I: Yes.

Gordon: There needs to be an explanation there about
- what it means and
- why we are utilizing it.

I: Yes.

Monica: All the same, why, when you put $n = 2p$ what on earth is p? I mean, later on I understood. Well, I still don't know if it's legitimate. I suppose I accept that it's okay to do it.

I: It's just another way of saying that m is even ... We can manipulate these symbols much more easily than words. I could have said all that in words. I could have said m is two times something. I could have said two something by something.

Monica and Lesley: Yes, that would have helped.

John: In a sense though you're putting into algebraic form what was written in words. Writing m is even, is the same as writing $m = 2p$. It's an identical statement ... But you've put it in a form that is manipulable algebraically.

Gordon: But the kid having trouble with maths wouldn't see it that way, would he? He wouldn't see that the two statements are the same?

Monica: I didn't see it.

John: No, the student having trouble wouldn't see it. Because that's the very nature of his trouble.

Gordon: Well, how do you get round that?

Shirley: The nature of his trouble could be as in my case that you're still questioning some previous statement. Because I'm still sitting here wondering about $c = \sqrt{2}$ long after you started you explanation.

John: Because you haven't learnt the language.

Lesley: Or you're feeling anxious when the figures start coming so your mind isn't able to concentrate as it should because there are emotional things happening and you begin to panic.

Chris: Yes. And my technique was always to write things down and then think "I'll work this out later."

David: You talk about a kid in trouble. What sort of kid did you have in mind, Gordon?

Gordon: Well. Myself, in grade 9 when we started doing logarithms.

David: You're not proposing you'd explain all this in grade 9 are you?

Lesley: Well, they do things like this in grade 9.

David: The whole point is the great debate: How much do you explain? I mean there is a difference between rote learning and understanding ... You can write Pythagoras' theorem and prove it. And you can prove it trigonometrically, or by cutting up paper or whatever, and show a few Pythagorean triples ... But you don't need to know any of that to *use* Pythagoras' theorem. The question is: How much do you explain to children?

Monica: But even when you're explaining it to people like me, you say "That's obvious, that n itself is even, is the same as $n = 2p$." But it's not obvious. I didn't know that.

John: That seems to be important learning to be done. How do you translate English sentences into mathematical sentences? Once you've learnt it, it's obvious. It's obvious to me ... As soon as I hear someone say m is even, I think of it as $m = 2p$.

Shirley: But if you taught kids little tricks like that, so they could convert things that they were saying into things like $m = 2p$ they'd love to be able to do that just like they like using invisible words and symbols between themselves. Wouldn't they?

The messages conveyed by this excerpt are largely negative and pessimistic. Mathematics is difficult. It is anxiety-provoking. It produces two classes of citizens: those who acquire key concepts, special terminology, and master the rules of the game, and those who never quite learn to play and who are still struggling with step 1 when the instructor has finished with the final step of the explanation. Since mathematics teachers tend to belong to the former group they all too often alienate even willing students whose difficulties they fail to grasp. Yet, must it be so? It seems appropriate to let one of the instructors have the final word:

"This group was, of course, far more responsive, critical and articulate than a normal student group. Even so, I was impressed with the way non-mathematicians were able to recognize the real difficulties and get to the bottom of them by intelligent questioning. This seems to imply that the number of people who can understand and enjoy mathematics is far greater than the number who do at present."

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A Social View of Mathematics —Implications for Mathematics Education

Stephen Lerman

In recent literature one increasingly finds the proposal that we take a more social view of mathematics, but the intention can vary considerably. On the one hand, it can mean a recognition of the social nature of teacher/pupil interaction and the significance of the social context for mathematics education, perhaps the last school subject to concern itself with anything other than content and the manner of its presentation. On the other hand, it can be a recognition of the invasion of the mathematics classroom by controversial issues. In Britain recently, the Prime Minister complained that children in our schools are learning anti-racist mathematics instead of arithmetic. In another instance, the conservative press complained about a public examination question that contained several parts asking pupils to read from a graph of arms expenditure by Nato and the Warsaw Pact. It ended with a question concerning the number of weeks of arms expenditure that would be required to feed the starving peoples of the world. It is obvious which part of the question was considered objectionable.

The intention of this paper is to propose that there are distinct consequences of a social view of mathematical knowledge and to briefly present two examples. In order to do this I will first indicate my use of the notion of a social view of mathematics. I take this to apply to mathematical knowledge itself, in that the history of mathematics is not one of the gradual revelation of absolute truths, but, as with all knowledge,

the consequence of people's ideas, interests, conflicts, and patronage, and is culturally and temporally relative. Mathematical knowledge is a social construction, the meaning of a concept such as 'polyhedron,' for example, following Lakatos, is negotiated and adapted according to convention and agreement, through proofs as explanations, leading to basic refutable statements. It is not the case that there exists, in some universal sense, a concept called 'polyhedron,' which merely needs discovery and explication. This equally applies to notions of proof, truth, and rigor, by which we justify particular areas of mathematics. Consequently, there is no natural or logical necessity to the state of mathematical knowledge at present. Undoubtedly, we have a body of mathematical knowledge, that generally works, but it is in the nature of a library of accumulated experience, rather than universal truths. In any case, the 'it' refers to that collection of books on the shelves of the particular libraries that we frequent. Bishop, D'Ambrosio, Gerdes, and others highlight our culturally restricted view of mathematical concepts. I suggest that the consequences of a social view of mathematical knowledge itself are far-reaching, including:

(1) that there are alternative mathematical concepts, the direction of mathematical development is not a necessary one;
(2) that, since mathematical truths have always been taken as the paradigm of true propositions in philosophy in general, taking this last bastion of certainty as itself relative is quite fundamental to our whole notion of knowledge;
(3) that there is a full sociology of knowledge, dealing symmetrically and impartially with 'true' mathematics as well as 'false';
(4) that the world 'out there,' including the mathematical, is unknowable in any universal *a priori* sense. This notion was proposed by the radical constructivists at the last PME conference in Montreal, and it is important to recognize that this is a central problem for philosophy today as well as for mathematics education.

Taking this alternative view of mathematics, there are many possible consequences for teaching styles, curriculum development, etc., and I have described these elsewhere (Lerman, 1983; 1986; 1987). I will develop here just two illustrations of implications for mathematics education, namely, political and social education through mathematics and the notion of ability.

Firstly, I suggest that teachers of mathematics can no longer sit in the school staffroom, believing that values enter every classroom except the mathematics one, and this not simply and solely because of arguments such as that education is everyone's responsibility. Since mathematics is as much a social construction as any other form of knowledge, it is culture-bound and value-laden. A strong case can be made for characterizing mathematical values as sexist, for instance. Further, sociological analyses such as those of Freire, Apple, and others propose that knowledge is power, i.e., that different conceptions of knowledge reflect different forms of social relationships and control. Freire, for instance, describes the 'banking concept' of education, whereby students' activities are restricted to storing, filing, and retrieving, as against the 'problem-posing' concept, whereby people see themselves as owning the mathematics and empowered to both pose questions and propose solutions. This latter notion resonates strongly with the ideas of Stephen Brown and others in mathematics education.

Mathematics indeed serves a central function as a tool of government and power groups, since it is used to justify all sorts of policies and decisions, including closing coal mines, fighting inflation rather than poverty, under-funding social services and health, and in Britain disbanding the Inner London Education Authority. Educating for a critical mathematics places power in the hands of people to have some control over their own lives, and, in particular, to help people become aware that they can and actually do have such control. Perhaps we have, in the end, more responsibility than any other school subject, not less, for political and social education.

Secondly, notions of ability in mathematics are dependent on theoretical interpretations of learning and understanding and are not in themselves fixed, certain, and value-free. It is a form of Platonism, that 'understanding' is a description of a particular completed mental state, much like the recall of forms known by the immortal soul. However, if concepts are themselves social constructions, determined by their use and consequently negotiable, the notion 'understanding' has a quite different meaning. Generally, we tend to see mathematics as, to quote Hart and others (Hart, 1981), a "very difficult" subject, that some people seem able to do, and others not, despite many years of learning mathematics. If mathematics is about certain kinds of interactions with the world around, the application of certain ways of thought, or a particular language game, there is no reason why it should be very difficult. We have all encountered instances of children and adults performing sometimes complex mathematical tasks successfully, and more important comfortably, in everyday life, but failing to repeat those same tasks, achieve the same success, or indeed feel comfortable, in the mathematics classroom. This brings into question in a quite fundamental manner our notions of ability and demands discussion, re-thinking, and new directions of research. Clearly, new ways of learning call on new ways of assessment and interpretation of 'ability.' Such very different directions as those described by, e.g., Cobb (1986), focusing on the child's constructions, which of necessity originate in the understanding that children bring into the classroom, cannot be assessed by the usual traditional methods which in general examine children's grasp of things taught by the teacher. This latter notion of children's understanding is the focus of much on-going discussion and research, but the point being made here is that the concept 'ability' is related to the concept of 'understanding' with which one is working and not some absolute concept. If we encourage children's understanding of mathematics through independent work in investigations and prob-

lem posing, and on computers, through Logo, for example, we need different ways to assess their progress. 'Understanding' by these approaches does not mean the successful application of a learned algorithm and thus cannot be identified by a traditional pencil and paper test, developed within a Piagetian framework of hierarchies of concepts. Yet we adhere to this mode of assessment of children's mathematical ability.

In conclusion, as long ago as 1972, René Thom suggested that "all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics." This paper is a contribution to an examination of the ways in which alternative philosophies bear fruit in mathematics education.

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A Hippocratic Oath for Mathematicians?

Chandler Davis

I would like to tell you today about an international campaign for a special kind of ethical statement, a pledge for mathematicians analogous to the Hippocratic Oath in medicine. I would *like* to tell you about it, but I cannot: I do not know that there *is* any such campaign. I will describe at the end a proposed Oath for all scientists. Before that, I will report on a number of related ethical statements—not of quite the same nature, but parallel in some way. Maybe we will equip ourselves to think about whether it might be and what it might be, this nonexistent mathematical Hippocratic Oath.

Such a discussion runs counter to strong prejudices. We are constantly told that moral concerns are political and therefore not professional; and that political questions must be addressed by political organizations which we join in our private capacity, while professional organizations which we enter as mathematicians and teachers must deal only with professional matters. Until recently it has been hard to get a hearing in a professional forum for questions of moral responsibility of the mathematician to the mathematical community or of that community to society.[1]

If physicians can talk about such problems, why can't we? Because mathematics does not affect people's lives? Not as intimately as medicine, to be sure. But they call World War I the chemists' war, World War II the physicists', and World War III the mathematicians'. At least it is plain that mathematics is used more and more essentially in explanations of vital problems (and in hocus-pocus purporting to explain them[2]). Or maybe we cannot have ethical debates because we do our work in independence of each other? This illusion too is dissipating. Or is it just that mathematicians are too conservative to acknowledge their embeddedness in society? Well, I do not see us as *that* much more conservative than physicians.

Or, to take an example closer to home, engineers. Electrical engineers in the USA got their first professional code in 1912.[3] Such codes have been adopted and rewritten by many engineering societies since then. Activists complain[4] that most members have never thought carefully about the meaning of the codes, do not keep them in mind in deciding their own conduct, and mostly forget the codes exist. But when-

[1] One of the few extended discussions is the bold book by E. & J. Hørup, *Mathematics and society* (which I read in J. Lamperti's unpublished English translation).

[2] See discussion of this point by N. Koblitz (1981) in L. A. Steen (Ed.), *Mathematics tomorrow*. New York: Springer-Verlag.

[3] Unger, S. H. (1982). *Controlling technology*. New York: Holt, Rinehart & Winston. This is a general text on engineering codes of ethics, and I have drawn several points from it. Further bibliography in M. S. Frankel (Ed.) (1987). *Values and ethics in organization and human systems development*. Washington: AAAS.

[4] Conversations with S. H. Unger, J. R. Tyndall, and others.

ever there is somebody to point to the code, its authority is conceded.

You see how even a conservative profession can impose such a code on itself. It is advantageous to be able to say to a client, "You can trust me, because I am a professional engineer." This is supported by being able to say, not that no professional engineer has ever been unethical, but that unethical behavior is unprofessional and is repudiated by the profession. In practice, the code is not used to expel shady engineers (as law societies occasionally disbar shady lawyers). It is used to defend honest engineers. When Virginia Edgerton, a computer specialist employed by the City of New York, was fired in 1977 for insisting that her department needed a study of the danger of exceeding the capacity of the computer in use, a special committee of the Institute of Electrical and Electronics Engineers investigated and found her conduct perfectly justified by the code. They were not able to do anything about her job, but the honor of the profession was defended.

This case had the special feature that the interests which might have been endangered were simultaneously those of the engineer's employer and of the public. More commonly, private and public interests are or seem to be in conflict. The employer or client may be tempted, as in the case of the DC-10 passenger plane or the space shuttle, to conceal design defects, and an ethical design specialist may insist on making them known to the purchaser or the public. Or, as in the Karen Silkwood case, an employer may think economics dictates cutting corners on safety of the workplace, and an ethical technician may feel obligated to insist on safe working conditions. Many accepted codes explicitly stand behind the engineer who in such situations defies the employer's orders. The code of the Association of Professional Engineers of the Province of Ontario says,

> A professional engineer owes certain duties to the public, to his employers, to his clients, to other members of his profession, and to himself ... A professional engineer shall regard his duty to public welfare as paramount ...

This code is fundamental policy of the organization to which every practicing engineer in the province is expected to belong.

There cannot be any insurmountable barrier to creating generally accepted codes in other fields. There is at least one barrier in the case of mathematics: habit. When 33 constituent scientific unions of the ICSU were surveyed about ethics and social responsibility, the International Mathematical Union (I notice without surprise) was not among the 14 which answered the inquiry[5].

Now if we want to encourage public commitments to moral conduct, our efforts may go toward strong statements by individuals or small groups; or toward the formation of voluntary organizations advancing a principle of responsibility (on the model of the Society for Social Responsibility in Science, the World Federation of Scientific Workers, the Federation of American Scientists, Science for the People, or Physicians for Social Responsibility, to name only a few); or toward getting an organization representing the profession as a whole to take a position; or, strongest of all, getting such a professional organization to promulgate a credo to which each individual member ought to be bound. Even this strongest level exists already for physicians and engineers. My summary today jumps between the different levels of statement, because I found in surveying the subject that they do not stay separate. Initiatives at one level inspire actions at another. For example, until recently efforts to demand non-discriminatory treatment of minority and women engineers were made only at the individual level, but now the principle is included in some society codes.

Much of the modern discussion of scientific ethics goes back to the movement after World War II to renounce scientific work on weapons of mass destruction. There was a principled exodus from American nuclear bomb work; Pëtr Kapitsa refused to participate in hydrogen bomb research in the Soviet Union; in 1957, a number of German physicists made a public refusal ever to help make nuclear weapons[6]. The most prominent mathematical example is Norbert Wiener, who severed his relationship with the military[7] and announced that all his work thenceforth would be communicated at once to A. N. Kolmogorov, to guarantee it would not become an American monopoly. In later generations other mathematicians have made individual decisions on principle to leave employment with weapons contractors; I think especially of Marvin Shinbrot and Rodney Driver. None of these followed G. H. Hardy in his radical attempt (unsuccessful, as we know) to work only on problems without *any* potential application; on the contrary, they all continued to do applied research.

The individual gesture risks being ineffective. If (say) a ghastly new weapon requires three mathematical ideas, the authors of all three may feel responsible for their part in it, even if two did not know there was such a potential application. Few of us want to keep our research secret, to forbid less ethical scientists to read our papers, or to stop teaching. Individuals who renounced direct destructive work have sometimes tried also to avoid indirect complicity. Several scientists, most notably the physicist Charles Schwartz, have at some time tried to insist that their students also undertake not to do war work. The physicist Eric Fawcett and some others attach to published papers the following:

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[5]Frazer, M. J. & Kornhauser, A. (Eds.) (1986). *Ethics and social responsibility in science education* (p. 249). Oxford: ICSU/Pergamon.

[6]Jungk, R. (1958). *Brighter than a thousand suns* (esp. p. 336). New York: Harcourt Brace.

[7]Wiener, N. (1947, January). A scientist rebels. *Atlantic Monthly, 179*, 46.

Endorsement

It is the author's wish that no agency should ever derive military benefit from the publication of this paper. Authors who cite this work in support of their own are requested to qualify similarly the availability of their results.

Individual exercises of conscience do not perturb the establishment terribly. When you try to get concerted action you hit closer to home. Here is one that enraged some otherwise mild-mannered conservatives.

The following advertisement was published by a group of mathematicians in the *Notices of the American Mathematical Society* during the Vietnam War.[8]

Mathematicians: Job opportunities in war work are announced in the Notices, in the Society's Employment Register, and elsewhere.

We urge you to regard yourselves as responsible for the uses to which your talents are put. We believe this responsibility forbids putting mathematics in the service of this cruel war.

The formulation of responsibility here was inspired by earlier statements by the Society for Social Responsibility of Science. The last sentence was thought too weak by at least one signer, Henry Helson: "*This* cruel war?" he said. "All wars are cruel."

In those days the columns of the *Notices* were full of recruiting ads by Lockheed, Litton, the National Security Agency, and so on. We had to put our statement in as an advertisement, too. Fate (or a sympathetic AMS employee) saw to it that it appeared right next to one of the recruiting ads it referred to. We ran our statement several more times, the list of signers growing to almost 400. Of course, in the most recent *Notices* the National Security Agency is still saying, Uncle Sam needs you.

Some signers were on military contracts, including the famous statistician Jerzy Neyman; they presumably did not consider their research on weather modification to be serving the war, though their clients surely did. The US Army threatened to exclude them from further research contracts, but a flurry of protests (and the military value of the weather research?) led it to drop the threats.[9]

This appeal perhaps does imply a pledge not to do war work oneself; even if one takes it as a pledge, it surely does not imply that anybody else should judge whether Professor Neyman had lived up to it. Though joining a collective affirmation of an ethical tenet, one did not by signing delegate any of one's autonomy. The same is true of some important recent pledges which I want to recall, involving other sciences.

For a year now, a voluntary organization called the Committee for Responsible Genetics, led by the MIT biologist Jonathan King among others, has been circulating this statement internationally:

We, the undersigned biologists and chemists, oppose the use of our research for military purposes. Rapid advances in biotechnology have catalyzed a growing interest by the military in many countries in chemical and biological weapons and in the possible development of new and novel chemical and biological warfare agents. We are concerned that this may lead to another arms race. We believe that biomedical research should support rather than threaten life. Therefore, WE PLEDGE not to engage knowingly in research and teaching that will further the development of chemical and biological warfare agents.

The operative clause here is quite simple and broad. On the one hand, *some* violations of it would be plainly recognizable. On the other hand, since it seems to be a commitment not to teach students who later make biological weapons and not to publish research later applied by weapons-makers, it raises again the question of how to escape indirect complicity.

The Jonathan King pledge was inspired by the example of a group of physicists, John Kogut and Michael Weissman of the University of Illinios and Lisbeth Gronlund and David Wright of Cornell University, who circulated a pledge in the physical sciences beginning in 1985. This pledge, however, is much narrower: It is simply a commitment not to solicit or accept money for research on the so-called Strategic Defense Initiative (SDI). The success of the pledge was unprecedented: There are over 12,000 signers, most of them in the US, though Japan, Canada, and other countries are well represented; there are 112 departments in physical and engineering sciences at 71 US institutions where a majority of the department's faculty have signed.[10] It makes an impression, because the rush of money for SDI has been so great. On the other hand, the pledge does not generalize, because people signed it for different reasons. Some would not work for any build-up for fighting nuclear war; others reject more specifically a build-up in an area, Anti-Ballistic-Missile countermeasures, which was deemed destabilizing and renounced by treaty in 1973; others oppose SDI merely as a waste of money, having heeded scientific evaluations saying that the techniques proposed would not work.

Mathematicians too, perhaps encouraged by these precedents from other disciplines, have recently criticized the level of military funding. The immediate target of our criticism was the "Washington office" run jointly by three large US mathematical organizations. It seemed to us that this operation, paid for by members' dues, was dedicated to finding government agencies to pay mathematicians, heedless of the effect of the contracted work on society and on mathematics. An influential article by William Thurston of Princeton[11] was followed by literally dozens of letters and three well-attended discussions at meetings. Two resolutions were offered to the American Mathematical Society; after a bit of haggling, their text took this form:

Motion 1. Many scientists consider SDI (commonly referred to as Star Wars) incapable of achieving its stated goals and dangerously destabilizing. Participation by universities and professional organizations lends a spurious scientific legitimacy to it.

[8] *14*, 509 (1967); *15*, 843 (1968). Some of us including myself were Canadian residents, and one opponent of the action berated me for taking it "from foreign sanctuary"!

[9] *Notices of the American Mathematical Society, 15*, 859–860, 985–986 (1968).

[10] These figures are from a special issue of *Science for the People, 20*, 1, (1988) which has many articles bearing on the present subject.

[11] *Notices of the American Mathematical Society, 34*, 39–44 (1987). Later articles in vols. 34, 35, passim.

Therefore the AMS will lend no support to the Star Wars program. In particular, persons representing the AMS shall make no efforts to obtain funding for Star Wars research or to mediate between agencies granting Star Wars funds and people seeking these funds.

Motion 2. The AMS is concerned about the large proportion of military funding of mathematics research. There is a tendency to distribute this support through narrowly focused (mission oriented) programs and to circumvent peer review procedures. This situation may skew and ultimately injure mathematics in the United States. Therefore those representing the AMS are requested to direct their efforts towards increasing the fraction of non-military funding for mathematics research, as well as towards increasing total research support.

The motions were submitted to a well-publicized referendum of all the 21,000-plus members in January 1988. Over 7,000 responded (about twice as good a rate as in elections of officers). The results were 4,034 yes to 2,293 no on Motion 1; 5,193 yes to 1,317 no on Motion 2.

So you see we achieved quite a definitive assertion of policy by the Society (whether we can hold the officers to it, time will tell). But in another sense these motions were much farther from the model of the Hippocratic Oath: There was no undertaking by anyone *individually*, even the proposers of the motions. To show that the idea of an individual Oath still has vitality, I will close with two more non-mathematical cases.

Students at Humboldt State University in northern California organized an extraordinary initiative last year.[12] Upon receiving the diploma, each 1987 graduate student was offered a card reading

I _____ pledge to thoroughly investigate and take into account the social and environmental consequences of any job opportunity I consider.

The authorities of the University, the faculty, and the student government had formally approved the exercise; but there was no penalty for not signing. This pledge was a voluntary recognition of an association between education and a sense of social responsibility.

As you can imagine, the activist students whose campaign led to this action had considered longer and more explicit statements. But they did not feel it was a loss to settle for this minimal one. The possibility of a commitment to moral exercise of the profession was raised, and in such a way that one at least conceived that such commitment *could* be universal.

Still more recently, a group in Argentina organized a conference of scientists from many countries, including representatives of the World Federation of Scientific Workers and the Federation of American Scientists. The Argentine astrophysics student Guillermo LeMarchand proposed a Hippocratic Oath for scientists. He and his colleagues would have included a pledge not to work on weapons of mass destruction, but were persuaded by the conference to be content with this more general formulation:

Aware that, in the absence of ethical control, science and its products can damage society and its future, I pledge that my own scientific capabilities will never be employed merely for remuneration or prestige or on instruction of employers or political leaders only, but solely on my personal belief and social responsibility—based on my own knowledge and on consideration of the circumstances and the possible consequences of my work—that the scientific or technical research I undertake is truly in the best interests of society and peace.[13]

It is not too late, I think, to propose amendments to the wording.

Do you think it will ever be true that our teaching is regarded by everyone the way medical teaching and engineering teaching are regarded (at least nominally)—as something given in trust, on condition of being used honestly and for the good?

Sociology of Institutionalized Mathematics

Mathematics and the way that mathematics is conceptualized, taught, and learned, is dependent both on institutions and on political decisions which were discussed under this topic. Eduard Glas, the Netherlands, presents an historical example by discussing some cultural differences within the Paris Ecole Polytechnique 1794–1809. Renate Tobies, German Democratic Republic, describes the influence and the activities of Felix Klein in the Teaching Commission of the 2nd Chamber of Prussian Parliament. Emma Castelnuovo, Italy, speaks about the teaching of geometry in Italian high schools during the last two centuries as a steady attempt to introduce geometry to a larger sector of society and to connect it more with reality. Michael H. Price, United Kingdom, offers some reflections on the role of associations in mathematics education and their relationship to and influence on other political and social pressure groups within the context of national educational systems.

Social Determinants of Mathematical Change: The Ecole Polytechnique 1794–1809

Eduard Glas

Introduction

Since mathematical definitions, propositions, and proofs are free from any appeals to criticizable empirical knowledge, mathematics is irrefutable and seems to depend exclusively on the logical canons of rational thought. How then could the social structure of institutionalized mathematics influence the cognitive structure of the discipline itself? This is only

[12] *San Francisco Chronicle*, 14 May 1987, p. 6. Update on the spread of the campaign to other campuses in *Los Angeles Times*, 22 June 1988, Part I, p. 3.

[13] *F.A.S. Public Interest Report, 41*, 5, 1–4 (1988, May).

possible when conceptual variations occur that cannot unequivocally be settled by rational argument alone. Such divergencies may (at least temporarily) challenge the consensus of the mathematical community and thereby invoke the external selective pressures of the social environment. I intend to present such a case of alternatives in mathematics and will clarify the reciprocity between social and cognitive factors in the light of this case.

Alternatives in Mathematics

The opposition between the analytic and the synthetic approach to mathematics in the first half of the 19th century is well known. In mathematical physics, Gratton-Guinness has traced a dichotomy between what he has called "analysis" and "calculus/descriptive geometry" minded types. Less well known is that both distinctions are rooted in a cultural clash during the revolutionary period between the established analytical tradition guided by Lagrange and Laplace and a new geometrically oriented approach swayed by the revolutionary upstart Monge.

Lagrange, Laplace, and Monge had developed their educational programs in response to the governmental assignment to normalize and rectify mathematics to a perfectly transparent language, free from all remnants of human speculative imagination—regarded as the source of enduring obscurantism and ideological forgery. Lagrange and Laplace seem to have adhered rigidly to the terms of this assignment. They set out to purge the discipline from all informal, quasi-empirical, or intuitive elements—which was their object anyway, with or without official assignment. They saw mathematics as a computational instrument for the edification of the mechanistic System of Nature, the legitimatory model of a "free" society based on Natural Law, in the service of Enlightenment.

Monge, too, emphasized the linguistic uses of mathematics, as required by the terms of his assignment, but expressly rejected the reduction of mathematical reasoning to a formalism. He insisted on the indivisibility of form and content and denied that "real" (research) problems could be tackled effectively by the application of formal rules. For him, analysis was not a language closed in itself, but merely a "script" for the notation of reasonings about informal contents. In other words: What for him counted as "rectification of mathematical language" was not formalization, but the improvement of semantic possibilities of representing problem situations in empirical reality.

Monge's geometry was in the first place the exact and systematic theoretization of the still incoherent collection of empirical rules of thumb and disconnected graphical techniques of mathematical practitioners. It was certainly not a mere application in the sense of a derived and theoretically inferior activity. On the contrary, it furnished invaluable new tools for the exploration and exploitation of problem fields with a considerable bearing on the most intricate features even of pure analysis. The geometrical program was progressive in that it could solve problems far outside its intended domain proper, problems that it had not specially been designed to solve. Lagrange and Laplace rejected not only the informal ways in which Monge had obtained his results, but especially his enduring reliance on geometrical arguments in the demonstrations and proofs thereof. So the same mathematical "facts" were perceived very differently. Any purely analytical argument was read by Monge as a discourse about spatial transformations, whereas Laplace and Lagrange read the same argument as a mere series of formal rearrangements of uninterpreted signs. The latter regarded a geometrical demonstration as mere pictography for the use of illiterates in analysis, but for Monge it represented what mathematics was all about in the first place. Because their perceptions were so different, there was no agreement about what were genuine problems of mathematics and hence no common basis for assessing the relative merits of the two approaches. In nearly every respect they entertained divergent perspectives on the nature of mathematics (formal language vs. quasi-empirical science), its object (formalization of reasoning vs. exploration of the world), its methods (guided by formal prescriptions vs. imaginative skill), and its meaning (logical perfection vs. functional expansion of knowledge). The two approaches to mathematics were *incommensurable* in the following sense: *No party disposed of logical reasons sufficiently independent of biassed underlying presumptions to force the other to agree on penalty of irrationalism.*

This definition of incommensurability does not imply that there were strictly formal or logical impediments to reaching a form of consensus; neither were opponents really unable to understand each other's arguments. Lagrange and Laplace, when officially assigned by the government to expound the proper methodology of mathematics, naturally took their own accredited accomplishments, competences, and skills as paradigmatic for the framework that was to be developed. Evidently, they evaluated methods, approaches, developments, etc., in terms of their consequences for the value of their personal intellectual assets. Their perception of mathematics was severely constrained by their evaluative framework, which incorporated their own precious achievements. Monge's position embodied a different set of skills and competences, to which they could not reasonably lay claim. Because they could not improve upon Monge's accomplishments, conceding to the latter's point of view would have damaged their own credibility without bringing any visible gain for mathematics as a whole. Therefore it was rational for them to stick to their own precepts. Likewise, it was rational for Monge to maintain his point of view, because it gave the greatest credit to his own attainments and abilities. No rational arguments could force one party to concede to the other on penalty of irrationalism, since the two programs could not be measured by the same standard.

It should be noted that the differences must not be interpreted in terms of the distinction between pure mathematics and its application or finalization. The Starnberger have defined "finalization" as a socially oriented horizontal differentiation within a closed theory or paradigm. After a theory has been completed and closed as a "paradigm," it might still be further developed with a view to application in other disciplines or in practice. In such a case of non-theoretical, mission-oriented progress, they speak of "finalization." However, in our case we have almost the inverse state of affairs: the theoretization of a socially oriented praxis that gave rise to vertical differentiation and hence to an alternative paradigm. Since the new paradigm was incommensurable in the sense defined above, it cannot be construed as an application or finalization. The analysts were as much concerned with applications as Monge, and the geometer was no less interested in purely theoretical developments than Laplace and Lagrange. But they differed in their appreciation of where exactly the theoretical interests of mathematics lay and of what sort of applications were legitimate landmarks of mathematical progress.

When pointing to the usefulness of mathematics, the analysts always referred to its applicability in rational mechanics, never the practical purposes of industrial and economic life. They conceived of mathematics primarily as a computational instrument for the exploration of nature's phenomena, whereon the exploitation of nature's resources was taken to be totally dependent. They envisaged no possibilities to apply mathematics directly to industrial and technical problems, whereas that was precisely the object of the Mongian alternative. Monge's geometry was in the first instance an exact and systematic theoretization of the informal rules and techniques of practitioners. His methods typically bespoke the practitioner by their appeal to unformalizable and inalienable skill and craftsmanship and by their emphasis on the personal transmission of proficiency through precept, example, and exercise. The confrontation was nothing short of a cultural clash between separate traditions, competing for the recognition of their own occupational activities and skills as the sole professional body of knowledge and competence to be relied upon to render disinterested services to the community. While tacitly laying claim to political and intellectual superiority over the ancient ruling class, the two groups appealed to different sorts of technical credentials and social motives. Thus the conceptual variation can be comprehended in terms of professional interests of groups that are in possession and control of different competences as resources in a competitive environment.

The Social Process of Selection

It now stands to reason that the main selective factor under these circumstances is the competitive control of stable and institutionalized opportunities for recruitment and training of students. For these are very likely to remain committed to the framework wherein they have been schooled, since it confers the greatest value to their acquired proficiencies. Because of their nature as resources in a competitive environment, the training and early professional experience of mathematicians will continue to affect their professional judgments. The rise of the Mongian geometric school turns out to have been due to:

(a) his temporary control of the *Ecole Polytechnique* owing to political exigencies and military imperatives. As a recognized expert in military technology and instruction, Monge was personally called upon by the government to set up this new School;

(b) the consequent opportunity to expound his approach at once as standard educational material at this most prestigious institution of the revolutionary epoch;

(c) the splendid career opportunities for his trainees, in military and civil practice as well as in fundamental mathematical research. Monge deliberately suggested sketchy ideas to his pupils, to be worked out by them in a spirit of friendly emulation, thus stimulating them to expand the new vistas over fresh fields. In this respect the Lagrange–Laplace approach had virtually nothing to offer to young ambitious mathematicians;

(d) the great propagandistic value of Mongian geometry as invaluable tool for the new practices based on division of labor and industrial organization, in particular for the exact planning of works of art and technology, and the derivation of structural details as required by engineers, artisans, workers, supervisors, etc.;

(e) the acquisition by his pupils of influential teaching positions throughout Europe in the wake of the Napoleonic conquests, for instance, at the technical institutes, modelled after the *Polytechnique*, that were to arise in European capitals such as Vienna, Berlin, and Prague.

Thus the new approach rose to eminence owing to a complex of political, military, educational, and professional opportunities. After 1806, however, the climate was changed in favor of the Laplacian approach, which among notable mathematicians had continued to be considered of higher standing. Prestige being Bonaparte's main object, the School was reformed and militarized, and placed under the supervision of Laplace. Monge resigned in 1809.

It is the irony of history that Felix Klein later should attribute the decline of Laplacian science, and of the *Ecole Polytechnique* with it, to pre-occupation with practical questions at the cost of the theoretical development of pure mathematics ...

This then is the way in which the sociological structure of institutionalized mathematics may influence decisively the developmental course of the discipline.

The Activities of Felix Klein in the Teaching Commission of the 2nd Chamber of the Prussian Parliament

Renate Tobies

On 17 February 1908, Klein was ordered to become a member of the 2nd Chamber of the Prussian Parliament as the representative of Göttingen University. The Teaching Commission was constituted on 19 March 1909. Klein accepted the job of vicepresident. As the referee of the Teaching Commission Klein also represented the interests of the German Commission of Mathematical and Scientific Instruction (Deutscher Ausschuß für den mathematischen und naturwissenschaftlichen Unterricht). Besides, he used the international experiences; from the same year on he was also the president of the International Commission of Mathematical Instruction (Internationale Mathematische Unterrichtskommission).

Klein used his function in the Parliament in order to overcome certain contradictory tendencies, which had become obvious in the fields of mathematical and scientific education, already during the 1890s. One of those contradictions was the fact that new educational institutions (technical colleges and a new form of secondary school, the so-called "Realschulen"), which were created to meet the requirements of industry, did not enjoy the same rights as the traditional institutions (i.e., the universities and the humanities-oriented secondary schools called "Gymnasien"). Klein helped to overcome this conflict by supporting the technical colleges in their struggle for the right to confer doctorates and the efforts of the "Realschulen" to obtain equal legal status with the other secondary schools.

A second contradiction existed between the demands of industry for mathematical teacher training and the education at the universities and schools, which was out of touch with real life or practice. Klein could win over representatives of industry and members of the Prussian Ministry of Education for his goals. First he founded an association which united scientists from Göttingen University with representatives of industry from the whole of Germany: the Göttingen Association for Applied Physics and Mathematics. My latest findings after intensive studies in the Göttingen archives have been that the industrials who were engaged in founding the association, considered the improvement of teacher training as the essential motive for founding the society. In agreement with those representatives of industry, with engineers, and with the members of the Prussian Board of Education, Klein reached his goals. Particularly, he could enforce in Parliament that the new mathematical curriculum (Meraner Lehrplan) was tested in all three types of secondary schools, and applied mathematics became obligatory in teacher training.

The third antithesis must be seen in the following: Activities in various professions of the economy needed increasingly to be able to use mathematical functions. Mathematical education at school did not provide enough preparation for this, however. The Euclidian tradition still dominated in secondary mathematical education. The turning point in the history of mathematics—the use of analytical methods in geometry and the introduction of infinitesimal calculus—had not yet found a permanent place in the curriculum of the schools. Klein was very active here to bring about changes.

If we look at Klein's efforts in the Prussian Parliament, it is good to know these main lines. The emphasis of his endeavors in the Teaching Commission of the 2nd Chamber of the Prussian Parliament was on:

- the improvement of women's education in mathematics and science;
- changes in all mathematical and scientific education, particularly in elementary schools;
- a better knowledge of foreign languages.

In the following I shall enlarge upon the first and second of the above items.

Women's Education

Women's studies had not yet been generally acknowledged, although the number of women students had more than trebled from 1908 (1,108) to 1913 (3,436). Klein had supported women's studies ever since his first trip to the United States in 1893. He acknowledged that women in principle have the same mathematical capability as men. In 1905 the first German women students listened to Klein's lectures. In 1908 a new order for women's mathematical and scientific education was issued so that the number of female students could increase. Twenty-one women attended Klein's lecture on infinitesimal calculus in the summer semester of 1911. From the winter semester 1908/09 Iris Runge, the daughter of Carl Runge (1856–1927), who had been the professor of applied mathematics in Göttingen since 1904, attended Klein's lectures. From the winter semester of 1909/10 Klein's youngest daughter Elisabeth also took part in his lectures.

Klein's first speech in the 2nd Chamber of Parliament dealt with the education of women. He delivered this speech on behalf of the Teaching Commission on 21 May 1909. And it was characteristic that he did not only speak about things, but always took measures to achieve practical results. He proposed in his speech to arrange a holiday course for women teachers of secondary schools. Klein knew that teacher training was the prerequisite for the better education of women in mathematics and science. In the autumn of 1909 Klein organized the first such women-teacher-course at Göttingen University. One of the most controversial issues in Parliament was the question of coeducation. The Teaching Commission with Klein as its speaker did not support the idea of mixed education for boys and girls. Some of the deputies regretted this. Although Klein was not really progressive in this re-

spect, he called for the same reform in the mathematical education of women as in that of men.

Changes in all Mathematical and Scientific Education, Particularly in Elementary Schools

It is not possible to discuss here the question of whether changes in this area really took place. One can only emphasize Klein's great efforts to bring about changes in the law, in examination requirements, and curricula as well as in teacher training as the essential prerequisites for effective changes in classroom practice. Klein explained the relevant demands of the Teaching Commission in three speeches: on 21 May 1909, on 27 May 1910, and on 7 April 1911. In the first two he spoke about the existing institutions of secondary education. The third speech was dedicated exclusively to elementary schools. This special emphasis on elementary education was unusual for a university professor at that time. Klein criticized the encyclopedic training of teachers for elementary schools, asked for the introduction of courses for theoretical training, and for financial means to be made available for improving it. Klein's demands for scientifically trained teachers for elementary schools surpassed the ideas of many of his contemporaries. Klein's views corresponded in this respect to those of the German social democrats, although he was a loyal subject to the existing state. The ideas of Klein remind us of the demands of progressive bourgeois pedagogues, which were also utopian under the existing social conditions.

The activities of the Teaching Commission and Klein's function as vicepresident ended with the breakup of the German empire, but the results of their work were able to be preserved. Klein's interests in mathematical and scientific education and in education generally were not dependent on the political situation. He wrote a letter to the Ministry only a short time after the November revolution offering his help in the planning of education. The Ministry of Science, Art, and National Education answered gratefully in a letter of 18 January 1919. Among Klein's papers there is preserved a draft of a lecture entitled "Mathematical instruction at the various types of schools (and debate)." It dates from 7 January 1919 and shows his continuing determination to accomplish the demands for reform under the new regime. For this purpose he formulated a number of theses in his draft:

- Comprehensive public education should form an ideal unity, its separate areas being adequately proportionate to each other.

By this, Klein wanted to make clear the relationship between pedagogic and subject-specific instruction, between general and vocational schools.

- Klein saw the necessity for promoting such subjects as philosophy, psychology, and for creating new chairs of general pedagogics and of didactics in the various subjects of instruction at the universities.
- New ideas were to be put into practice to teach future research scientists and teacher students separately.
- Klein insisted on doing everything possible to achieve higher appreciation for mathematics and science at schools and in public.

In promoting those aims, a new institution was founded on 6 January 1921: the "Association of German Mathematical Clubs and Societies."

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The Teaching of Geometry in Italian High Schools During the Last Two Centuries: Some Aspects Related to Society

Emma Castelnuovo

My subject concerns Italian high schools during the last two centuries. As this period is very long, I will emphasize only those points which, in my opinion, are rather original.

First of all I will begin talking about a French book of the 18th century which had a great influence on our teaching of geometry. The book is *Eléments de géométrie* by A. C. Clairaut, the great mathematician and astronomer; it was written in 1741. The idea of Clairaut was to present geometrical properties in such a way, that beginners would be stimulated to investigate. He says, in his preface, that beginners are not able to follow a book like Euclid's Elements, starting with axioms and abstract properties. He bases his work on examples taken from agriculture, the economy of his country being essentially agricultural: the problem of how to measure fields of different shapes, ...

But Clairaut's Elements never influenced the teaching of mathematics in France. On the contrary, this book, translated into Italian, was used for many years as a textbook in some technical schools of Lombardia, a region in northern Italy. Without any doubt the book was used because of a strange coincidence: The governors of Lombardia thought that Clairaut's book would be very suitable for the technical-agricultural

schools because many examples were connected with the measurement of fields. It is interesting to observe that Clairaut had foreseen this misunderstanding; in his preface he says, with humor: "I do hope that people do not think that this book has been written for students of agriculture; the problems of measuring fields only represent a stimulus to investigate further." However, Clairaut's Elements, interpreted rightly or wrongly, were widely used in our country in the first half of the last century.

The unification of Italy in 1861 led to the establishment of national curricula in our schools. The curricula of mathematics in secondary schools were published in 1867 by three distinguished mathematicians: Cremona, Betti, Brioschi. The ideas inspiring these curricula are clearly described by their preliminary directives; they say: "The fundamental aim of mathematics is to accustom pupils to reasoning and deduction." It is clear that the application of mathematics were excluded. I would like to underline that, even without an explicit declaration, these curricula were conceived for students of a high social level. But a very unusual fact happened in Italy: In 1881, that is 15 years after the publication of new curricula, a very open-minded Minister of Education, a doctor named Guido Baccelli, declared that the curriculum of geometry was too abstract for pupils 11 to 14 years old. He succeeded in convincing the mathematicians to introduce into the junior secondary school the so-called "Intuitive Geometry." He thought, particularly, of lower social level children. Thanks to this Minister, Italy was the first country to introduce Intuitive Geometry. But this new course was not very original: It reproduced the senior course of geometry leaving out the traditional proofs.

At the same time, curricula of senior schools have always been inspired by Euclid, even if, at the beginning of this century, some mathematicians declared the absurdity of such a purism in a modern society. I would like to quote the declaration of Guido Castelnuovo, pronounced in 1912 during a national congress on the teaching of mathematics. He says:

> In our schools we drive students to idolize a perfection which is illusory, instead of encouraging them to work with approximations. We must accustom them to compare theory with practise in order to prepare people able to participate in the life of our country.

But voices like this always remained unheard at the political level.

After the last war many initiatives flourished on education. For what concerning the teaching of geometry in junior secondary schools, a critical study of Clairaut's old book gave brilliant ideas. The examples on fields of different shapes and their measurement were replaced by up-to-date examples and problems, and geometry was strictly connected to the other branches of mathematics in order to investigate practical and theoretical problems. In 1979 a new reform of the first secondary cycle officially recognized the value of these experiments. A geometry motivated by various real problems became a part of a brilliant and widespread course of mathematics. A "dynamic" course whose principal aim is to sensitize pupils to the function concept.

I have to point out that teachers have always had full freedom in their teaching. It is this freedom, perhaps, that saved us from the so-called "modern mathematics."

When, in the 60s, most countries were introducing at all levels mathematics inspired by set theory, Italy was continuing with its curricula and methodology, completely oblivious to a fashion which was invading the world. Why? Some foreign mathematicians say that our behavior was due to the influence that Galileo's thought has had on Italian people: Galileo never proceeded without experimenting or without foreseeing the result which one experiment could produce. I really should like to think that this interpretation reflects the truth!

Some Reflections on the Role of Associations in Mathematics Education

Michael H. Price

My interest in this topic springs from some research I am currently undertaking to produce a commissioned history of the Mathematical Association in England, from 1871. This project is developing along the lines of a case study in a growing field of educational inquiry: the history of the curriculum. Investigating the role of associations generally in the shaping of curriculum is one aspect of curriculum history. Thus, my interests now embrace not only the Mathematical Association but all forms of association having a significant stake in English mathematics education, as well as comparisons *across* the curriculum, and particularly in the case of science education. There may also be some interesting comparisons to be made with other countries, where national patterns of educational organization and curriculum development may vary considerably. Furthermore, the patterns in one country may change dramatically over a comparatively short time, as is the case in England under the present Conservative Government.

In relation to mathematics education, this short paper will only consider the following aspects of the development of associations and their roles:

(1) Variety of forms;
(2) A framework for investigating associations;
(3) Growing complexity.

Variety of Forms

The roots of the Mathematical Association are to be found in the establishment of the Association for the Improvement of Geometrical Teaching (AIGT) in 1871. This was the first of the academic *subject*

teaching **associations to be formed in England. This type of association may be regarded as occupying the middle ground between that of two other types of association, which developed in number and variety during the 19th century. Their principal concerns are as follows, with examples in each case:**

Table 1

A Advancement of Subject Knowledge	B Curriculum and Pedagogy	C General Educational Policy and Practice
British Association for the Advancement of Science (1831) London Mathematical Society (1865)	AIGT (1871) Modern Languages Association (1892) Geographical Association (1893)	College of Preceptors (1846) Headmasters' Conference (1869) National Union of Elementary Teachers (1870)

Associations of type A and C did, from time to time, become involved in specific concerns of type B, but the distinction by type is a clear one nonetheless. By 1907, type B associations had also been formed for subject teaching in science, classics, history, and English, with a separate association for women in science from 1912. By the First World War, the existing number and range of associations of all three types in England is already striking. I might also add that the first International Commission on Mathematical Instruction was set up in 1908 as an off-shoot of the International Congress of Mathematicians. I will return to this theme of growing complexity shortly.

A Framework for Investigating Associations

In any investigation of the role of an association, a global distinction may be made between *internal* and *external* developments, though, to some extent, they are interrelated. The following framework may be used to analyze various aspects of an association's role in relation to the curriculum and its development over time and within different national patterns of educational organization and curriculum control.

The range of concerns of an association and how these are accommodated internally, as well as realized externally, i.e., an association's *policy*, is another major aspect to consider. The evidence concerning the history of the Mathematical Association well exemplifies the theme of growing complexity in terms of both internal and external developments spanning over 100 years of educational history.

Table 2

Internal Developments	External Influences and Relationships
1. *Constitutional:* organization (central and local), membership and hierarchy, regulations, administration, and finance.	3. *Influences:* consultation and advice, curriculum development, examinations, local and central government initiatives, etc.
2. *Social:* forms of professional support, e.g., meetings, working groups, publications, library and resource facilities, course provision, and accreditation.	4. *Relationships:* coordination and collaboration with other associations (types A, B, C); links with institutions in relation to 3 above, industry and commerce, etc.

Growing Complexity

There are *two* major aspects to the historical theme of complexity in relation to mathematics in education. These are shown in the following diagram:

Table 3

Agendas for Policy and Action	Professional Organization
Key Questions, e.g., Mathematics for whom? How much mathematics? What kind of mathematics? What teaching methods? How assessed?	1. *Internal:* developments in the structure and functions of an association
Professional Knowledge in Mathematics Education	2. *External:* number of different associations; range of interests represented; relationships
External Changes and Responses	

In England in the 1870s, the professional world of mathematics in education was relatively simple: just *one* specialist organization—the AGIT—concerned exclusively with *geometry* in a *liberal* education. To illustrate the theme of growing complexity, consider the situation in England 100 years later. Analysis of the submissions of evidence to the National Cockcroft Inquiry (1978–81) from specialist associations in mathematics or mathematics education reveals 15 active associations at this time, 7 of which are principally concerned with school mathematics. The mathematics education group, with the dates of formation of individual associations, is as follows:

(1) Mathematical Association, 1871;
(2) Association for Teaching Aids in Mathematics, 1952
(now Association of Teachers of Mathematics);

(3) Mathematics Section of the Association of Teachers in Colleges and Departments of Education, 1959
(now National Association of Teachers in Further and Higher Education);
(4) University Departments of Education Mathematics Study Group, 1961
(now Association of University Mathematics Education Teachers);
(5) National Association of Mathematics Advisers, 1974;
(6) British Society for the Psychology of Learning Mathematics, 1978;
(7) Girls and Mathematics Association, 1981
(now Gender and Mathematics Association).

Other recently formed associations include the Institute of Mathematics and its Applications (IMA, 1963) and the British Society for the History of Mathematics (1971). Also, the Royal Society now acts jointly with the IMA through a Mathematical Education Joint Committee.

To attempt to bring some coordination into the professional worlds of mathematics and mathematics education, the Joint Mathematical Council (JMC) of the United Kingdom was also formed comparatively recently, in 1963. Here, associations 1 to 5 are represented but these associations have also endeavored to coordinate further amongst themselves by forming a Standing Committee of Associations concerned with Mathematics Education in Schools (SCAMES). There is now some evident tension between professional concerns for mathematics as a subject as opposed to wider concerns for mathematics in education generally. In the latter case, the comparatively recent refinement of professional concerns and the differentiation of specific interests has yielded a complex organizational network, with the contemporary agenda in part reflected in the actual associations themselves. This network has itself supported the internal development of *mathematics education* as a field of endeavor and contributed to its growing status and professionalism. However, in relation to *external* developments and the politics of mathematics in education, the present situation in England is problematic.

Space does not allow any detailed assessment of recent developments concerning the National Curriculum (5 to 16) in England. The pace of change, which is being dictated from the center by politicians, is unprecedented, and the effects on the curriculum in schools are still to be felt. It is clear that political and professional motives and interests are to some extent in conflict. In relation to external developments, complexity may now be regarded as a *problem*, both in terms of organization, in relation to political effectiveness, as well as current agendas, professional knowledge and judgment, in relation to the public debate. I am currently investigating parallel developments in other school subjects in the United Kingdom, particularly science. International comparisons may also prove illuminating. I hope that the framework I have presented will provide a basis for discussion and further investigation of the topic of professionalism in mathematics education.

The Mathematics Curriculum as a Social Issue

In the history of mathematics education epistemological and socio-economical conditions of mathematics learning are strongly related. The issues surrounding this relationship were considered under these topics from different points of view. Michael Otte, Federal Republic of Germany, in his presentation "Mathematics for All and the Epistemological Problems of Mathematics Education," argues that "the dichotomizations current in didactics are both false as far as practical, aesthetical, theoretical, philosophical, and many other kinds of experiences are equally essential for the development of the personality,—and productive, as far as the rationality of human learning and human knowledge is no 'flat world' but includes belief systems, convictions, dogmas, ties to contexts." Neil Bibby and John Abraham, United Kingdom, through considering a few cases of controversy in the historical development of mathematics, analyze the implications of studying controversies for mathematics education. Bernard Charlot, France, discusses social controversies in mathematics education by means of the introduction of the "modern mathematics program" in France.

"Mathematics for All" and the Epistemological Problems of Mathematics Education

Michael Otte

Mathematics for all can mean two things. First, it can mean that the same kind of mathematics—and it will inevitably be the popularized version of scientific mathematics in this case—shall be taught to all pupils in the same way. Second, it can also express the intention that each individual should have access to some kind of mathematics.

If we assume that every knowledge must have a knowing subject and an objective content, the first interpretation focuses on the content of mathematics, and the second on its respective individual subject.

The decisive thing now seems to be that these two different interpretations belong to different levels of reality, or, as could be said, to different logical types in the sense of Russel's theory of types. Since the so-called "New Math" reform movement we have witnessed a shift in emphasis from the first to the second understanding of that statement "Mathematics for all" and together with this shift a change of the type of ra-

tionality on which our thought and reflection about education and science is based. G. Bateson has repeatedly treated such applications of the theory of types in his epistemological writings, always stressing

> that there is a deep gulf between statements about an identified individual and statements about a class. Such statements are of different logical type, and prediction from one to the other is always unsure.

In one of the chapters of his book *Mind and nature* which is headed by the appropriate remark "every schoolboy knows ...," we find the following text:

> This matter has a number of sorts of relevance to the theory of history, to the philosophy behind evolutionary theory, and in general, to our understanding of the world in which we live.
>
> In the theory of history, Marxian philosophy, following Tolstoi, insists that the great men who have been the historic nuclei for profound social change or invention are, in a certain sense, irrelevant to the changes they precipitated. It is argued, for example, that in 1859, the occidental world was ready and ripe (perhaps overripe) to create and receive a theory of evolution that could reflect and justify the ethics of the Industrial Revolution. From that point of view, Charles Darwin himself could be made to appear unimportant. If he had not put out his theory, somebody else would have put out a similar theory within the next five years. Indeed, the parallelism between Alfred Russel Wallace's theory and that of Darwin would seem at first sight to support this view.
>
> The Marxians would, as I understand it, argue that there is bound to be a weakest link, that under appropriate social forces or tensions, some individual will be the first to start the trend, and that it does not matter who.
>
> But, of course, it does matter who starts the trend. If it had been Wallace instead of Darwin, we would have had a very different theory of evolution today. The whole cybernetics movement might have occurred 100 years earlier as a result of Wallace's comparison between the steam engine with a governor and the process of natural selection. Or perhaps the big theoretical step might have occurred in France and evolved from the ideas of Claude Bernard who in the late nineteenth century, discovered what later came to be called the homeostasis of the body. He observed that the milieu interne—the internal environment—was balanced or self-correcting.
>
> It is, I claim, nonsense to say that it does not matter which individual man acted as the nucleus for the change. It is precisely this that makes history unpredictable into the future. The Marxian error is a simple blunder in logical typing, a confusion of individual with class. (p. 46–48)

Some remarks to that: The idea that learning and knowing is a self-regulated activity and a royal road to knowledge is impossible corresponds to the idea that history is unpredictable. Knowledge as the subject's independent construction correlates with the contingent character of history. Or, in other words: Knowledge can be taught only as far as the world will not surprise us with anything essentially new.

The close relationship between education (Bildung) and science in Humboldt's conception is based on the assumption that the world as a whole becomes a *means* of growing cognition and knowledge. In this sense, there is no important difference between empirical facts and formal mathematical structures. Both are recognized or perceived according to the potentials of the knowing system only.

This conception of the world as a means of knowing, however, has two sides: education and science as a basis of technological applications. First, the world becomes a means of the subject's own autonomous activity (educational function, the knowing subject becomes, so to say, the object of its own activity). Second, an application of the world to itself takes place in the subject's activity in the sense noted by Schelling when he wrote that man owes his rule over nature

> again to nature itself which he would in vain strive to rule if he were unable to push nature into conflict with itself and to use its own forces against it. (Schelling, F. W. von, 1907, *Ideen zu einer Philosophie der Natur, Werke, 1*, p. 170. Leipzig: Meiner)

This yields that the object of cognitive activity is means and representation on the one hand and resistance and opposition on the other. Again, it was Schelling already who reduced this to a formula by defining nature "as something simultaneously autonomous and as well determined by the I." (That Schelling is quoted here instead of Humboldt is due only to the fact that he happened to be that of Humboldt's contemporaries to whom fell the task of formulating the philosophy of German idealism with regard to nature.)

The common open horizon of education and real history has been lost from view with the increasing dissemination of successful scientific achievements and with the scientification of all walks of life. Science has adopted the self-image criticized by Bateson, and education has become permeated by pedagogical formalism and didactics. In fact, however, science as research and science as education or application have got into a growing conflict. Or, in other words: Science has ceased to understand its own development. It is no accident that a work which was the first to develop the idea of discontinuity of the development of science, an idea which was later so immensely popularized by Thomas Kuhn's *Structure of scientific revolutions*, Ludwig Fleck's *Entstehung und Entwicklung einer wissenschaftlichen Tatsache* of 1935 also pointed out that aspects like certainty or simplicity of knowledge will develop only in popular accounts (p. 152). And Kuhn, too, held textbooks responsible for the false, cumulative, and continuous self-understanding of the growth of science.

In fact, a general and compulsory school system is unimaginable without a theoretization of knowledge which is conceived of as generalization. Thus, the double meaning of the title "Mathematics for All" discussed above proves to be an expression of the contradictory or quasi-paradoxical character of scientific knowledge itself. It is this contradictory character which Bateson tries to show by means of the theory of logical types. The important point which shall be elaborated further must be seen in the fact that these contradictions, like the contrast between continuity and discontinuity within the evolution of science, cannot be productively grasped in the "flat world." If we consider science, for instance, only on one level, identifying it, for instance, with "theory" there is no choice but to state with Kuhn the incommensurability of different paradigmatical theories, with interrelationships between them being just as impossible as interrelationships between Leibniz' monads. In the present context, Bateson intends to use the example of the contingencies of history to point out the impor-

tant difference between the world and logic, the difference between territory and map. In the context of another of his works, *Steps to an ecology of mind*, he attempts to apply the theory of types to the problem of communication and learning. I shall not quote this here at length, but should like to recommend to everybody to read these texts in part III of this work.

In Bateson's types of learning, a fact emerges which would seem essential to me. It can be described by the proposition that there are changes of the type of rationality of thinking upon transition from one system level to another (superior or inferior one). An individual's subjective sensation of pain, for instance, can, as a rule, not be related to a medical theory of pain, but is quite another type of consciousness. Another example is the debate about radioactivity limits for foodstuffs which arose after the nuclear plant catastrophe of Tchernobyl and which turned out to be, after all, only in part a matter of experts. Or, to give a third example which is directly associated with didactics: the question discussed in connection with the problems of the computer: "Can machines think?" All arguments sceptical of the assumption that computers could be able to think one day are based on a "confusion" of logical types. Thinking, on the one hand, is seen as an internal experience. The fact that somebody thinks is accessible only to himself and to his introspective intuition. According to this conception, thinking and feeling cannot be described, but only carried out or experienced. If, on the other hand, we talk about the thinking of a computer, we must of course assume that a computer simulation of thinking or feeling presupposes a description of these things.

This shift from one level to another, then, serves to prove automatically that computers can neither think nor feel. Another kind of distinction between man and machine is based on the claim that while thinking can be described, the subject basically can not be determined from his/her thinking. This line of argument does not differ from the former in its fundamental character.

I consider it important, however, to note that this "confusion of logical types" and the shift in rationality it implies is not opposed to the objectivation of human knowledge and human learning, but, conversely, inseparably linked to these. As far as each knowledge and each insight requires a subject, and as far as this subject cannot be objectified by its own theory, but at the utmost by the latter's application, or, in other words, as far as every theory has relative a priori which it cannot thematize itself, but on which the meaning and the objectivity of its propositions are based, this shift of rationality seems inevitable even where it displays paradox features.

Social History of Mathematical Controversies: Some Implications for the Curriculum

Neil Bibby and John Abraham

In the paper from which this summary is extracted we explore the notion of controversy in mathematics. We contend that controversy is inherent both within and between the social segments in which mathematical activity is pursued, that in course of time the controversies are resolved, that the renewal tendency in mathematics ensures that the controversies are eventually forgotten, and that the consequence for institutionalized education is that pedagogy is impoverished. The argument is that, at all levels, controversy has the potential to enrich pedagogy, and that its inherent place in mathematical history justifies a didactical approach which accords it a legitimate role.

Popular Myths

Lakatos has commented on 'infallibilist' accounts of mathematics in which logico-deductively arrived-at results are rendered relatively devoid of meaning and tend to convey an authoritarian incontravertability. At best the effect is that the learner does not feel in a position to criticize, only passively accept what is presented. At worst, in addition deep levels of anxiety can be induced. Lakatos' arguments demand what Restivo has called the demystification and demythification of mathematics. Restivo further argues for a social constructivist view of the subject in which "mathematical representations are socially constructed around social interests" (see Restivo, 1984, p. 66). He goes on to contrast this stance with the Platonist stance:

> *Mathematical* knowledge, then, like *all* forms of knowledge, *represents* the material experiences of people interacting in particular environments, cultures and historical periods rather than pure, eternal truths residing in a Platonic realm of Ideas or waiting out there in naked splendor to be discovered. (Restivo, 1984, p. 73)

In this view, then, the phenomenon of controversy in mathematics can be regarded as an intrinsic component of the social process of the resolution of competing assertive representations grounded in differing social interests. Thus, as Davis has pointed out, installed representations are essentially *conventional* "in the sense that mathematizations other than the ones installed are quite feasible" and *provisional* "in the sense that alternative schemes are often installed which claim to do a better job" (see Davis, 1988, p. 11). The following case study illustrates a controversy in which these conventional and provisional aspects are clearly evident.

Quaternions and Vector Analysis in the 19th Century

In Crowe's (1967) outstanding study, a detailed account is given of the controversy last century between

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the defenders of Hamilton's system of quaternions and those advocating a simpler form of vector analysis which in essence was derived from Hamilton's. Hamilton (1805–65) had unsuccessfully sought for over a decade an extension of the (two-dimensional) complex number system to three dimensions, and finally in 1843 at the height of his professional fame developed a four-dimensional system of quaternions (of the form $a + bi + cj + dk$ with $i^2 = j^2 = k^2 = ijk = -1$) which inherited all of the properties of complex numbers except for multiplicative commutativity. Hamilton's eminence ensured great interest in his system, and he himself believed that it would stand out as the great discovery of the 19th century. By the 1870s the careers of many younger mathematicians had been built upon pursuit of the quaternion system, notably that of Tait (1831–1901).

Following the appearance of Maxwell's *Treatise on electricity and magnetism* (1873), which included attempts at the application of quaternions, a younger generation of men with both an interest in applications and familiarity with Maxwell's work began during the 1880s to vociferously criticize the quaternion system. Foremost amongst them were (in the USA) Gibbs (1839–1903) and (in Britain) Heaviside (1850–1925). In their systems the separately defined scalar and vector products replaced the single, but non-homogeneous, full quaternion product, and formal niceties such as closure, associativity, and uniqueness of division were cast to the wind. They invoked in support the by then posthumously celebrated name of Grassmann (1809–77) who, while working in obscurity as a schoolmaster, had proposed a similar system in his *Ausdehnungslehre* of 1844. The controversy raged from 1880 onwards, reaching a peak in 1890–94, and can be regarded as being essentially grounded in the differing evaluative criteria of, on the one hand, the quaternionists, whose concerns were formal, and on the other, those advocating a Grassmannesque system whose concerns were primarily with applications, as indeed Grassmann's had been.[1] The latter group, for instance, regarded it as absurd that the square of a vector in the quaternion system should be negative, itself a necessary consequence of the formal preoccupation with finding a higher-dimensional extension of the complex number system.

Hamilton's position within the hegemonic mathematical establishment of the day ensured both an absence of threat to his social interests and a credibility for his system which would attract future adherents, such as the younger Tait. Gibbs and Heaviside were even younger men whose careers were in the making and were working in the period after Maxwell's *Treatise* with an essentially applications orientation in the context of the burgeoning field of electrical technology. Thus, the pragmatic ideology of Gibbs and Heaviside became the dominant one by the turn of the century and the influence of the post-Hamiltonian quaternionists gradually faded, Maxwell's work, and much else in physics and engineering, having been successfully recast using the new analysis.

Curricular Implications

Wheeler has commented on the rule-bound nature of school mathematics. He goes on:

> But where do these rules come from? Are they like the rules of a game, or like the laws of nature, or like the commandments of God? Could they be different? ... [these] ... questions have been argued throughout the history of mathematics, and some of them continue to be argued today. If then, the behaviour of numbers is known to be controversial, why is not more of this controversy apparent in school? (1974, p. vi)

More strongly, Davis has advocated that:

> Since we are all consumers of mathematics, and since we are both beneficiaries as well as victims, all mathematizations ought to be opened up in the public forums where ideas are debated. These debates ought to begin in the secondary school. (1988, p. 13)

This is not the place to go beyond a justified advocacy of the role of controversy in mathematics education. It is one thing to advocate, another to convince, and yet another to implement. It is however unlikely that many teachers will feel comfortable with controversy in their classrooms while a perception of another quality, irrefutability, is what attracts them to mathematics in the first place (see Backhouse et al., 1980, p. 4 and p. 13). It is this aspect of cultural reproduction that must be addressed if teachers' perceptions are to change so that they welcome a classroom in which controversy plays its full role as part of an "open, flexible and developing" mathematical experience (see Steiner, 1987, pp. 8–9).

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[1] Grassmann's earlier work had been on tidal flow—*Theorie der Ebbe und Flut*, 1839.

Institutional and Socio-Economic Context of the "Modern Mathematics" Reform in France

Bernard Charlot

In the late 60s and early 70s, France set forth an important reform of its teaching of mathematics known under the name "modern mathematics reform." This reform was at the order of the day since the early 50s. Thus, in 1956, the mathematician Gustave Choquet compared the teachers of mathematics to "museum guardians" who show dusty objects of which the most part lacks interest.

This reform movement enters in an international context. In 1958, the O.E.C.E. (which then had the name of O.C.D.E.) created a bureau of scientific and technical staff, and, in November 1959, it organized the Conference of Royaumont which had the object to promote a reform of contents and methods of mathematics teaching in secondary schools (12–19 years old). The mathematician J. Dieudonné cried out a "Down with Euclid!" that soon became well known.

Being so started, the reform movement developed in the 60s, especially in France, Belgium, Canada, Great Britain, and in Poland. The I.C.E.M. organized a conference in Utrecht in August 1967 on the subject "How to teach mathematics in order that they would be useful?" and reunited in August 1969 in Lyon, its first International Congress for Mathematical Teaching.

In France, the movement was brought forth by the Association of Mathematics Teachers (A.P.M.E.P.) which created a commission of reflections in 1964 and published in 1968 a project of reform named *The Chambery Charter*. Parallel to that, the Ministry of National Education reunited a commission called Lichnerowicz Commission which published its first report in March 1967.

The students' and pupils' movement of May 1968 accelerated this process. The reform was set forth in 1969 in the secondary instruction and, in 1970, in the primary one. The reflux movement did not delay. So, the opponents protested. Thus, in March 1973, J. Leray stated before the Academy of Sciences: "The reform in course jeopardizes seriously the economic, technical and scientific future of the Country." But the partisans of a reform were worried, too: The A.P.M.E.P. manifested its concern in 1972 and in 1973. So, G. Choquet wrote: "I am bewildered by what I noticed as to the instruction in the primary school as well as in the first cycle of the secondary one."

Indeed, under the name of modern mathematics reform there was set forth a very formal and selective teaching, cut off from every intuitive support. This "perverse effect" proceeded from an institutional, social, and economic context as well as from the mathematics conception that the reform subtends.

The reform of modern mathematics enters clearly in a policy of development and economic modernization. The occidental world was worried about its technological delay, facing the USSR that launched, in October of 1957, its first sputnik. More generally, the occidental countries entered in a period of industrial expansion which took the place of the postwar period of reconstruction.

Now, mathematics are considered to be the key—like every scientific formation.

France lacked scientists, engineers, specialists, and teachers and had to raise the qualification level of its workers. Besides, France gets more urbanized and modernized and is impregnated with the ideology of technical progress. Parallel to this evolution, the idea spreads that mathematics teaching must be reformed in order that the formation would be adapted to the needs of modern society. *Modern:* This is the keyword, the magical one. But also the ambiguous word which functions in a vague system of thought: Mathematics teaching for a modern society, modern teaching of mathematics, teaching of modern mathematics. This reform, moreover, was elaborated and set in a school system which was in a course of restructuration. The Berthoin reform of 1959 prolonged the obligatory scholarship up to the age of 16 and opened the doors of secondary instruction to youngsters who had not access to it previously. This opening, justified by the modern society's needs of qualification, required a pedagogical reform. So, many people thought mathematics teaching would be more democratic than Latin and the classic humanities. The reform of modern mathematics was thus brought forth through faith in the technical progress and through a will to democratize the instruction. This reform, however, turned out to be very selective. This time, the economic, social, and institutional context makes possible a better understanding of what was going on. Indeed, behind the ideology of technical progress that invades the collective conscience, there functions the reality of a growing Taylorization of work organization. The latter got organized more and more around two poles: technicians and "specialized" workers (that is, non-qualified, unskilled ones). The French society gets modernized, but it preserves and even accentuates hierarchic distances.

Being promoted as a forming discipline *par excellence*, mathematics were to be invested likewise with a function of selection and hierarchization of youngsters. Since modern rationality turns out to be essentially mathematical, the mathematics teaching must pick out the "abstract" minds who will manage the production, and the "concrete" minds who, for want of sufficient rationality, will be confined in executive tasks that are less skilled. The key ideas of reformers, however, were generous. The reform is based upon three main principles:

(1) "There are mathematics everywhere, *la mathématique partout.*" Mathematics are presented at the same time as a tool in everyday life and as a way to a privileged access to the scientific and technical thought. These mathematics are conceived as structures study, symbols system, and as language.

(2) One must teach the mathematics of our time, living mathematics of which the unity is profound and which constitutes a universal language. One must then rid the instruction from dead notions which lost their usefulness, their topicality, or their importance. The model is here one of an organism that is ever regenerating and then may be but unitary and modern.

(3) The educational methods of mathematics teaching have to be reformed. The pedagogy must be active, open, without any jargon, and without any precocious axiomatization so that the mathematics teaching may become at one and at the same time useful and democratic.

Nevertheless, modern mathematics teaching will be rapidly victimized by the jargon, the dogmatism, and by an extreme symbolism which, for many students, will conceal the meaning. The promoters of the reform remarked herein a perversion of their intentions. But, in fact, this perversion was contained in their very conception of modern mathematics. Indeed, they define mathematics as a language, not as an activity, and they consider mathematics teaching as a precocious access to fundamental structures. Ever since, students, including the youngest ones, would be asked to locate and nominate structures of which they did not understand the meaning. The mathematics teaching would be an abstract discourse on symbols cut off from their meaning. This teaching would be condemned by itself as a social selection tool and as an ideological alibi of the control of those who are supposed to master the basis of modern rationality over those who are but performers without having access to meaning.

This reform would be slightly corrected in the course of the 70s, but mathematics teaching remained formal and selective. The C section of secondary schools (mathematics, physics) would become a hyper-elitist section which attracted the best students, including those who intended to continue studies of humanities or medicine arts so that, paradoxically, France may lack scientists and mathematicians for it selects the "best" of its youngsters on the basis of mathematics criteria! For ten years, the Ministry of National Education had tried to correct this evolution and thus elaborated in 1985 a new reform which is focused on an *active* teaching of mathematics. But the situation is too difficult to be set right, for teachers of many generations have been formed in an elitist and formal prospect and the "great schools" (les Grandes Ecoles) enlist their students among those who came from the C section of secondary schools.

The reform of the 70s which had to serve the economic development of the country, involved rather some consequences that slackened this very development.

Non-School Alternatives for Mathematics Education

Expanding social needs for mathematical qualification and within-school obstacles preventing the development of a broader mathematical culture, call for new solutions to the problem. John D. Volmink, Republic of South Africa, discusses some examples of non-school mathematics education. Such examples are the more important under conditions of political systems that exclude systematically, for racial, political, religious, or sexual reasons, certain groups of the population from adequate qualifications. Sixto Romero Sanchez, Spain, reports on the experiences with a very successful and socially acknowledged radio program to popularize mathematics developed and conducted by a group of mathematics educator at the polytechnical University of Huelva. Virginia Thompson, USA, discusses the role of the already famous American "FAMILY MATH" Project; and Jeffrey T. Evans, United Kingdom, analyzes, what sort of statistical skills are needed by adults in non-specialist work contexts and how these skills might be developed in more informal settings outside the educational system.

Non-School Alternatives in Mathematics Education

John D. Volmink

To know and to understand is a basic human right. Mathematics, maybe more than any other subject, explains things and helps us to come to know our world. It provides us with the means to think thoughts and to create and examine ideas that we otherwise could not. It also helps us to articulate these ideas and images, which would not be expressible in any other way. Mathematics is therefore a significant means of empowerment. To deny some students access to the process of mathematics is also to predetermine who in society will move ahead and who will stay behind. But at the same time mathematics as it is taught in schools has been disempowering. Indeed D'Ambrosio (1983) points out that

> mathematics has been used as a barrier to social access, reinforcing the power structure which prevails in the societies (of the Third World). No other subject in school serves so well this purpose of reinforcement of power structures as does mathematics.

Both the mathematical content and the methodology used in developing countries are coming under criticism and scrutiny. In his opening address to this conference, Nebres (1988) questioned the "remarkable uniformity" of school curricula all over the world. Damerow (1985) has argued that the canonical school curricula for mathematics was designed for an elite (European elite) and not for the masses in the Third World. Any attempt to 'universalize' these mathematics curricula would be unfeasible and inappropriate. As

we approach the year 2000, it is my belief that while developed countries become more responsive to the dictates of technology, mathematics curricula in developing countries will respond increasingly to the aspirations and purposes of the people.

The traditional made of teaching mathematics in these countries also needs reexamination, since not only the content, but also the methodology used in these contexts have created misconceptions about the subject and about the role of the learner. For so long, learners in mathematics classrooms have been socialized to believe that their own experience, concerns, curiosity and purposes are not important. Mathematics is seen as being devoid of meaning, bearing no relevance either to their every day experience, or to the pertinent issues in their societies. Learning mathematics for these students partakes more of the nature of obedience than of understanding. By contrast, Munir Fasheh (1982) claims that

> ... the main object in teaching mathematics (or any other subject) in developing countries, is to doubt, to enquire, to discover, to see alternatives and most important of all, to construct new perspectives and convictions.

Mathematics is not neutral. It is not independent of people and their purposes; neither is it free of political implications and moral consequences. There is always a mismatch between the intentions of a mathematics curriculum and the actual outcomes, between the needs of technology and the aspirations of people. This is a threat to society wherever it occurs, however, it is particularly harmful in a situation such as South Africa, where the dominant ideology is not referenced to the aspirations of the majority of the people. South Africa is an example of a society which continues to draw a sharp distinction between the few and the many as a fundamental assumption of social organization,—where the ruling class constitutes 1/6 of the entire population, but still represents 96% of the group responsible for all economic decisions and actions—where some are arbitrarily privileged and others deliberately deprived.

The oppressed majority in South Africa have resisted the pressure to become participants in their own exploitation. In the aftermath of the 1976 Soweto riots, a critical awareness has developed in the workplace, the church, and the school, about the importance of self-determination and a firm sense of identity. A resolve has been made not to merely identify with the choices and actions of those in power. Events during the last twelve years have heightened the political consciousness of the majority to the point that black children in South African schools are much more politically astute than their counterparts in white South Africa and in many other parts of the world. They hold up to scrutiny everything read in school texts or which they hear from the teacher. Since the early 1980s, bridges have been built between parents, workers, and students, and the struggles of students have become linked with the daily issues in their communities.

The formal education system for blacks in South Africa was traditionally seen not only as promoting servitude, but also to force the indigenous people to despise their own traditions, aims, and culture and to embrace those of the ruling class. Although this education system at present is firmly controlled by the nationalist government, it should be borne in mind that long before 1948 the education system had its origins in the British Colonial regime with its usual disregard for the legitimate aspirations and culture of the people. Over the years, black education leaders have objected to this lack of fit between the socio-political realities and the imposed educational goals. Students have also joined in this protest, and there have been many school boycotts since 1976. In response to the crisis in education, several non-school alternatives have been sought—some by the private sector, and some by progressive education movements. Those concerned either directly or indirectly with mathematics education are

(a) Private commercial colleges

There has been a proliferation in recent times of private, non-racial colleges, also known as "cram" colleges. These profit-generating institutions are accessible only to a wealthy elite who can afford to pay the exorbitant fees. No challenge to the traditional approach of mathematics education is offered and pupils are occupied solely with the task of performing well enough in mathematics and science in national examinations in order to gain entrance to university.

(b) Supplementary programs funded by the private sector

A number of alternative programs are being run by 'big businesses' corporations in order to help students cope better with the formal school curricula, particularly in science and mathematics. These programs are designed to run parallel to the formal system, without replacing it. While some of these programs could be applauded for the freedom and flexibility they give to the teachers, there are nevertheless others which are clearly suspect. For example, one such program supports a small hand-picked group of black students at exclusive private white schools for post-high school education in mathematics and science. These students come to see this opportunity as a ticket to wealth and status while avoiding the socio-political realities in their own communities. Such blatant attempts to create an apolitical black middle class is viewed with great suspicion by the community.

(c) Peoples' Education Movement

One of the consequences of the struggle in education was the launching of the Peoples' Education Movement at the end of 1985. This was not a response only to the crisis in education, but was also to provide guidelines and a foundation for education in a non-racist, united, democratic, post-apartheid South Africa. While Peoples' Education itself defies definition, it should be seen as a dynamic process, a set of actions which aims to transform the educational institution and will find final embodiment only within a new political structure. A cross-section of the community is

involved in decisions about the quality and content of education. It aims to achieve education for all. The gap between theoretical knowledge and practical life must be bridged and individualism and unhealthy competition should be replaced by collective input, active participation, and critical analysis.

While alternative schools were not seen as realistic possibilities, alternative educational programs were to be pursued both within and without the formal school system. Commissions were set up in Peoples' English, Peoples' History, and Peoples' Mathematics. These were to provide guidelines for a change in content and methodology, student and community participation, and education in the community and workplace. In mathematics, also in History and English, materials were to be developed that would be non-racist, non-sexist, and relevant. The government has recently declared all such materials undesirable and its use within and without schools is prohibited. Some of the leaders of this movement have been detained and are being held in solitary confinement. Although people can be banned and imprisoned, ideas cannot be. The concept of Peoples' Mathematics has taken root in the community, and work in this regard continues in various contexts. For example, in a preservice teacher education course at the University of Cape Town, students of all races cooperatively produce worksheets for use in schools. In these worksheets some information concerning an aspect of the socio-political context of South African society is given as a focusing point. Mathematical questions are then designed which disembed certain specific features usually involving some aspect of inequality or oppression. Some of the issues covered are living conditions in squatter camps, transportation problems, unemployment, working hours, and deaths in townships due to police action.

In summary, in reflecting on the range of non-school alternatives discussed above, it is clear that the first two do not address the question of power. It is only in the third case that the center of control and concern is shifted to the community. It is therefore clear that the last alternative offers the best if not the only possibility of transforming the South African society.

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The Necessity of Popularizing Mathematics via Radio Programs

Sixto Romero Sanchez

Introduction

... They looked for ways of doing what I told there and they found the answer in such a way that they discovered the most interesting and different methods of doing so ... This convinced me that the only valid education is that in which young people participate actively ... Competing young people to draw circles, lines and angles makes their minds more accurate and precise and creates an active energy which will aid their studies and future knowledge. (Pestalozzi, 1.801)

These words, written by the so-called *"Beethoven* of education" serve to introduce and present our experience with the idea we like to call *active learning*.

We are convinced of the importance of presenting lessons, and this applies particularly to mathematics, within a social context which motivates the framing of problematic situations which have to be resolved. The difficulty arises in how to organize and arrange activities so that, within the active learning method, these *problem-solving* strategies can be compared and redirected.

As far as we know, there has not been any attempt to relate mathematics students and the public, with any kind of continuity, in an attempt to popularize mathematics, as up until now the daily press has been the most popular method used. We believe that our break-through is the use of the enormous influence of the *radio*.

For the past three years, four teachers from three different levels of education
- primary (Romero, J.)
- secondary (Merino, M.)
- university (Romero, S. and Contreras, L.)

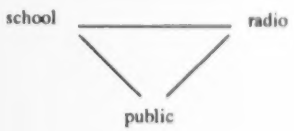
have carried out the experiment which we hereby present and which can be found within a work plan, the aim of which is to familiarize and make mathematics more popular to society in general, using one of the most important methods of communication—the radio.

Our Aims

In general terms, our aims are as follows:

(a) The basic concepts of mathematics cannot be transmitted by too formal and too traditional methods. One must use an important and often forgotten aid, *children's natural curiosity*.

(b) Create in students from 13 to 15 years old an interest in a different approach to mathematics, a pleasant, attractive, and creative way of learning.
(c) Is it not true that a lack of *historical background regarding mathematics*, something often forgotten in traditional teaching methods, is a negative factor in students' progress?
We, in an attempt not to fall into the trap previously mentioned, use the history of mathematics as a prop and as an aid, so that students are able to benefit from a new and different accessory to learning.
(d) Make mathematics more popular by means of the radio, creating a perfect triangle:



(e) Create a team in which students work together collaborating and striving for a common goal, but at the same time bettering and improving their individual progress.

Experiments Carried Out

The work we present is the result of an interesting experiment consisting of 40 radio programs, including the participation of 40 different schools within our province.

Basically, each program is divided into two different parts: The first part consists of a run-down in the history of mathematics according to *Thales of Mileto*, using, wherever possible, the vocabulary and expressions of our pupils. The second, is that in which we present six problems from our Central Control at the radio station, using a mobile unit, which in turn is situated in one particular school. The aforementioned problems cover all relative points necessary to those pupils taking part in the experiment (13–15 years old).

Conclusions

Various surveys made among teachers, students, and listeners of our program have enabled us to draw the following conclusions:

(1) That the pleasant, attractive presentation of mathematics gives much better results than the traditional style used by many teachers.
(2) Difficulties with mathematics are a social/cultural phenomena. We would ask teachers *to try and use a more colloquial, more modern vocabulary as much as possible when teaching, and not to use those formal terms so frequently.*
(3) The mathematics teacher must be very careful when planning his class. He must make sure that the less capable pupil does not find reasons to reject mathematics more than he does already. Encouraging pupils, both the more intelligent and the less capable, to work together in a team, enables the student to participate more actively in class.
(4) This type of experiment opens up new possibilities in dialogue, within the subject of mathematics, giving the student a new way at discussing, agreeing or disagreeing with other opinions.
(5) And, lastly, we have deduced that, at this level (13–15 years) the student begins to discover his own rate of development, both *formative* and *cultural* features which are *perfectly applicable through* the use of aids and different methodologies in the most varied fields.

FAMILY MATH: Linking Home and Mathematics

Virginia Thompson

Parents and their children doing mathematics together and enjoying it! An unheard of event? Not in a FAMILY MATH class or at home after a family has attended such a course. How did this come about?

Background

The program began as an outgrowth of the EQUALS Kindergarten to grade 12 mathematics inservice program, based at the Lawrence Hall of Science, of the University of California, Berkeley. The EQUALS educators work to encourage all students to continue with math courses when they become optional in high school in the United States—particularly those students from groups that are underrepresented in math-based careers. These educators asked EQUALS to develop a program that would also involve parents in addressing this issue.

FAMILY MATH pilot courses began in low socioeconomic neighborhoods in Richmond, California in January 1982 with funding from the U.S. Department of Education. Parents attended class with their children to learn "hands-on" problem-solving activities that would reinforce and supplement the classroom mathematics curriculum. The activities require only inexpensive materials and are designed to be repeated at home. Classes met 1 1/2 to 2 hours once a week for six weeks. They focused at particular grade levels such as K–1, 2–3, 4–6, 7–9 and were made up of 8 to 15 families.

As reported in 1984, although recruitment was the greatest challenge for the program, once parents came to their first class, they tended to come back. Not only did parents who were already active in school activities attend, but many who had not attended prior school events came regularly to FAMILY MATH classes.

The most successful recruitment method to date has been a personal invitation from another parent or community liaison of the same ethnicity. Other methods include recruiting through church and other community based organizations, doing a few activities in the classroom and encouraging the children to bring their parents for more, offering to present a parent meeting on "How to Help Your Child with Math at Home" and recruiting from the audience, and what some might call out and out bribery. An example of the latter case occurred when an instructor's first class had only two families; during the next week, he met the school buses with a large jar of peanuts. "Do you want this? Come to FAMILY MATH. The family who makes the closest estimate to the number of peanuts will win the jar." The next week 17 families came.

Follow-up surveys showed that 80 to 95% of the parents used FAMILY MATH activities at home, talked to their child's teacher, and/or bought a math book or puzzle for their child. In addition, 15 to 20% of the parents took a math course or bought a math book for themselves.

In the fall of 1983, EQUALS developed a two-day inservice workshop to prepare parents, teachers, and interested community members to instruct FAMILY MATH courses in their own schools and communities. During the following year 322 teachers and educators attended these inservices and 103 courses were taught for nearly 1,200 families and over 2,400 parents and children. By 1986, these figures had risen to 2,300 educators, 600 courses, 7,500 families and over 15,000 parents and children. Today the program has spread to 27 states in the U.S. as well as Puerto Rico, Canada, Australia, New Zealand, and Sweden. Many people have initiated classes on their own after reading the FAMILY MATH curriculum guide. Because many of these classes have not been reported, we can only estimate the number of parents and children served through June of 1988 to be 34,000.

FAMILY MATH continues to be well-received and spread to a wide variety of communities and cultural groups, ranging, most recently, from rural northern Louisiana with a large Black population to communities in Arizona educating Indian and Hispanic students to Lund and Luleå in Sweden.

What are the characteristics that generate such interest in the program? One way to answer this question is from the perspectives of the parents who attend classes, their instructors, the people from different ethnic groups involved with the program, and the curriculum itself. We will begin with the latter.

Curriculum

The FAMILY MATH curriculum was developed from a collection of good classroom activities and new ideas. The emphasis was on selecting activities that stress problem solving and encourage talking about mathematical ideas, together with activities that provide information about why math is important to a child's future.

The distinction from the classroom is that FAMILY MATH materials can be repeated at home and require only inexpensive materials. They can be presented in a non-threatening, supportive manner and are of interest across extended age groups. Their oral mode of presentation (backed up with written instructions) creates a comfortable atmosphere regardless of the parent's educational background. The class presentations model a way for parents to work with their children. FAMILY MATH does not attempt to teach parents how to teach their children mathematical algorithms, but rather to facilitate their doing math activities and games with their children and talking about mathematical ideas at home.

The FAMILY MATH book, funded in part by the Carnegie Foundation (also available in Spanish and Swedish), not only presents the curriculum, but provides guidance for starting FAMILY MATH classes. A 17-minute film/video also documents the program.

Another reason for the rapid spread of the program is the ownership that instructors feel for their courses. They feel free to add new activities and take off in new directions. The Northwest EQUALS site in Portland, Oregon has developed a Family Science program. The FAMILY MATHs Project Australia has received funding to develop science and technology activities for their country. At the Lawrence Hall of Science the EQUALS staff is planning a Family Computer inservice.

Parents

Parents want to help their children with math, but often do not know where to begin. Limited education, poor experiences in learning mathematics themselves, and confusion about the "new" math often leave parents feeling that they are incapable of helping their children.

They come to FAMILY MATH classes expecting to watch their children learn their multiplication tables and are surprised when they find themselves enjoying the class and learning along with their children.

Anonymous comments on follow-up surveys and interviews from a new National Foundation Study (The Families of FAMILY MATH) indicate that parents are also surprised to see their children take a leadership role in the family learning. Watching their children think through problems is a new opportunity for many parents. Parents also report that they leave classes with a broader concept of mathematics—that it is more than arithmetic algorithms—and how it can affect their child's future work and education. For many, especially single parents, the most valuable aspect of the class is the "quality" time spent with their child during class, even when they continue to use activities in mathematical terms.

One parent who had gone to community college to make up the math he did not take in high school

wrote, "It's too late for some people to go back; well, it's not too late for my children, that's why we have to try."

Instructors

FAMILY MATH instructors include parents, teachers, and community members. They choose to conduct classes because they want to help more parents become involved in their children's mathematics education, to convey the importance of mathematics in a child's schooling and the importance of the equity issues that surround the subject.

Those who are school teachers also want to improve school-home relations, to get to know parents better, and to inform parents of changes in the mathematics curriculum (away from the rote learning of arithmetic to an emphasis on problem solving across a wider set of topics). Some hope to enlist the parents to bring pressure for further curriculum changes. Many classroom teachers also see the program as an opportunity for professional growth.

All of the instructors have been surprised at the enthusiastic response of the parents and how the classes enriched communication between family members. The classroom teachers see an increased interest in math from their students together with more motivation to study. Some claim that test scores improve for students who had attended FAMILY MATH. The school-based programs do improve parent involvement, frequently reaching families who have not previously been active in school affairs.

However, the most impressive and rewarding outcome for the instructors is how much the parents appreciate the classes and their teaching. Parents have known that they can help with reading and language arts by taking their children to the library, talking about the books their children have read and reading bedtime stories to them; all pleasant activities. But what have parents known to do to help with math? Help their children memorize the multiplication tables, usually with flash cards, and insist on homework being done before playtime? These interactions frequently do not turn out to be so pleasant. FAMILY MATH instructors are providing parents with math activities that engender feelings more like those of reading and discussing bedtime stories. No wonder parents are grateful.

Ethnic Diversity

The first FAMILY MATH classes included Black, White, and a few Spanish-speaking families. Later classes have included Hmong, Mien, Laotian, Vietnamese, Greek, Chinese, Native American, and more Spanish-speaking families in the United States and Canada. The activities have been well-received by all of these groups. However, more intense recruitment methods such as those mentioned earlier were necessary for parents for whom English is a second language.

In 1985, the program received a National Science Foundation grant for dissemination via five minority community-based organizations: the Oregon Indian Education Association; the National Urban Coalition and Indianapolis Urban League, both of which work with inner-city Blacks, and two Hispanic organizations, Alba in San Diego, California, and Valle del Sol in Phoenix, Arizona. These groups saw FAMILY MATH and its relationship to their function in two distinct ways. For some, improving math education and increasing the pool of children who might become scientists and technicians was the goal and the involvement of families the vehicle. For others, family education was the goal and mathematics the vehicle. Despite these differences, all of the organizations report that the program worked with their constituents and that the activities needed few adaptations.

One organization leader commented that FAMILY MATH is successful because it is one of the few programs that respects her constituents.

> You are offering us something to share with parents rather than telling them that they are deficient and you have the cure. A subtle but important and essential distinction to make a successful project.

Some examples from the organizations will illustrate how they fit FAMILY MATH to their situations. The Native Americans stress that the cooperative family approach fits very well into their culture. They easily found cultural examples to illustrate many of the mathematical principles in the activities, such as discussing how someone who does beadwork on three-dimensional objects would be aware of how the height and circumference of a cylinder would compare.

The Urban Coalition and Urban League work with families in church settings, recruiting their instructors from the congregation or parents themselves. When a child cannot find an adult to bring to class, church members serve as surrogate parents, often initiating significant changes in the child's behavior and attitudes.

The Hispanic groups have presented many classes in Spanish. In San Diego teams of one teacher and four parents taught. The workshop to prepare these instructors was given in Spanish to ensure that the correct Spanish mathematical terms were used in the classes.

In Arizona, one potential instructor came from a community of Yaqui Indians and Spanish-speaking parents. At first she was skeptical, saying it sounds too simple. But after one successful class series, she was convinced. Next she organized a series for senior citizens (many of whom were grade-school dropouts) to work with 5 to 8-year-olds and one for high school students to work with 9 to 12-year-olds. All in an area where everyone says, "you'll never get parents to do anything."

Conclusions

In closing, perhaps we should ponder a comment from a FAMILY MATH instructor:

Most parents would not expect children to become good readers if they never read to them at home. They don't seem to have the same idea about math because nonprofessionals doing math is not in the [U.S.] culture. In that respect these classes can play a role; they point out that doing math at home is important and that parents have a part to play.

For those of us from the United States, what does this say about our culture and us as mathematics educators? How should we be addressing these issues in our work?

For those from other countries, are these questions relevant to your situations?

Mathematics for Adults: Community Research and the "Barefoot Statistician"

Jeffrey T. Evans

I use the idea of a barefoot statistician in two senses in this paper. The first sense draws on the idea of a "barefoot doctor" and refers to adults doing research in their local community. The second sense I will argue provides a metaphor for the competences we might expect from a student not specializing in mathematics, but taking one or two courses in mathematics or statistics at tertiary, or late secondary, level.[1]

Introducing the 'Barefoot Statistician'

Barefoot doctors or village health workers were first trained in China in the 1930s. Since then, there have been attempts to spread the idea to other parts of Asia, Africa, and Latin America. The "idealtypical" barefoot doctor provides health care based on disease prevention and health education (about e.g., nutrition and sanitation), depending on the expressed needs of the local community. In some settings, they also undertake to diagnose and to treat the most common complaints, using a simplified conceptual framework and set of treatments. However, barefoot doctors know how to recognize a set of symptoms that they should *not* attempt to treat—but should refer instead to the nearest clinic or hospital. They have minimal formal qualifications (except for literacy), and training normally takes place in the rural community itself (based on Doyal, 1979, pp. 288–290).

This suggests parallel aspects of what might be a "barefoot statistician's" role. They will take a lead in producing and analyzing information, normally using very basic techniques, and responding to the needs of their community (e.g., as part of pressure groups in a neighborhood or work groups). They need to be able to present information in terms that are both comprehensible to their community (see Evans, 1986), and powerful in the discourses within which they may sometimes need to argue (e.g., those of state bureaucracies). Barefoot statisticians will normally have at most very basic training in numeracy and perhaps statistics, and will probably be (self-)selected into any further training.

Are there any "barefoot statisticians" in existence? Examples may be hard to find, because people may not recognize themselves in such a role. But community research may be characterized as a way for members "to participate in the construction and implementation of social policies," by producing facts and figures or accounts of experiences, so as to present a picture of needs, and of the adequacy (or otherwise) of resources to meet those needs (Cooper, 1986).[2]

Here are some examples:

- a survey of local feelings about dog excrement on the footpaths, done by a woman to put pressure on the local authority to enforce the regulations;
- a study by a social science student on placement of the evolving "psychological identity" of young people for her own ethnic community group (see Cooper, 1986);
- a Mozambican farm-worker seeking to estimate, without scales, the weight of a cow, so as to determine the proper amount of medicine to administer (for this and other examples, see Gerdes, 1985).

I would identify the person(s) doing such work as likely to be barefoot statisticians (in the sense above). What they have in common is minimal training (if any) in mathematics or statistics, the use of very basic techniques, and a commitment to service to the community.

Content of a Barefoot Statistician's Training

Of course, by no means all barefoot statisticians are conscious of wanting further training, but some definitely do (Cooper, 1986, p. 44).

Because their needs are varied, only guidelines to topics likely to be important can be given here:

(a) the *production of data*, including how the process must be shaped by the context; e.g., the way decisions about whether to use questionnaires or interviews depend on the resources available;
(b) knowledge of the availability, and means of accessing, of *sources* of relevant information; for example, official indicators (national and local); institutional files; oral history;
(c) familiarity with ideas of *research design;* e.g., sampling and the resultant limits on generalizing results;

[1] The issues in this paper are viewed from my position as a resident of the "North," but I have tried to develop a perspective which remains open to examples, and lessons from the "South."

[2] This characterization of community research has much in common with "participatory research," as widely discussed in the developing world (see Society for Participatory Research in Asia, 1982).

(d) *decision-trees* for use while gaining knowledge and experience; for example, how levels of measurement (nominal, ordinal, interval, ...) determine which summary statistics to use (see e.g., the inside front cover of Blalock, 1972);[3]
(e) experience with *computation technology* appropriate to the resources available—and enhanced access to it, where possible;
(f) a variety of methods and languages for the *presentation of data* to particular audiences.

It is important that *intuitive conceptions* and methods of barefoot statisticians be engaged with and, where possible, built on. For example: if it were necessary to show how to calculate compound probabilities, we might use the knowledge of bookies (based on memory or tables of permutations and combinations) (Acioly & Schliemann, 1986, in Brazil)—or those of participants in local betting games (Wang, 1987, writing of Malaysia).

Thus, the barefoot statistician's training should be both *drawing out* of some of their existing ideas and methods, and a *challenging* of them.

Learning Resources for Barefoot Statisticians

Many educational institutions offer courses, which might meet the needs of barefoot statisticians. For example, my own offers *Using Data*—a one-semester course available in term-time or at summer school. It concentrates on data sources, graphical and tabular presentation, Exploratory Data Analysis, simple line-fitting (sub-batch median method) and "intuitive" inference.

However, we must realize that most barefoot statisticians will be unable or unwilling to attend a conventional course. They need training facilities which are flexible and immediate advice sometimes.

It is important, therefore, that a range of different resources be available. Here are some examples from North London in the UK:

- *Statistics in Society* (Open University, 1983): an institutionally paced "distance" course during the Open University academic year; the materials are accessible in many libraries and bookshops; addresses a range of topical problems in the economy, education, and health.
- *POSE* (Project on Statistical Education) (Schools Council, 1980): a set of resources including teaching units, teachers' notes and pupils' resource sheets; widely used in schools, and in some tertiary institutions; addresses problems arising in the immediate context of students' lives, as well as national policy problems.
- *Radical Statistics Group:* a national network of statisticians and social scientists in the UK, committed to:
  - the critique of statistics as used in the policy-making process (see e.g., Radical Statistics Health Group, 1987; Irvine Miles & Evans, 1979);
  - support for oppressed groups (e.g., Radical Statistics Race Group, 1989); and also
  - the competence-building of potential barefoot statisticians; for example:
    * practising teachers (Radical Statistics Education Group, 1982);
    * workers subject to occupational health hazards (Radical Statistics Health Group, 1982).
- *Community Research Advisory Centre* (CRAC): a center established at the Polytechnic of North London; carried out a survey of 220 groups in North London to identify the origins, conduct and outcomes of community research; offers advice, workshops, and short courses on basic research methods, statistics, data processing, information searches, and dissemination of project results (Cooper, 1986).

Finally, it is valuable for the barefoot statistician to establish links with a "consultant" statistician or researcher who can offer *advice* concerning a problem at hand, and *competence-building* in connection with approaches which fall outside of the barefoot statistician's training, e.g., the use of certain statistical modeling procedures. Both CRAC and Radical Statistics offer such support.

Barefoot Statisticians in our Educational Institutions?

The first sense of "barefoot statistician" discussed above may be somewhat restrictive: There are not all that many barefoot statisticians, practising or potential, in any community, and so there cannot be that much institutional involvement in planning their training. However, a great proportion of mathematics courses in tertiary and secondary education are taken by students who are not mathematics specialists.

Many of these recruits to our courses are reluctant. Yet they need knowledge and skills in producing (or retrieving), analyzing and interpreting information that is relevant to their "adult lives"; that is, (a) their everyday activities, (b) their (paid or voluntary) work, and (c) the protection/promotion of their social and political interests. This is a somewhat broader sense of "community research."

I would argue that "community research" knowledge and skills might be developed by a mathematics course with an applied statistics flavor. Applied statistics would constitute an "appropriable mathematics" in the three senses set out by Papert (1980, p. 54)—that is, it should be personally meaningful, empowering, and have cultural resonance.

Thus, in designing courses in mathematics for adults, it may be helpful to think of these adults as *barefoot statisticians*, and of community research as an appropriate "microworld," or pedagogic context, in which to locate as meaningful the contents of the

[3]There may also be a role for computerized "expert systems" (Hand, 1987).

course. This may be useful for various courses in higher education, in further education, or at school. (For example, there are many similarities between the course *Using Data* described above and current GCSE courses from the Midland Examining Group and the Northern Examining Authority in the UK.)

What would be distinctive about these courses for barefoot statisticians? The relationship between the course material and the students' adult lives can be illustrated and strengthened by:

(a) encouraging them to bring problems from their daily activities;
(b) showing the importance of the context and methods of data production for the interpretation of the results;
(c) using teaching and assessment methods which encourage the active use of investigations and projects, preferably on issues which can be fed back into their adult lives.[4]

Of course, there remains the question of whether to "begin from" the practice of community research, or from the mathematics and statistics. This will depend on the teacher's experience, commitments, and confidence—and the students' as well.

For those who choose to begin with community research, and thereby to "motivate" the mathematics, it is clear that the contents of an applied statistics course include a number of issues with potential for leading to statistical and mathematical investigations; for example: measurement scales, leading to properties of sets and number systems; frequency distributions and modeling, leading to graphs and functions; distributions and sampling, leading to probability; and hypothesis tests leading to logic (truth tables) and probability.

Conclusions

(1) The idea of a "barefoot statistician" is useful, either to describe a number of adults who spend some part of their time engaged in community research and who might wish additional training in applied statistics—*or* to designate some of the large number of students who, though not specializing in mathematics, take one or two—often compulsory—mathematics or statistics courses at post-16.
(2) Guidelines for the training of a barefoot statistician (in the first sense), can be given as well as an indication of some of the resources available (for North America, see Frankenstein, in press; for the developing world, see, e.g., Society for Participatory Research in Asia, 1982).
(3) One particular need of barefoot statisticians is for access to a "consultant" statistician—and, by the same token, "community consulting" is a potential growth area for statisticians. Universities, colleges, and schools must encourage staff (and students) to do "community consulting," and allow barefoot statisticians' access to information technology. (CRAC at the Polytechnic of North London provides a model.)
(4) Many potential barefoot statisticians (in the second sense) might be attracted to the sorts of skills and competences that could be developed in a community research context; once these students are "won over" to an applied statistics course, many of the issues have the potential of leading to topics of interest for mathematical investigations.

[4]For example, at Middlesex Polytechnic, recent community research projects done by 2nd-year Social Policy students include: students' income and spending patterns; the effects of closure of a children's ward at a local hospital; the need for greater spending on books in the site library.

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The Mathematical Demands of the Economy

An important social constraint on mathematics education is given by the various demands of the economy with its challenges and its risks. Guida Maria Correia P. de Abreu and David William Carraher, Brazil, show that, although the general mathematical abilities of the Brazilian sugar cane workers vary greatly because of their poor schooling, they use knowledge and techniques quite well to measure and calculate which are cultural in nature, having their roots in the social traditions of the sugar cane communities. Wang Chang Pei, China, gives examples of differences in mathematics education between rural and urban areas in China, despite the national uniform curriculum. Howard Russel, Canada, presents the "Generic Skills/ Economic Development Project" as an attempt to match the demands of the economy. On the other hand, Bernhelm Booß-Bavnbek and Glen Pate, Denmark, shows how a rapidly growing number of mathematical models and simulations turn out to be sufficient for the design of workable technological improvement, but many of them are not sufficiently transparent to provide the necessary means for their technological and social control. This places new and urgent demands on public education in mathematics for a democratic society.

The Mathematics of Brazilian Sugar Cane Farmers[1]

Guida Maria Correia P. de Abreu
and David William Carraher

While most of the sugar cane produced in the Northeast of Brazil is from large plantations, over 90% of the growers work on small farms[2], where they personally participate in the activities of land measurement, fertilization, payment of workers, as well as dealing with the sugar mills, to which the cane is sold. These activities engage farmers in mathematical problem solving. While the use of mathematics by farmers depends upon the cognitive abilities of individuals, the knowledge and techniques used to measure and calculate are cultural in nature, rooted in the sugar cane community's traditions. This interplay between personal meaning and cultural representations, which are part of a group's knowledge, can be found in the activity of calculating areas, which we focus upon here.

Clearing the land, tilling, and planting are paid by the area worked. In order for a farmer to determine the area of a plot of land, it must conform to one of the 'valid' geometric shapes (quadrilaterals and triangles) or be divided into valid regions, which are individually computed. The area of a plot of land is determined by going from a linear measure (the *braça)* to an estimate of area obtained through formulas known by farmers. The *braça* refers both to a unit of standard length—2,20 m—as well as to an unmarked wooden rod used for measuring, which in reality may depart slightly from the standard length.[3]

We will follow in this paper the process of abstraction involved in going from linear measures to measures of area, which is necessary in many of their everyday activities.

Measurement in the field begins by pacing off the sides of the plot in *braça* lengths. The farmer grips the *braça* in the middle and briskly walks from one corner to the next in an approximately straight line, establishing the distance as follows. One tip of the rod is placed on the ground ahead of the measurer, while the other tip begins at about 80 cm above the ground at the corner. The rod is then revolved upward and forward from the rear around the front point. Finally, the rod revolves in the same direction around the grip, such that the tip touches the ground and the other raises. One braça has been measured. Since the rod is not laid flat on the ground, measurement is rough. However, the farmer only wishes to obtain a result in

[1] The authors wish to thank Terezinha Carraher, Analúcia Schliemann, and Paulo Figueiredo and for their very helpful comments and suggestions regarding the present paper.
[2] Small scale production refers to farms with an output of less than 5,000 tons of sugar per year (cf. de Araújo et al., 1987).
[3] Taken literally *braça* means "arm span." The *braça* measure also appears among mat weavers from the same region; however, in this case, it is indeed a measure of arm span which, evidently, varies from person to person.

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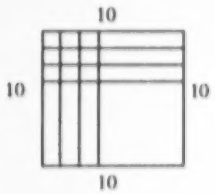
whole numbers of *braças;* slight imprecisions are either unnoticed or tolerated. When the farmer reaches the second corner, he has the length of the side in braças; the same procedure is used to determine the lengths of the remaining sides.

Cubando a terra, literally, "cubing the land," refers to determining the number of 'cubes' (squares of 1 braça length sides) in the surface. The relationships among meters, *braças*, cubes, and *contas* are explained below by Sr. José, an unschooled farmer, aged 72:



Interviewer: What's a braça?

J: A meter is 100 cm and a braça ... braças accepted by agronomists are 2 m long, but for people from the countryside the braça is 2 m and 20 ... It's a stick like that one standing up over there [he points to it]. Well, we measure for example a *conta* 10 by 10 *[braças]*, so it's not necessary to cube because 10 by 10 is 100, 100 cubes is a *conta* and this here [the figure] ... nobody ever made this for you, m'am!? Look here, 10 *braças*, 4 sides of 10, here there are 100 cubes inside. Now the cubes are these things [pointing to a square].

E: This is a cube?

J: Here inside the square you'll find 100 squares like this.

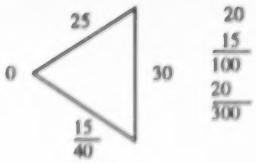
E: Each one is a cube?

J: It's a cube, it's one *braça* like this [runs finger around the perimeter of a cube].

The area of a quadrilateral is found by summing the opposite sides, halving each sum, and then multiplying the two resultant numbers. This corresponds to the formula: Area = ((side 1 + its opposite)/2) times ((side 3 + opposite)/2). This formula happens to be correct for rectangles but incorrect for other four-sided figures; nonetheless it provides a reasonable approximation for most practical purposes.

The areas of triangular plots are calculated by multiplying the average of two "opposite" sides by one half the length of the remaining side. Some farmers spontaneously expressed the view that the triangular plot is a four-sided figure with one side of zero length.[4] In their drawings they would label the three sides and mark a zero next to one of the vertices. Determining the average of two sides is viewed as sharing braças between opposite sides so that each has the same amount; some farmers explicitly refer to the process as "lending." Sr. Manoel explains his drawing (see figure) and calculations as follows:

[4]The procedure for determining the area of triangles is formally incorrect, normally resulting in an overestimation of the actual area.



Right, the triangle, right, here I'll measure this area of land, here there's 30 braça, now here there's 25, here on the side there's 15 and now here at the termination (vertex) it's zero. Now let's see how many cubes there are here ... Five and five are ten, ten, carry the one, three become 4. That means it [the sum of two sides] gives 40 braças. 40 braças divided in the middle, that means 20 braças. Now here, this 30 braça, I have to give 15 for here [one side] and 15 for here [the vertex with the zero]. Now [multiplying the averages] 5 times nothing is nothing. 5 times 2 is 10. One times nothing is nothing, nothing and 1 times 2 is 2, nothing, and 3 ... that is ... 30 ... 30 cubes ... 300 cubes.

Although *braças* times *braças* yields cubes, farmers wish to obtain results in *contas* (100 cubes), the area unit measure according to which fertilization requirements and work production are assessed. The conversion is straight-forward yet noteworthy. 378 cubes, for example, become "three contas and 78 cubos." This expression represents the area as the sum of two natural numbers, each referring to distinct units of known relative magnitude (100:1) rather than as a single measure ('3.78 contas,' or '3 contas and 78/100 of a conta,' for example). Such a representation maintains meaningful referents for areas of less than a whole conta. Furthermore, the representation avoids the need for a general system for expressing rational numbers, since fractional parts (cubes) are always expressed in units one hundredth the magnitude of the larger unit. Hence, the farmers can calculate areas without being hampered by difficulties of decimal and fractional notation (Resnick et al., 1988) inherent in understanding, for example, why 3.78 contas is greater than 3.555 contas.

In calculating the areas area of land, farmers tended to eliminate halves which arise when averaging sides of odd and even lengths.[5] This appears to be part of a general tendency to avoid non-integers. In determining the average weight of tied bundles of sugar cane[6] weighing 11, 7, and 8 kg, for example, 22 farmers of 26 gave integer responses; the 4 exceptions were all "8 and one half kg," including the response of a farmer who used a calculator. Conversion from their measures to the metric system is likewise rounded off: all farmers interviewed treated 20 *contas* as equivalent to

[5]However, the following protocol of Sr. José displays how he recognizes that fractions of braças may play a part in the determination of an area. In multiplying 11.5 by 7.5 (which he does without using a decimal point) he obtains 8625, which he explains as follows:

"Here it looks to us like the answer is 8625, but it's not *that* number ... you have to cut out these two 'letters' [2 and 5] which are only good for the proof 5. 11 [multiplied] by 7 only gives 77, so with these two halves it moved up to 86. There's 9 more. These 9 [cubes] are what made use of the two halves."

[6]Cutters are normally paid according to weight. The farmer samples two bundles, weighs them, and uses their average to estimate the weight of the cane cut.

one *hectare* (10,000 m^2), although the area actually falls short by 320 m^2.

Concluding Remarks

The sugar-cane farmers of northeastern Brazil have their own measures and procedures of determining land area, which are not taken into account in Brazilian mathematical education. While their procedures might at first appear to be simply incorrect, our interviews show that farmers indeed understand how measures of linear magnitudes are related to areas. The imprecision accepted by farmers appears not to be due to carelessness, but rather motivated by practical considerations and the preference for working with integer values. The estimates of farmers in the South of Brazil on diverse mathematical tasks (Grando, 1988) have been found to be much closer to correct answers than those of students, who tend to lose touch with the problem and producing answers off by orders of magnitude. T. Carraher (1988) has discussed these findings in terms of the distinction between as problems 'generated by meaning' and 'problems generated by syntax.'

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Differences in Mathematics Education Between Rural Area and Urban Area in China

Wang Chang Pei

Background

The original motivation for presenting this paper is based on the following considerations:

China is a well-known old agricultural country with more than 800 million rural population, which makes up 80% of all Chinese on the mainland.

It is clear that one of the keys to Chinese education lies in the rural areas. There are remarkable differences between town and country in terms of economic development, social structure, life-style, and ways of thinking. But, what about education? It may be necessary to understand how those factors affect education, and mathematics education in particular.

Moreover, it is said that many deep and great changes have happened since 1979 in China. Much evidence shows that the recent reforms of economy have already produced some impact on education both in urban and rural areas.

It is suggested that we have taken a dynamic point of view, when we try to understand the differences mentioned above.

It is a challenge for me to discuss the differences between town and country in relation to mathematics education. Hopefully, some issues identified and discussed in existing literature have served to develop different frameworks for understanding the complexity of mathematics education, which is a process developing under the influence of factors, which are themselves of a social nature and in continual change. It is helpful to encourage us to discard the previous naive and simplistic mode of describing the interaction between mathematics education and those influential factors. In this paper, I try to choose some issues which are not only relevant to our topic but also "hot" subjects in today's China.

As far as this paper is concerned, so-called "mathematics education" refers only to mathematics education in primary and secondary schools. All schools, in China, are classified into the following categories: city schools, town and county schools, and rural schools. But, I will describe both city schools and town and county schools as urban schools. The figures below may help me to explain why we have to pay more attention to education problems in rural areas.

Primary Schools	Number of students
city schools	11 million
town and county schools	10 million
rural schools	115 million

Junior Secondary Schools	Number of students
city schools	6 million
town and county schools	6 million
rural schools	27 million

Senior Secondary Schools	Number of students
city schools	2.2 million
town and county schools	2.7 million
rural schools	2 million

That is to say: In rural areas, there are 110 million primary school students, which account for 87% of the total population of primary school students in China.

Most children of primary school age are able to receive 5 (or 6) years of primary schooling. Recent statistics also show that most adolescents (12–14 years) are receiving junior secondary education, but in rural areas junior secondary schools can only accept 60% of adolescents (13–15 years old).

Nature of Recent Developments in General

The major changes which have happened in China are not only characterized by so-called "modernization," but also by a series of deeper reforms such as: development of a commodity economy, formation of markets and introduction of competitive mechanisms, etc. These developments are so new that all Chinese have to adapt themselves to meet the changes. In a sense, it is a challenge for all Chinese: "The Big Pot" has been broken, what am I going to do now? Moreover, I could list a few characteristics of rural areas in comparison with urban areas. These are:

- agricultural methods which are traditional;
- feudal economy, self-supporting;
- ill-educated—200 million of the population are illiterate;
- ways of thinking are more conservative, etc.

Rural development could be characterized by: traditional agriculture being transferred to modern agriculture and feudal economy being replaced by large-scale commodity economy.

How do these changes affect education and mathematics education in particular? First of all, I would like to invite you to look at the government's policy.

Multiple Levels

The government has formulated a series of strategies to reform education: It decided to put the 9 years of compulsory education into effect. But the old "uniform development of all people" has been given up. The arguments are: The education is conditioned by limited resources, vast territory, uneven development, etc.

The gap between town and country has been taken into account. That is to say, the authority has worked out different schedules and levels of criteria for rural areas.

(1) Modest criteria for rural areas
The authority defined "literacy and numeracy" by means of different levels. The modest standard is: Possess capacity to know 2,500 Chinese characters and matter the four fundamental operations of integers. And, these are also taken as the criteria for primary school graduation. But, this just is equivalent to the standard of 4th grade in ordinary urban schools.

(2) Schooling and streaming
Urban schools have a 6–3–3 (6 years primary, 3 years junior secondary, 3 years senior secondary) schooling system, while rural schools are asked to use a 5–3 schooling system. It is expected that 5 years of primary education is acceptable even to poorer farmers. It enables most children to fulfill modest, but completed primary education. But, the 5–3 policy has not been accepted by all rural local authorities. Many rural schools have changed to the 6–3 system.

The schooling system results in different steaming. For urban schools, students are steamed after they fulfill junior secondary education. Then they can continue secondary education, or choose vocational education. Since the steam is so important for the children's future, there has been a highly competitive examination to filter students. This also affects primary school education. For most urban schools, reputation and quality is decided by the proportion of students entering high status junior schools called 'key schools.' As far as mathematics education is concerned, the goal for teaching mathematics, in some sense, has become passing the final examination.

Moreover, in rural areas, some students have to stop their education after they graduate from primary school. And, the rest of them are streamed again after they complete their junior secondary education. This is to say, most of them have to go back to their village and work with their parents. Although most rural schools follow the policy of urban schools in working towards examinations, less pressure is put on rural students.

(3) Teaching program
There are different teaching programs for urban and rural schools. Briefly speaking; urban schools are asked to offer all subjects (up to 12 subjects during the six years' schooling), while rural schools are allowed to adapt the teaching programs, or reduce the number of subjects to be taught in school. What is more, some rural schools known as "simple schools" are allowed to run two subjects only, these are mathematics and Chinese. Since there are fewer subjects in rural schools, more class hours are used for teaching mathematics.

(4) Curriculum and textbooks
Although the curriculum and syllabus are still controlled by the central education authority, people have

been allowed to produce different kinds of textbooks for choice by schools. It also aims to encourage more competition and different styles of teaching. But under the control of examination pressure, the teachers have to change the curriculum in these books, in accordance with their experience, examination guide books, etc. Even students and parents can exert pressure on their teachers to force them to change their way of teaching.

Obviously urban schools, the key schools in particular, play a leading role in such examination games.

(5) Administration

The administration of primary and secondary schools has been transferred to a lower local level. For example, rural primary schools are run by village councils (the lowest administrative agency of government in rural area). This will strengthen the linkage between rural school and local development.

It is clear that the government adopts a flexible strategy for developing primary education at different levels in accordance with local living conditions and environment, we have not mentioned many other external and internal factors, but it seems that many factors (including expectation, attitude of government) are in favor of creating a better quality of primary education in urban areas. And it is a fact that the gap exists between town and country. But we have a different picture in secondary school, where the government carries out a more uniform policy. As we are going to see later, there is divergence of views and expectations related to secondary education.

Different Attitudes

Government and educators have made great efforts to develop junior secondary education both in rural and urban areas. Now more than 60% of primary graduates are able to go to junior secondary school in rural areas. But many adolescents and their parents do not take advantage of the opportunity to get advanced education.

It was recently reported that 40% of adolescents left school before they completed junior secondary education in rural areas in one of the north provinces. What is more, of the rest of the students who stayed at school, 60% of them were failures, who did not reach the standard of graduation.

In 1988 another investigation which covered nine provinces was undertaken. It demonstrated the same result: many adolescents (in rural areas) left schools, without finishing their junior secondary education. Although, a few local authorities resort to compulsion to keep the majority of students at school, many students hate studying. The investigation found that more than 20% of the students dislike studying. There is more data to demonstrate that a negative attitude towards education in general is consistent with a negative attitude towards education in general is consistent with a negative attitude towards mathematics study. For example, in the province mentioned above, 74% of the students were failures in mathematics examination; they could not achieve the graduate standard in the final mathematics examination of junior secondary school. Furthermore, the percentage of students whose scores were below 20% was 31%. Overall, this evidence tells us that many students and their parents vote with their feet to reject the "kind offer" by policy-makers and education.

An analysis made by a survey from the south of China revealed that most adolescents who left school felt that it was easy to find a "well paid" job, since local village and town enterprises were expanding rapidly. At the same time, some of them were failures in schools where teachers and hard-working students despised them. In fact, rural schools are also in favor of "able students." Now, rural education has become a "hot" subject in Chinese society, even in parliament. People complain about:

- poor quality of teachers in rural areas;
- short-sighted farmers;
- the present curriculum, which has nothing to do with rural and local development;
- schools which do not meet the needs of society and individuals, etc.

In urban areas, a similar phenomenon could be found. Although few students leave schools early, their attitude towards education is passive except for in a few "key" schools. One of the examples is that more and more parents and students are interested in vocational education rather than senior secondary education (which is for preparing students to go to the university). It might be good news for developing vocational education, but it could result in people holding a negative attitude towards education. Now newspapers call this "the second wave" of "rejecting study." The "first wave" dominated Chinese schools for about ten years during the so-called "cultural revolution."

Rural and urban areas have something in common. Yesterday, people believed in the observation "those who hold a pen earn more than those who wield a hoe." Now, the joke is "those who shave people's heads (a barber) earn more than those who operate on people's heads (a surgeon)."

The present economic development has proven to be a deep and complex process. Interaction between economic development and education is complex too, although it is unanimously agreed that they help each other forward. Moreover, if the reform is going to break the old economic structure and value system, chaos might happen. It is true in today's China.

As far as secondary mathematics is concerned, some pilot programs have been developed in order to make secondary mathematics more usable in rural development. But they only emphasize content, such as measurement, use of abacus, etc. But the improvement of mathematics education in rural areas is far more than that. Is it not? What is more, do we need to develop different programs for rural schools and urban schools? Do we also need to take into account that half of the students are going to go to vocational schools in urban areas? Does the present curriculum prepare them for that?

What is the teacher's attitude when our mathematics education is facing these challenges?

We have conducted an investigation to look at this issue. The results are interesting. For example, the majority of school mathematics teachers considered "knowledge transfer" as the most important goal for teaching mathematics both in rural and urban areas. But it is different from those lecturers of normal schools (or colleges)—who are teacher trainers. They considered "encouraging logical thinking capacity" to be the most important goal for teachnig mathematics. On the other hand, a research report published in 1987 said that most education innovations were limited to the improvement of teaching methods. That is to say, the majority of teachers are only concerned with how to teach the content in the traditional curriculum. Few of them address deeper questions, such as 'Why do we teach this?' 'What should be taught under today's circumstances?' etc.

In rural schools, many teachers are fighting for survival in the classroom. As a matter of fact, many of them are poorly prepared for teaching. For instance, a class teaching observation study in 1988 showed that 60% of observed classes were labelled as "the teacher made some mistakes in knowledge," while 20% of them are defined as "the teacher made serious mistakes of knowledge." It is clear that the poor quality of teachers and their traditional philosophy of mathematics education might block any effort which aims to readjust school mathematics to meet the needs of development in the economy and society.

Although there are many things which have to be mentioned here, I should like to draw attention to the fact that the relationship between students, teachers, and parents has shown some change too. Since the whole of society has become more open, more democratic, and able to communicate, young teenagers, even children, have tended to be more independent and more concerned about individual values, etc. This is also a challenge for teachers. Is it necessary to encourage more discussion and more negotiation in the whole precess of teaching and learning?

All in all, "mathematics education for all" is a real challenge for all of us in today's China. It might imply a series of deep changes and reforms in terms of curriculum and teaching methods, as well as in the whole philosophy of mathematics education.

The Generic Skills Economic Development Project: The Mathematical Demands of the Economy

Howard Russell

There are many forces impacting on the mathematics curricula of today's schools and the ones in focus in this paper are the requirements of the workplace. The name of the project to be described is "Generic Skills/Economic Development," and it has been underway in Toronto, Canada since 1984. Its influence to date has been almost exclusively in the corporate training programs of the host corporations, but it is likely to begin to have some significant impact on school curricula within two years. Whatever that impact may be, it is clear even at this early stage of the enterprise that the needs of employers as identified by GS/ED will only be met if they are also the needs of the workers, who are the primary clients of the educational system. The incentive that the workers have in seeking a closer connection between the substance of school curricula and the requirements of the workplace is to increase their capability to produce better goods and services, which they themselves can share through increases in their paychecks.

The GS/ED project focuses attention on the question "What are the most serious mismatches between the job requirements of workers and worker capabilities?" There is a clear assumption in this enterprise that a society can improve its international economic competitiveness if it reduces the most critical mismatches through a reorientation of both school curricula and out-of-school training programs, as well as through the tailoring of new economic activity to match worker capability already established. The pilot phase of GS/ED is now complete[1] and the plans for a continuing enterprise are well advanced. The Ontario Ministries of Education, Colleges and Universities, Skills Development, Labour, and Industry Trade and technology, have initiated the project for the Government of Ontario and the founding host corporations involved are Canadian Tire Corporation, bank of Nova Scotia, Royal Bank, Northern Telecom, and the Ontario Public Service.

The impetus for the project came from the Ontario data of the Second International Mathematics Study which showed that the amount of education service in mathematics, delivered by the Ontario schools is really among the highest in the world. This is noted by Westbury in his review essay *A nation at risk* (p. 440), and it was especially important to note in view of the popular reporting of mean p-values which showed Ontario t be down towards the middle ranks among participating nations. Business leaders who noticed the high delivery capability were curious

[1]Russell, H. H., Vafa, R., & Kenny, J. B. (1987). *Generic skills/Economic development*. Ministry of Education, Ontario, Toronto.

enough to want to determine why this superlative capability appeared not to be converted into economic advantage. Their scanning of the items on the SIMS pool suggested the hypothesis that the mathematics that is "delivered" may not be relevant to the issue of economic competition. Hence the GS/ED project was established in an effort to see what mathematics, and indeed other subjects, were actually used by people at work.

The project operates in rough conformity with a model shown in a diagram which corresponds to an identifiable stage of the project. Box 1, for instance, is labelled "Target Occupations" and it is the stage of the project at which the host corporations choose the occupations to be analyzed. These occupations are called "critical" occupations within the project and they are selected because they need special consideration. Sometimes an occupation is critical, because there is a significant decline in the need for people for that occupation, whereas in other cases there is a significant increase in the need. Indeed some critical occupations are selected because the skills needed by the people within the occupations are changing dramatically, even though the number of workers remains the same. In other situations there is no known change taking place, but there is speculation that changes could be made that would produce beneficial consequences.

The second stage of the project is represented by two boxes, 2a and 2b. Each box represents one formal mechanism for encoding the tasks that are performed by workers within the target occupations. Functional Job Analyses[2] is a technique developed by Sidney Fine and it has been the primary means of analyzing work during the Pilot Phase of GS/ED.

The third stage carries the label "Task Banks," and this is essentially an analysis stage within which the very specific information from the deliberations between Sidney Fine and the six or seven incumbents from the critical occupation in focus, is synthesized. The result is a comprehensive list of tasks that covers more than 95% of the work done by the incumbent. The task banks then consist of lists of things people do at work, and in a sense each of these tasks may be seen as an objective for a training program or even a part of the school curriculum. The scaling of the tasks (box 4) is a mechanism by which the importance of the task of the job is assessed.

Box 5 refers to two distinct activities which in the earliest phases of GS/ED were considered to be just one activity, namely creating survey instruments to reflect the work done by people in the occupations under consideration. Indeed the first instruments were constructed directly from the task banks, but that process left the connection between tasks and instruments rather ill-defined. The survey instruments are intended to be analogous to the items in the SIMS pool. The difference is that the GS/ED survey instruments are based on things people do at work, and the SIMS pool items are based on mathematics curriculum content.

Twelve survey instruments have been developed during the Pilot Phase of GS/ED, and they have undergone very limited pilot trials both in schools and among incumbents in the host corporations. The validation procedure that is under development for these instruments is quite different from standard psychometric procedures. Since the GS/ED purpose is to estimate the proportion of people who can perform a task and those who can not, the instrument should possess a type of "rough" discrimination capability. Thus, job incumbents should, as a group, have a higher proportion of passes than a group of workers at a lower point on the occupation hierarchy. Indeed, this very simple discriminator was the basis for the validation procedures of the pilot trails of the survey instruments, and it will be the basis for field trials yet to come.

The work of boxes 6a and 6b, namely to use sample surveys to estimate both need and availability, has not been attempted formally during the Pilot Phase of GS/ED. However, the plan calls for such surveys of workers, and students still in school, in order to estimate the total number of people in a corporate group or in a society at large that passes the test item or the survey instrument in question. The proposed sample surveys are relatively economical to conduct, because the sample size can be very small compared with conventional samples. It can be small because the error of estimate can be much larger than customary practice suggests. The error can be large because there is no advantage to having it smaller than the error of estimate of the "need" data. The error of estimate for the need data will be large, because nobody knows how to make precise estimates of how many people need a particular workplace capability at any given point in time, let alone making estimates for the future. Thus, data with a low level of recision seems to be inevitable in this enterprise, and thus the concentration of the project staff can be focused on the face validity of the instruments and the informed guesses of the people, who must make the guesses with or without supporting data.

The seventh stage involves the merging of the data from boxes 6a and 6b, and the result should be some large deficits and some large surpluses. In the case of deficits there is a decision to be made at a very high level, namely that the responsibility for remedying the deficit is within the corporations, or a government training program, or a particular level or levels of the educational system. When that decision is made, then the training programs or the various curricula can be modified accordingly.

Stage 8 consists of four boxes, none of which has come into operation within GS/ED as yet. But the labels are clear and the kind of actions anticipated are

[2]Fine, S. (1984, May). *A. Functional job analysis*. Chapter 9, Advanced Research Resources Organization. Fine, S. (1986). A. Functional job analysis. In S. Gael (Ed.) *Handbook for job analysis*. New York: J. Wiley and Sons.

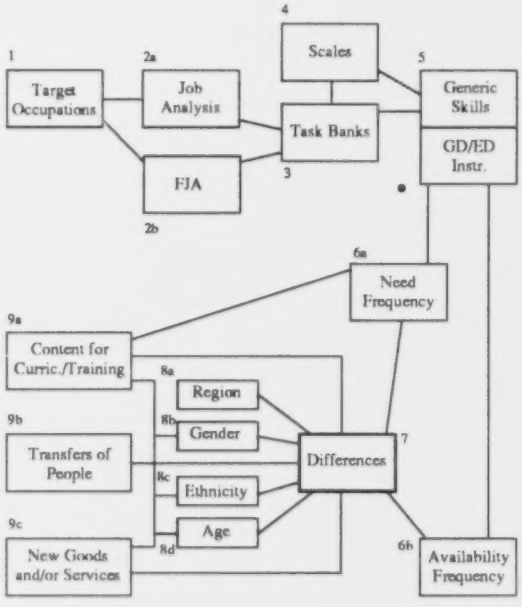
worthy of attention. In the case of regional data it may be true that there are regional differences that balance out from one region to another, and those do not show up as differences in nationally aggregated data. When regional differences of this nature emerge, there are two solutions to the problem that emerge solely from the regional data. One solution is to transfer people from the region with the surplus to the region with the deficit. The second solution is to move the jobs to the people. There is really not much new to these two solutions, but to have quantitative data available to document the problems may lead to more effective solutions.

Some general comments about the various features of GS/ED are in order. The rationale for the enterprise is admittedly based on the premise that marketplace effectiveness, and therefore money earned, can be influenced by changes in the curricula of the schools and other educational agencies. This premise is already confirmed in the actions of the host corporations, and it is readily accepted by the host school systems that have agreed to participate in the next phase. There is no need for employers to control the curricula, nor is there a need to change the structure of educational institutions. The GS/ED data should be available to students teachers, workers, employers, and, indeed, everyone. The interest of mathematics educators in "mathematics for all" may find in this enterprise a mechanism by which the objectives, and thus the content, for a "mathematics for all" curriculum may be developed.

Another observation with both practical and philosophical implications is that GS/ED type data are more compatible with data from the second International Mathematics Study than they are with data generated in every-pupil surveys that use national examinations or standardized tests. The reason is that the test items used in SIMS are in a sense operationalizations of objectives of curricula. The fact that they come in the form of a list of discrete items may seem to trivialize what is involved, but the test of time seems to suggest that there is considerable substance to such items. The GS/ED survey instruments are also "test items," and they too come in the form of a list of discrete items. A cursory look at such items may suggest that they too are perhaps trivial, but a careful study suggests that they incorporate objectives which are indeed market- relevant objectives. They are fundamental objectives which will warrant careful attention by educators at all levels, and they will also warrant some considerable debate by virtually all stakeholders in education. Their form and their accessibility will facilitate debate, and the fact that they have obvious face validity may indeed make the debate intelligible to virtually everyone.

There is one further observation that has far-reaching practical consequences. The GS/ED data and the format they share with SIMS tends to shift attention away from people themselves and towards what people do. It is away from how hard they work and toward what they work at. The changes that are called for as a consequence of the surveys in question are changes in curricula primarily. The title of the U.S. report of SIMS was "The Under-achieving Curriculum," not the under-achieving student and not the under-achieving teacher. With such a focus for these survey enterprises it is now possible and indeed probable that significant changes in curricula can produce significant changes in teaching and learning which in turn will more adequately fulfill society's hopes for the schools as a preparer of productive free citizens.

GS/ED Data Flow



1. Target jobs selected by host corporations.
2. Job analysis done by S. Fine using FJA.
3. Task banks for target jobs created by S. Fine in collaboration with host corporations.
4. Scales created from task banks by staff and advisors.
5. Generic skills (assessment instruments) created by staff.
6a. Need frequencies simulated from Smith data.
6b. Availability frequencies reported from file data.
7. Differences not yet available, i.e., differences between 6a and 6b.
8. Special differences not yet available.
9a. Three host corporations have training programs based on project data.
9b. One host corporation has transferred people based on project data.
9c. New goods or services not yet established.

Expanding Risk in Technological Society Through Progress in Mathematical Modeling

Bernhelm Booß-Bavnbek and Glen Pate

1. Needs and Consequences of Growth Economy

It is coming to be conventional wisdom that the currently celebrated and criticized high technology is essentially mathematical technology. Conclusions for democratic mathematics education should not, how-

ever, be drawn already from this very general insight; instead we need to distinguish various types of mathematical modeling of fundamentally different character:

(A1) Finding a geostationary orbit for a telecommunication satellite is a mathematical exercise in elementary celestial mechanics, which could have been done by hand by Newton himself with the required accuracy. Such well-founded applied mathematics generates prestige, which is inappropriately generalized to support quite different applications.

(A2) Determining optimal rules for fish feeding and harvesting in a pond, for fiscal policy to control unemployment, budgetary deficits and monetary problems, or for the optimal use of various chemicals in agriculture and forestry may be a similarly elementary exercise from a mathematical point of view. These calculations, however, cannot currently be based on a solid theoretical foundation. The clarity and precision of the mathematical derivations here are in sharp contrast to the uncertainty of the underlying relations assumed. In fact, similarity of the mathematical formalism involved tends to mask the differences in the scientific extra-mathematical status, in the credibility of the conclusions, and in appropriate ways of checking assumptions and results. And the ease of programmed calculation with the added prestige of the information technology used, for the calculation supports the proliferation of this kind of mathematical reasoning—however dubious it may be. This unwarranted prestige is enhanced, moreover, by the unfortunate tendency to transfer the prestige which mathematics itself has accumulated in thousands of years to such applications.

(B1) A quite different andcertainly not better situation arises when the mathematical means used are outside the kernel of mathematics, which is theoretically well-founded, and which is moreover generally well-understood by its users. Intricate inverse problems, e.g., are raised in the analysis of geoelectrical measurements needed to separate regions of different specific resistance in the underground, to get an impression of size and shape of the various geological layers. Still, we do not understand all relevant aspects of the transport of electrical charges in porous media, nor the whole sophisticated mathematics of three-dimensional inverse problems. Nevertheless, by combining geological knowledge and efficient machine calculation, one can obtain quite reasonable "guesstimates," e.g. of the distribution of fresh water resources in the underground. Mathematical modeling as heuristic means, however limited in its truth content, can save many expensive drillings and will be adequately tested when wells are drilled. Recurrent failures will, moreover, support continued awareness of the weakness of the mathematical method.

(B2) The situation is again quite different if foreseeable consequences offailure would not be acceptable. The same parties of the previous example, perhaps municipal authoritiesand consulting agencies, may now perform the same type of geoelectrical measurements and apply the same program package for determining the geological structure and possible liquid permeability of the underground in the search for safe depositories for toxic waste. The only difference is that the validity test is postponed to later years or decades when the drinking water—perhaps for a large population—may be contaminated. The identity of methods and procedures masks the total diversity of situation and encourages the indiscriminate use of certain non-validated mathematical methods in totally unacceptable contexts, as well as in acceptable, productive contexts. The imperatives of the growth economy induce haste, precluding knowledge of perhaps long-delayed empirical validation or invalidation as a basis for decision.

(C) A third type of mathematics applications—and this is the worst case—is the support of devices or processes in cases where reliability is completely unattainable, due to the coincidence of missing theoretical foundation in the application domain, as in example A2, and missing mathematical insight, as in example B2. Such is the case e.g., in the attempt of nuclear engineers to model the growth of cracks in concrete or in metal alloys under high temperature, pressure, and radiation; in the psychological modeling of the behavior of operators in the control center of nuclear energy stations under crisis pressure; in the design of steering automata for directionally unstable high-speed container ships. For most of these situations, it is characteristic that the calculations may be acceptably descriptive under "normal" circumstances, but worthless under special circumstances, as witnessed in Harrisburg, in Tchernobyl, in ship losses due to control failures, or in the recent loss of a fully loaded new model Airbus during its maiden flight. In some cases the technology failed; in other cases system failure was due to human failure induced by misguided confidence in the technology.

We conclude part 1: Growth economy demands the expansion of production, demands continuously increasing productivity. As long as these goals are assigned highest priority by the dominating population sections in higher-developed countries, one has to expect that any progress in mathematical modeling will be accompanied by expanding risk in technological society arising from ruthless creativity, both military and civilian. A highly sensitive indicator for abuse of mathematical modeling is the degree to which responsible parties maintain consciousness of the real limits of the knowledge, on which their decisions can be based, rather than succumbing to the temptation to use in particular IT-based mathematical models to mask ignorance, as an opiate to ease the pain of felt or real need to act in an excessively interlaced environment of inscrutable complexity!

2. *How Does Mathematics Education Relate to Destructive Technological Developments?*

The methodological guidelines with respect to the software production process and the nature of the resulting products, which are equally accepted in military and dominant civilian circles, are profoundly military in their origin and character. The conven-

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tional systems characteristically reflect the nature of the organizations which employ them: They support rule-based human activity embedded in hierarchical structure, i.e., bureaucracy which characteristically denies the human agents adequate knowledge of the system goals, adequate knowledge therefore of the effects of their own activity, and leaves, therefore, no basis for individual responsibility for the effect of that activity. The nature of these systems and the position of the human actor within them leaves frequently inadequate scope for human correction of system failures or even adequate assessment of the risks involved.

The technocratic view of the situation is known as the "software crisis." Frequently, a more adequate description would be a failure of modeling on the one hand, and inappropriate definition of the role of human actors within the system on the other hand.

The task of creating a humanistically oriented civilian IT is only now being addressed by a small but growing, predominantly European, sector of the IT community. The initial efforts can perhaps be characterized by the slogans "Small is Beautiful," "Keep it Simple," "Support, Not Replace Cooperative Work," and "Give Careful Attention to Modelling Reality in Categories Well Understood by Users."

In the current social reality, the emancipation of IT from its comprehensive military pollution can hardly be the work of the civilian side, which has hitherto been completely dominated by the interest of international corporations, which have in general very military-like structures and are in any case closely interlaced with military interests.

As a pure mathematician specialized in mathematical physics and in theoretical aspects of other modeling situations, and as a consultant in practical IT system development and use, we call upon the educational institutions to help in establishing alternative ideals for the relationship between citizens and suitable IT systems based on adequate models, to contribute all their possibilities and experience in the more then ever decisive task of developing self-confident people who strive for working environments which permit them to know the consequences of their activity and thereby create the possibility for them to bear responsibility and encourage them to do so.

We need to support the subordination of the technology enthusiasm of the young generation to a disciplined humanism, counteracting the tendency to pernicious technocratic systems which may be the natural result of young peoples' tendency to test the boundaries of their mastery of technology. We must in particular support the comprehension and the ability to distinguish the various and fundamentally different types of applications of IT and of mathematical modeling, to develop sensitivity to the abuse of mathematical models to mask reality, or to facilitate avoidable increases of technology-induced risk.

Educational institutions may not be able to correct fundamental malorientations in society. But they can help to keep the eyes of the young closed or can help to develop antennas, ears, senses to accurately distinguish between true complexity and true simplicity even in face of contradictory superficial appearance.

Thus, in concordance with most thinking about modern mathematics and computing (Atiyah, Kahane, Abelson) the focus in mathematics education must be directed to the rich possibilities of traditional and modern mathematics to handle complexity:

(1) The pupils and students should learn to distinguish between the different status, foundations, controllability of mathematical calculations. Not all calculations are reliable, nor are all calculations arbitrary free constructions. Every educational level should achieve awareness that many models have no domain of validity and that every non-empty domain of model validity has its boundaries, which should be the object of persistent attention.

(2) They should be aware of the great potential of mathematics to reduce apparently chaotic surface phenomena to quite simple generating mechanisms, i.e., to reveal the true simplicity behind some complex appearance; they should learn not to accept the falsification of real simplicity by vested interest-motivated allegations of complexity.

(3) Students at the appropriate level should be aware of the great strength of mathematics to reveal the complexity of apparently simple relations. The students need to gain familiarity with non-linearity, with time lags in buffered systems, with the diversity of forms near singularities, near "flat points," with the occurrence of rare but in the long run repeating and accumulating "strange" incidents in number theory, probability, and numerical analysis.

This puts the currently and foreseeably required mathematical education in a delicate position, right between two well-established orientations in mathematics education, towards "pure mathematics" on the one hand and "applied mathematics" on the other. The pure mathematics-oriented view emphasizes the counter-intuitive side of mathematical concepts, the borders of applicability of definitions and statements, the surprising vigor and simplicity of some arguments, and the limitations and weakness of others. The proponents of pure mathematics have rarely undertaken serious efforts to define, explain, and justify their educational goals. They have often marketed their product simply as useful selection criteria for advancement to privileged positions in society; or they may just put forward the general view that pure mathematics is at the root of much in modern society; or they suggest that some non-specific qualifications, such as logical competence results from pure mathematics; or they just sell it as "brain building." The absence of testable goals is an insurance against failures.

The other orientation, the applications-oriented position, tries to overcome the obvious weaknesses of the often unaccepted pure mathematics-oriented position—problems with student motivation and criticism

of "users" of mathematical training—by incorporating carefully chosen examples of mathematics applications in the curriculum, perhaps by presenting examples or by guiding the student's project work.

Overcoming pure mathematics-oriented concepts, the applications-oriented position has a great appeal for many of the best, most creative, and most energetic teachers. However, the results are often poor and in fact inferior compared with the "pure mathematics-oriented" approach and evaluated with a view to the new trends in mathematics applications. Because of the pedagogically well-intentioned selection of the examples, those chosen are biased and generally very misleading: They are too simple, too smooth, giving the student a wrong impression of "mathematical concepts matching reality." The students do not see the true problems in modeling, in simplification and generalization, in the formulation, analytical treatment and numerical solutions of the equations, in the interpretation and control of the results—or they see them only in systematically reduced form.

The choice of context is, moreover, often misleading: Population biology or macroeconomic modeling are preferred choices and other fields which lend themselves to arbitrary simple modeling easily grasped therefore by the mathematics learner precisely due to the lack of mathematical tradition and quality standards such as those obtaining, e.g., in engineering, geodesics, and meteorology. In these fields with well-established traditions of mathematical modeling practical experience prohibits the use of simple, toy models. These more established subjects are more representative of real modeling of complex reality and therefore, alas, more difficult to teach.

A true humanistic alternative has to provide the students with much more insight into the true complexity of modeling in all its aspects. This will be a difficult task in practice: The curricula and the rules of examination demand testable knowledge and have difficulty with the required types of metaknowledge. Teacher education usually does not provide the necessary experience in modeling. The pupils and their parents feel uncomfortable and lost with lessons of talking about mathematics.

However, more efforts in that direction should be made. And in concrete situations where school administration and teacher qualification only permit the choice between the "pure mathematics-oriented" and the "applications-oriented" position one should hesitate to follow too readily the pragmatics of the "applications-oriented" or to blame too much the "pure mathematics-oriented" as elitarian. As in general technology assessment, the conservative approach may be the more humanistic and more soundly futuristic approach also in mathematics education.

Mathematics Education Under Different Cultural Constraints

The conditions of mathematics education in industrialized and in developing countries are very different. The implications of serious cultural differences are discussed by Siaka Bamba Kanté, Côte d'Ivoire, who reflects upon some critical issues concerning mathematics education in the Ivory Coast; Jens Naumann, Berlin (West), reports a partnership-project with Senegalese villages involving Berlin teachers organized in the German United Nations Associations, and practical aspects of basic mathematics teaching there; Diana C. Rosenberg, Argentina, presents her experiences in transforming the Dutch Hewet-Project into quite a different culture, "HEWET-Argentina." Munir Fasheh, Palestine, contrasts the informal mathematical practice socially used in daily life and work with the institutionalized formal mathematics, in order to analyze and criticize the role of "hegemonic education" implanted in developing countries.

Critical Issues of Mathematics Education in the Ivory Coast

Siaka Bamba Kanté

The Republic of Ivory Coast is a former French colony in West Africa; its official language is French, spoken by about a third of the population of some 11 million inhabitants. There are some 60 indigenous languages spoken in Ivory Coast, four of which (Akan, Agni-Baoule, Senoufo, Dyula) are of considerable quantitative importance; Dyula has the status of a lingua franca.

The school and university system has grown rapidly in the past decades, but illiteracy is still fairly important (almost 50% in Abidjan, 80% in the rest of the country), and even in the capital only some 20% of the population attained more than primary education.

More specifically with regard to mathematics education, we can observe that the government tries to follow an overall policy of emphasizing and encouraging—as witnessed by the "Prix Mathématique" for students created by our President, Houphouet Boigny. Yet, for some 25 years now, the average yearly number of mathematics graduates of the university able to teach the subject in all grades of the secondary school has not surpassed three, while the deficit of secondary mathematics teachers currently is about 900 (with another 700 needed for teaching physics). Admittedly, the number of graduates of the Teacher Training Institute is much higher, but they are only entitled to teach in the lower four grades of the secondary school.

But apart from the politics and problems of training sufficient numbers of qualified teachers another issue

has gained in saliency, that of linking the teaching of school subjects to the local languages and cultures.

A major first step in this direction was a law passed in 1977 calling for the study and eventual introduction into the school system of local languages; another important step was an unprecedented recent parliamentary debate of the matter. Practically, the Institute of Applied Linguistics of the University of Abidjan was charged with the task. Some remarkable progress has already been achieved: Some local languages are now taught at the University. Eventually, mathematics education should and, hopefully, will be affected by these efforts.

Slowly, the existence and the forms of local traditions of mathematics, of numeric systems, weights and measures are rediscovered and become documented. One example is the study of the sociology professor Niangoran Bouah about the traditional Akan gold weights: The multitude of forms and geometric designs represent, in fact, an exact and detailed system of weights of practical use and relevance in the Akan community.

An engineer of the Ivory Coast, Mr. Traugucye, has systematically studied the numeric systems currently used in the country besides the decimal system. He found a multitude of systems, with a basic unit ranging from 5 to 20.

Further, various linguistic studies have spelled out and documented structural differences between local languages and French, besides differences in the pragmatic meanings and connotations of certain concepts and words. Thus, some of the difficulties of pupils forced to learn new concepts and operations in a language they hardly master (French) become evident.

Finally, let me mention the experimental school of Professor Ait Hamou Khaled in the rural province of Korhogo in the north of the country. It is a pre-school program for children between 4 and 8 years of age, designed to provide an introduction to the basic reasoning of the exact sciences, the humanities, and the social sciences. The program is entirely run in the local language spoken by the pupils. Thereby, it liberates the children from the heavy constraints of the typical public school program which mechanistically assigns contents and learning steps on the only criterion of age of the pupils with complete disregard for their cultural background. I can assure you, the results are convincing! It is often claimed, that the mechanistically administered public curriculum is based on the insights of Piaget in cognitive developmental processes: If that were so in the case of our country, the results of the experimental school would, in fact, refute basic tenets of Piaget's theory. Currently, the work and the results of this experimental school, which for some years has been supported and financed by our university and some international institutions, begins to have an impact on the region and beyond (in fact, I brought along a video film documenting the work).

The adoption of French as the official language of the Ivory Coast was and remains a political and cultural necessity. Of equal importance and a practical necessity, however, is the acceptance of our cultural heritage. It falls upon our government and our intellectual leaders—among them the eminent African mathematicians—to pave the way towards this goals.

Maybe, one day, we will no longer wonder why, in our higher education, we have so many posts for mathematics and not a single pedagogical one facing up to our very specific practical problems of how to teach mathematics in our schools to our children.

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Practical Aspects of Basic Mathematics Teaching in Senegalese Villages

Jens Naumann

From comparative empirical surveys of student achievement (at the secondary level) we know that

- achievement levels of students in Third World countries (especially in Africa) tend to be considerably lower than those of students in industrialized countries;
- school quality and, especially, availability of school books and teaching materials, appear to have a considerable impact on student achievement in Third World countries.[1]

In this note, I shall describe some of the socio-cultural factors limiting school attendance and achievement at the primary level village school in the Diourbel area of Senegal, some 200 km southeast of Dakar. Some of these factors—like the provision of textbooks—could, in principle, be changed by politico-economic intervention, others are much less amenable to short-term interventions.

The information of the paper is based upon answers of four village school headmasters to a small questionnaire which I sent them in June 1988 and my general acquaintance with the villages which grew out of a small cooperative project between the Senegalese and the German United Nations Associations. This

[1] Fuller, B. (1987). What school factors raise achievement in the Third World? *Review of Educational Research*, 57, 3, 255–292.

cooperative project started in 1982 and involves, among others, a yearly 2-week study trip for Berlin teachers to learn about development issues and fund-raising efforts of the German side to help finance small projects and activities in the villages, both linked to the schools and beyond.[2]

General Information About the Villages

In the following, I shall refer, essentially, to four villages (Touré MBonde, Patar, Toky-Gare, NDoulo), all just 10 to 20 km away from Diourbel-City, the administrative capital of the Diourbel region, the most important peanut-growing area of Senegal (the fifth village of the cooperative project, Ngohe, did not return the questionnaire).

Two of the villages are rather small (Touré MBonde and Patar, with less than 700 inhabitants), Toky-Gare is of a medium size, while NDoulo (and Ngohe, with some 3,000 inhabitants each) is quite large.

The main economic activity (cash crop) is the growing of peanuts; besides that, millet is grown, and some goat and sheep are raised for local consumption; vegetable production is severely limited, due to the lack of sufficient sweet water in the dry period (November to July). Besides some smaller herds of cattle (a few dozen heads each) of passing nomads, there is only one large cattle herd in the area (some 3,000 heads), belonging to the wealthy marabout of NDoulo.

There is no private ownership of land; income differences (per capita) between the peasant families are small. The communities are (very) poor, but not destitute. Much more than 90% of the inhabitants of the four villages are Muslim (only the fifth village, Ngohe, has a substantial Christian minority, mainly belonging to the ethnic group of Sereer), a large majority belongs to the ethnic group of Wolof, and Wolof is by far the dominant language of communication (in Ngohe, the second very widespread language is Sereer). French, the official language, is more or less spoken and understood by some 5 to 10% of the adult population of the villages.

Some General Characteristics of the Senegalese School System

In overall terms, the school system is patterned after that of the former colonial power, France. It consists of a six grade primary school, followed by a secondary school, which in its academic branch consists of another six grades leading to the BAC, the university entrance qualification. Administratively and curriculum-wise, the system is highly centralized, the only language of instruction—from the 1st grade on—is French.

The enrollment ratios of all three levels (number of students as percentage of the official school-age population) have increased considerably over the last three decades, although at a decreasing speed in the more recent period, due to ever more severe economic constraints. For the country as a whole, the 1985 gross enrollment ratio was 55% for the first, 13% for the second, and 3% for the third level. A rather high 90% of those starting primary education actually reach the 4th grade.

Besides the public ("French") school system (and a few private "western" schools), there exist many, entirely "privately" run Koran/Arabic schools, a few of which offer a fairly general primary and secondary curriculum, beyond the basic religious instruction.

Education in the Villages—A Closer Look

The overall enrollment ratio differs considerably between the villages, ranging from a low of some 20% in Touré MBonde, to a high of 60% in Toky-Gare, with about 40% in Patar and NDoulo. Everywhere the enrollment of girls is very much lower than that of boys; classes tend to be very large, mostly with more than 45 pupils, reaching a high of 69 pupils; there is a teacher for every class of a public school (see Table 1).

Table 1
Schools in the Villages (1988)

	"French" Schools							Koran/Arabic Schools			
Grade	Number of pupils	girls	teachers	Percentage attending school total	boys	girls	Length of studies (years)	Number of teachers	pupils	girls	classes/ groups
Touré MBonde											
	96	12	2	20	35	5	3	1	30	5	1
2 (CP)	45	5									
3 (CE 1)	51	7									
Patar											
	62	17	2	40	60	20	3[6]	1	30	10	2
2 (CP)	36	14									
4 (CE 2)	26	3									
Toky-Gare											
	129	37	3	60	85	35	6	1	120	30	4
1 (CI)	50	21									
3 (CE 1)	46	8									
5 (CM 1)	33	8									
Ndoulo											
	356	116	6	40	55	25	3	1	60	20	3
1 (CI)	62	25									
2 (CP)	62	24									
3 (CE 1)	59	16									
4 (CE 2)	59	17									
5 (CM 1)	45	11									
6 (CM 2)	69	23									

The Koran/Arabic schools are, in terms of pupils, smaller than the "French" schools (with the exception

[2]Deutsche Gesellschaft für die Vereinten Nationen, Landesverband Berlin (1987). *5 Dörfer – unsere Partner im Senegal*, Part 3, 70 pp. Berlin: Eigenverlag.

of the one in Toky-Gare). They all have only one teacher. Interestingly, their sex-specific selectivity does not seem to be more pronounced than that of the public schools. In the last few years, the further expansion of the public schools was limited by the declining financial contributions of the state and the lack of village resources to take over a growing share of expenditures for construction, repairs, textbooks and materials, and school canteen provisions.

More specifically, regarding the curriculum and the use of textbooks/didactical materials the following can be observed: All public schools follow the same curriculum, use the same books (see Table 2) in virtually the same way and report identical problems. The main textbooks (in French) are, essentially, language-texts bought by the state, but in insufficient quantities. While in the first two grades, two to three children tend to share a book, there is usually one copy of the main textbook available for three to four students from grade 3 upwards. In the case of subject-specific texts like "Calcul quotidien," meant to be used in the 5th and 6th grades, one copy is used by four to six pupils. The textbook provision for the higher grades is worse than that for the beginning grades, because the books are three to four times more expensive than the texts for the beginners.

Table 2
Textbooks used in Senegalese Villages, 1988

"French" schools, all

Grade	Main textbooks	Supplementary textbooks
1 CI	Je lis et j'écris, 1st year	
2 CP	Je lis et j'écris, 2nd year	
3 CE 1	Afrique, mon Afrique, vol. 1	
4 CE 2	Afrique, mon Afrique, vol. 2	
5 CM 1	Afrique, mon Afrique, vol. 3	Calcul quotidien
6 CM 2	Afrique, mon Afrique, vol. 4	Calcul quotidien, Livre d'Observation IPAM

Special teacher manuals exist for
- basic mathematics, grades 1 to 4
- grammar, grades 2 to 6
- history and geography, grades 3 to 6
- observation, grades 3 to 6

Koran/Arabic school, Toky-Gare only*

Grade	Main textbooks
1 CI	Akhraw, 1st book
2 CP	Akhraw, 1st book; Amayatta (Koran); Khassaboul diadite, 1st book (math); Almouslima Sakkir
3 CE 1	Akhraw, 2nd/3rd book
4 CE 2	Kassaboul diadite, 2nd/3rd book
5 CM 1	Akhraw, 4th book; Khassaboul, 4th book
6 CM 2	AK Tilawaatou Al Khadiya, 5th book

* In the other villages, the teacher has two or three books while the students do not use any (with the partial exception of Patar-students).

All public schools report using the main textbooks "practically every day" and "to follow them very closely" (only NDoulo indicates a "from time to time" use in a more exemplary way of the main textbooks in the first two grades). The specialized math books in grades 5 and 6—while used every day—do not appear to stand as much in the center of instruction as the main language texts. All schools report that they are "unofficially" using Wolof to facilitate the comprehension of concepts and the acquisition of a basic vocabulary, particularly in the first two grades. In fact, "language problems" are always mentioned as one of the major impediments to basic math learning, although the lack of materials and books and the rather limited time reserved to math (one hour per day in grades 5 and 6) receive the major share of the blame.

With regard to the Koran/Arabic schools, it should be pointed out that all claim to teach their pupils how to read and to write (in Arabic), in three of the schools pupils usually do not use books of any sort (with the partial exception of Patar, where some Tunisian school books are used). The Arabic School of Toky-Gare is the exception in that it lasts 6 years, uses a variety of texts, and includes basic math teaching. It should, however, be pointed out that all Koran schools—entirely financed by the villages—are much worse off than the public schools. This is also true of the school in Toky-Gare.

The learning results of the Koran schools with the "normal" short program are usually held to be very limited. The predominant rote learning permits the pupils to recite, read, and write (parts of) the Koran with only limited (if any) comprehension. Transfer capacities are hardly developed.

Compared to that, the situation in the public schools is definitely better, although very far from satisfactory. Since improvements in the teacher–student ratio or class size are very unlikely in the next decade, a more adequate textbook provision appears to be the fastest and cheapest strategy to improve teaching and learning. The next promising step could conceivably consist in the improvement of the texts used. Such revisions should include some use of the actually spoken major language of communication (for the introduction of some concepts, some vocabulary lists, etc.), if only to legitimize incidental efforts of teachers to use the spoken language in the otherwise French instruction. Given the constraints, this would be a strategy to implement some basic premises of pedagogical differentiation needed to overcome the substantial rests of inefficient rote learning in the public (French) schools.

Knowledge Transfer From one Culture to Another: HEWET From the Netherlands to Argentina

Diana C. Rosenberg

If we ask ourselves if knowledge transfer from one culture to another is possible, there immediately arises the need for two definitions:

(a) the one concerning *culture;*

(b) the other concerning *knowledge.*

Let us start with the first. *Culture* should be understood as the way a certain social group lives, behaves, believes, and expresses itself.[1]

Argentina, as the rest of Latin American countries, has no national culture. It is a very young country where some old Indian cultures have survived, a country that used to have some Negro immigration that was eliminated during the war against Paraguay (1864–1870) and that at the end of the last century received a great European immigration (Italian, Spanish, and less British, Irish, German, Swiss, Yugoslavian, Russian, Libanese, Syrian, and Arabic).

Due to the impossibility of identifying a national culture, attempts have been made to look for a typology of the so-called subculture types[2] in Latin America.

For our case study we are going to circumscribe ourselves to the Province of Buenos Aires, situated in the east of Argentina with the Capital city La Plata. The national Capital, Buenos Aires, is not considered as part of the province. It extends over 307,571 km^2 and has approximately 13,000,000 inhabitants.

Following the typology of Wagley and Harris,[3] we could say that Buenos Aires is characterized by representatives of the Engenho and Usina Plantation Types, Town Types, Metropolitan Middle Class, and Urban Proletariat subculture type. As to the democratic tradition: Argentina has had only one period of democracy between the years 1914 to 1930.[4] Before and after that period there have been only some trends to establish limited democracies, as well as a type of authoritarianism based on the consent and support of the majority (1946–1955). After 7 years of horrible military government and 30 years of not being able to end any constitutional period, Argentines have—for the first time again—elected their president in 1983 and are hoping to have safe elections in 1989.

It is certainly not quite necessary to introduce the Netherlands. We all know that it is an old kingdom with a long democratic tradition, showing a clear cultural identity, due to a long and very well-"digested" history.

In Utrecht, at the IOWO (Institute of Research in Mathematics Education), Professor Hans Freudenthal started an important work in the 70s with the slogan "Mathematics for all." He fought for a teaching of mathematics from out of the "valley" and not, as has been done all these years, from out of the "ivory tower" of the mathematicians. He was sure that this way it would be possible to avoid the unjust fact that

> for whosoever hath to him shall be given, and he shall have more abundance, but whosoever hath not, from him shall be taken away, even that he hath.[5]

That means that he tried to promote the motivation and social participation of pupils and teachers in the process of learning, as one of the valid methods of exercising democracy since childhood.

It was not easy for him either in the tolerant Dutch country, and IOWO was reduced to OW&OC after a big struggle for survival.

The project we refer to (HEWET[6,7] or Maths from Reality for 16–19 years) was started in 1981 by Jan DeLange and Martin Kindt. It was constructed on the solid ground given by Freudenthal's philosophy of mathematics education, 1984 HEWET could be considered a "cultural fact" because it was introduced in all VWO (Colleges) of the Netherlands.

At the same time in Argentina great efforts were being made to return the educational system to be somewhat democratic.

In 1985 Jan DeLange visited Argentina and met the Director of Secondary Education of the Province of Buenos Aires. From what DeLange told her, she understood that the booklet of HEWET could fit perfectly into the reform of upper secondary they were facing. She wanted to get the materials, so as to transfer (?) the Project to teachers and pupils of the Province.

It was at that point that the question arose: "Is it possible to transfer[8] knowledge from one culture to another?"

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[1] Havighurst, R. J. (1973). *La Sociedad y la Educación en América latina.* Buenos Aires: EUDEBA.

[2] *Subculture:* they are called subcultures because they are variations of a larger cultural tradition and represent the way of life of significant segments of Latin American populations.
Type: because their content differs according to the environment, history, and destructive local traditions of nations or subregions in which they are found.

[3] Wagley, C. & Harris, M. (1965). *A typology of Latin American subcultures in contemporary cultures and societies in Latin America*, edited by Heath and Adams. New York: Random House.
Note. We think that there is research being done at present and that this typology is going to be actualized, due to the migration of social groups in the country. But for our goal, we thought this old classification could be of help.

[4] It was the first government of the Radical Party.

[5] Freudenthal, H. (1978). *Weeding and sowing* (p. 60). Dordrecht: Reidel Publishing Company.

[6] Kindt, M. & DeLange, J. (1985). *Realistic maths for (almost) all—The Hewet project.* Proceedings of the 37th Meeting of CIEAEM, 4–10 August. Leiden, the Netherlands.

[7] DeLange, J. (1987). *Mathematics, insight and meaning.* Utrecht: OW&OC.

[8] Maybe the word *transfer* is not the best for what we mean, because in the way we use it, the receiver is much more important than the giver.

We should now make clear what we understand by *knowledge*. We not only mean the bare scientifical knowledge, but also the philosophy that supports it, seen as a cultural process.

The developing and underdeveloped countries[9] are used to accept the "generous" gift or the sale of educational means, didactical methods, etc., without being allowed to know their genesis and—most of the time—without getting to know the philosophy behind it. We are therefore not allowed to evaluate if it can be of use to us, or if it really adapts to our context.

This type of "transfer" I would rather call "imposal" of knowledge or parts of it. So—as far as HEWET is concerned—the demand of the functionary was to get the "imposal" of the Project. But, luckily, we could convince her that an Exchange Program would be a better possibility of transfer.

The first part of the Program finally started in September 1987. Martin Kindt arrived in La Plata and organized a Workshop-Seminary of ten days, based on everyday-problems explanation of the genesis of the Project and active work of the teachers on the booklets. The group consisted of 26 teachers (13 were 25 to 30 years old, 11 between 30 and 40, and 2 between 40 and 50). One of the teachers came from 500 km south of La Plata and all the others from 30 to 50 km around La Plata. They were coming from rural as well as from urban schools. There were only 2 teacher trainers in the group. All of them were very interested in innovation of teaching mathematics by means of looking for "serious" applications, but starting from true "formal" mathematics.

I could observe their attitudes and resume in the following remarks: They behaved like a normal human group, with some of them trying to emerge from the group to be considered and others using the strategy of isolation with the same aim. They worked in small teams (3 to 5 persons), of which only one group previously knew each other.

The first four days a special resistance was shown to use intuitive methods or simple mathematical strategies. Mathematics was conceived as something superior, almost imposed from above, that had to be respected.

It was possible to detect an absolute lack of relying upon themselves and on their own intuition and reasoning structure (consequences of long years of repression or continuity of a cultural tradition?).

Aha!-experiences happened very often when they started from informal problems and finally got to familiar formal structures. As groups, they changed their attitude into a more participating one, they dared from day to day to make more creative contributions. It was possible to detect a good integration of the isolated ones, and they became motivated to learn from each other. So the change consisted in becoming productively competitive instead of the rather nonsense competition to be considered by the leader prior to this.

The "maximum" was arrived at when Martin Kindt brought an everyday problem inspired on his visit to the La Plata Cathedral. There was a feeling of "we can also do it in our surroundings."

After Martin left, the group met at the end of October 1987 for the first time in La Plata. Twenty-one teachers came. After solving an everyday problem together, everyone told his own experience of when they returned to school after the intensive HEWET days.[10] You could strongly feel that they had changed their attitude in the sense of the philosophical content of HEWET with enthusiastic results.

Now, the second part—the so-called indirect transfer—was to start consisting in the development of materials based on the HEWET booklets. That was not an easy task.

In December of 1987 the Province of Buenos Aires changes the government by elections. As we are used to in Argentina, a new government implies the removal of all that has been done. The reform was stopped, everything in secondary schools returned to the position of 1984 with the consequence of chaos and despair of the teachers.

The "HEWET-Argentina" group asked for one day per month to gather in La Plata. But the new authorities have given no answer until now, because they showed a great preference for using the original booklets or parts of it in school. The direct transfer had been so good, that the teachers were not willing to accept that demand, arguing that it could have negative effects (imposal?).

Since then, we came together six times on saturdays which meant a great sacrifice. Some materials on matrices[11] were developed and were "transferred" in a school in Ezeiza (56 km from La Plata) to 17-year-old pupils.

They started with a problem about the railways of their neighborhood. This net has an electrical and a mechanical railway. They worked with incredible enthusiasm, got to their operations on their own, discussed and argued about the prices and the convenience of purchasing a ticket, which is good for one month. The teacher was only in class to guide the learning and the discussion.

Important for the context was the fact that they discovered on the tickets that prices had changed in a short period of time. So they had to consider the rate of inflation when discussing the convenience of buying a ticket every day or once a month (this would have been of no interest in the Netherlands).

The teacher of this experimental class (a member of the HEWET-Argentina group) had taught matrices the year before in the same school to 17-year-old pupils, but with traditional methods—one motivating example and then start with definitions and operations. The reaction had not been participative.

[9]Argentina is considered a developing country.

[10]We do not include here the original affirmations of the teachers, but they will be communicated during ICME in Budapest, July 1988.

[11]DeLange, J. & Kindt, M. (1983). *Matrices*. Utrecht: OW&OC.

We do not know if this experience can be followed on, because the teachers that 'survived' (about 10) are very tired of being a sort of Don Quixote fighting against professional and socio-economic problems.[12] They want to refresh themselves by studying another package of booklets from OW&OC and learn more about their strategies.

We conclude from the above that knowledge transfer from one cultural surrounding into another seems to be possible as long as:

(a) the supporting philosophy is "help to help yourself" and not a paternalist one;
(b) it is understood as a knowledge feasible to integrate to any culture;
(c) the socio-political system to which it is transferred does not obstaculize the exchange because of the ignorance of their members or because they prefer the imposal, to stay dependent for the profit of the elites.

The loud demand that should be made—especially from the underdeveloped and developing countries that are doing their best to fasten democracy—is that the members of the educational community of the industrialized countries should help their equivalents from the Third World to "help themselves." The gifts, the vans with food, the credits, and stand-by only helps them to feel in debt and guilty, which makes the situation worse.

What does actually help, is to exchange methodolgy and strategies of independence that could be acquired by a good knowledge transfer, and why not, thanks to our "sterile, noble" mathematics and other science education.

This is, in my opinion, the only transfer that would update the possibility of letting all the countries of the world and their people grow healthy in order to preserve their own culture.

Mathematics in a Social Context: Math Within Education as Praxis Versus Within Education as Hegemony

Munir Fasheh

Unfortunately, it usually takes a crisis or even a disaster for people to realize or discover certain fundamental things about life, societies, or themselves. In my case, for example, it was the 1967 Israeli-Arab war that raised in my mind the first serious doubts about the kind of education (and later to the math) I had been given (and was teaching) at schools and universities. Those doubts gradually led to my awareness of my illiterate mother's math. In my paper I briefly trace the development of this awareness; describe the contexts and characteristics of both types of math—mine and hers; and touch, again briefly, on the meanings of hegemony and praxis. In conclusion, I argue that the stress in math education should not be so much on the ethno-cultural determinants or the differences in the conditions in industrialized and in developing countries, as on whether math is being learned and taught within the perspective of perceiving education as praxis or within the perspective of perceiving it as hegemony.

When the 1967 Israeli-Arab war broke out, I was 26 years old, already with a Masters Degree and 4 years, experience of teaching math at various levels. The war shook the foundations of my small, comfortable, and seemingly consistent and meaningful world, a world created by formal institutionalized education. The war revealed how little we—the formally educated—knew. Almost none of our conceptions, convictions, and expectations matched what was going on. Although I started questioning education in general almost immediately after the war, At that time I did not question the possible relation of the math and physics (which I had studied and was teaching) to what was going on in the world, nor did I question the fundamental assumptions upon which math and science were based. In fact, as a result of the war, I became more convinced that one task I had as an educator was to expand the use of logic and science in the world through teaching. I thought that all that we needed was more math, a "New Math," as well as better and more diversified ways of teaching it. Actually, I was formally involved for six years (1972–78) in math instruction at several levels and in different ways in the schools of the West Bank. The "New Math" I was in charge of introducing into schools was fundamentally alien, dry, and abstract. In order to overcome this, I encouraged the incorporation of cultural concepts, independent avenues of exploration, and personal feelings into the work. In addition to classroom teaching, I established and used math clubs, magazines, general discussion meetings and inservice courses. (For details, see my article 'Mathematics, culture, and authority.' *For the Learning of Mathematics*, November 1982, Quebec, Canada.) This revitalized the teaching, introduced both structure and logic, and was important in developing creativity and enthusiasm among both teachers and students. It did not yet lead me, however, to question hegemonic assumptions behind the math itself.

While I was using math to help empower other people, it was not empowering for me. It was, however, for my mother, whose theoretical awareness of math was completely undeveloped. Math was necessary for her in a much more profound and real sense than it was for me. My illiterate mother routinely took rectangles of fabric and, with few measurements and no patterns, cut them and turned them into beautiful, perfectly fitted clothing for people. In 1976 it struck me that the math she was using was beyond my comprehension; moreover, while math for me was

[12] A teacher works in many different schools, and if he has 30 hours he earns U$ 140,–. An average family needs about U$ 250,– to survive.

a subject matter I studied and taught, for her it was basic to the operation of her understanding. In addition, mistakes in her work entailed practical consequences completely different from mistakes in my math.

Moreover, the value of her math and its relationship to the world around her was drastically different from mine. My math had no power connection with anything in the community and no power connection with the Western hegemonic culture which had engendered it. It was connected solely to symbolic power. Without the official ideological support system, no one would have 'needed' my math; its value was derived from a set of symbols created by hegemony and the world of education. In contrast, my mother's math was so deeply embedded in the culture, that it was invisible through eyes trained by formal education. Her math had no symbols of power. Its value was connected to concrete and immediate needs and actions.

Seeing my mother's math in context helped make me see my math in context, the context of power. What kept her craft from being fully a praxis and limited her empowerment was a social context which discredited her as a woman and uneducated and paid her extremely poorly for her work. Like most of us, she never understood that social context was vulnerable to its hegemonic assertations. She never wanted any of her children to learn her profession; instead, she and my father worked very hard to see that we were educated and did not work with our hands. In face of this, it was a shock to me to realize the complexity and richness of my mother's relationship to mathematics. Mathematics was integrated into her world as it never was into mine.

I was attracted to math and physics because of what I felt to be their role in making the world more intelligible; because they could explain phenomena and predict events; and because they could make absolute statements and accomplish concrete things such as build bridges, make planes, and facilitate surgery. I was also attracted to math and physics because of such claims about them as neutrality, universality, precision, and bettering human conditions. I was aware of their role in producing bombs and pollution, but was convinced that the ethical, moral, and humanitarian dimension was the norm and the fundamental role.

What I started to see in 1967 were the practical limits of the education I had been given. The 1967 war started a process which made the real environment and power relations more visible. My sense of the intellectual, moral, and humanitarian dimensions of science and math gradually gave way to a sense of the central functions of science and math: creating power and generating hegemony. The stunning Isreali military victory in 1967 was a victory of superior science, math, and technology, and not a result of moral superiority or superior personal courage. The message of the highly sophisticated warplanes and bombs (mainly the prohibited napalm) used was loud and clear. Math, science, and technology generate a system of control, because they perform material functions successfully.

My mother's sewing demonstrated another way of conceptualizing and doing mathematics, another kind of knowledge, and its place in the world. The value of my mother's tradition, of her kind of mathematics and knowledge, while not intrinsically disempowering, however, was continuously discredited by the world around her, by the culture of silence, and cultural hegemony.

The most crucial issue the above discussion raises is that of the relation of education to the world it inhabits. The education I had been given fitted me to live in a world created by education and hegemony. It left me blind to its ideological dimension, to the relationship between the knowledge transmitted to me, and power. This blindness, characteristic of hegemonically educated Third World people, left me unfit to live in the real world, in the real environment. Hegemony is not only characterized by what it includes, but also by what it excludes: by what it renders marginal, deems inferior, and makes invisible. The effect of hegemonic education is such that it is almost possible to define the real environment by what formal education marginalizes or excludes.

Hegemony is to be understood here as a form of domination. It often precedes political and military conquests and continues after them. But unlike the situation in military conquest, hegemonic conquest permeates almost all spheres, and in it the dominated facilitate their own domination. Hegemony is always linked to an ideology which reflects the manners and interests of the invaders and their culture. This ideology embodies certain conceptions, values, language, relations, and interests, which are translated into daily practices. Crucial to the hegemonic relationship is the invader's belief that the life-styles and values of the hegemonic group are inherently, naturally, and objectively superior. Hegemony is successful when the invader's ideology is taken or even assumed to be universal and superior; when, like the math I valued, it is believed to transcend class, gender, culture, and national boundaries.

Because ideology is a world-view which embodies certain conceptions, values, language, relations, and interests, which are translated into daily practices and which produce certain consciousness, the role of intellectuals and institutions is of primary importance; the reproduction of a hegemonic ideology is achieved through them (among other apparatuses). Intellectual development in a colonial hegemonic context is designed to provide ideology without a basis in power. It allows intellectuals to participate vicariously in the moral, intellectual, humanitarian, and technical aspects of Western culture, as well as in educational, scholarly, and research activities. Individuals are sometimes acknowledged and rewarded for their participation and contribution. The training of colonial intellectuals directs them to derive their sense of worth and status from this vicarious participation, alienating them from their own culture, history, and people. The indigenous population often supports this tendency by giving status to such intellectuals. Hegemonic education produces, generally speaking, intellectuals who

have lost their power base in their own culture and society and who have been provided with a foreign culture and ideology, but without a power base in the hegemonic society. (An intellectual at Berkeley, MIT, or Harvard, for example, may actually participate in decision making concerning weapons production and in weapons production itself; or be involved in the policy decisions of multinational corporations; or with U.S. government policy concerning other countries.) Lacking a power base at both ends, Third World intellectuals tend to sharply overvalue symbolic power and tokens (titles, degrees, access to prestigious institutions, awards).

As with individuals, the institutional hegemonic connection does give an illusion of power. Moreover, in some Third World countries, these institutions do produce people who govern the country, compete in international business, and build bombs. In that sense the institutions have become part of the community of the oppressors rather than of the oppressed; they have moved from the Third to the First or Second World. It is precisely this possibility which directs Third World institutions to ally themselves with hegemony and with the dominant trend in education.

In the final analysis, the power of Western hegemony rests on the claims of superiority, universality, and ethical neutrality of Western math and (positivist) science and technology extended into claims of Western superiority in the social, cultural, moral, political, and intellectual spheres. Continuing to accept Western math and science as universal and authoritative is detrimental to creating a healthier and more humane world. Like other human activities, math and science need to be treated in a critical way, not only at the implementation or application stage, but also, and more importantly, at the level of the basic premises and values governing its conceptions, practices, and production.

Education perceived as praxis is the opposite of hegemonic education. Praxis is the combination of concrete conditions (social, cultural, and material), reflection and action in constant interplay. Education perceived as praxis

> implies the existence of two interrelated concepts: ... the context of authentic dialogue ... [and] the real, concrete context of facts, the social reality in which [people] exist.

The primacy of the learner's experience in this type of pedagogy is obvious. Within this perception of education, the essential functions of intellectuals and institutions within the Third World are to make sense of the world, of our experience, and of our culture; to respond to the challenges posed by the real environment; to empower people; to overcome the culture of silence; to remove obstacles to learning; and to release the human capacity to understand and act, in order to help transform consciousness and society. The purpose of both structure and teachers is to facilitate that praxis and that empowerment. Crucial to achieving this are: gaining a power base in one's own culture and society; starting with the perspective and experience of the indigenous population; creating an education relevant to the community's needs and geared towards production (not only in the material sense but also in literary, artistic, etc., fields); and having the community as a primary ally of institutions and intellectuals. It is also important to stress here that empowerment includes acquiring

> the means to critically appropriate knowledge existing outside of [our] immediate experience in order to broaden [our] understanding of [ourselves], the world, and the possibilities for transforming the taken-for-granted assumptions about the way we live.

The study of and interest in ethnomathematics should not be an end in itself. We should avoid the pitfall of reversing the previous attitude. Indigenous cultures and ways were generally ignored, marginalized, or simply considered backward in educational activities and thinking. We should avoid treating them as precious things to be preserved at any cost and celebrated at such occasions as international conferences, and we should avoid glorifying them out of context, out of the realities, activities, needs, and consciousness of the moment. I do believe that cultures should play a revitalizing role in math education; culture here refers to "the particular ways in which a social group lives out and makes sense of its 'given' circumstances and conditions of life." In order for culture to play a positive role, however, education perceived as praxis has to be stressed; linking education with the real environment is central.

Within the perspective of education perceived as praxis, the study of ethno-math becomes an avenue to recognize the fallacies and dangers of a universal conception of life and the world; to question established beliefs and convictions; to free our imagination from ready answers and solutions; to help us gain concreteness and new meanings of words and ideas through real life situations and personal experiences; to provide us with diversified ways of seeing the world and our environment and dealing with the fundamental problems we face; and to help us transform our assumptions and structures.

In this sense, the linear concept of progress has to be abandoned. The belief in a best and universal path is detrimental to the richness which is manifest in life and in communities, as well as to the production of conceptions, practices, and attitudes (including those in math and science) which are meaningful and relevant to the majority of people. The strongest case against hegemonic formal math, in my opinion, is that it is useless and meaningless to the majority of students.

In conclusion, the stress in math education should not be so much on the cultural determinants or the differences in the conditions in industrialized and in developing countries as on whether we perceive education as praxis or not. It is probably easier to do this in Third World communities, because the differences between the ideological environment created by formal education and the real environment are easier to see, and because other ways of doing math are still practiced. Thus, the basic challenge we face in math education, in my opinion, is how to teach math within the perspective of education as praxis.

Society as a Source of Ideas for Mathematics Teaching

Mathematics teaching, if it is to be relevant for the learner, has to take into account the broad range of sources offered by the specific culture of a society. Brian Hudson, United Kingdom, outlines the results of a project aiming to develop materials and approaches for the mathematics classroom from a global perspective and hence dealing with issues related to world development and military technology/arms race. Tadasu Kawaguchi, Japan, shows mathematical thoughts being latent in various artistic activities like literary and painting work, theater plays and movies. Diana Schultz, Australia, describes the theoretical framework and practical strategies for developing and implementing a mathematics education program designed to make mathematics more meaningful to a larger majority of the school population (PRIMITE: Projects in Real-life Integrated Mathematics In Teacher Education). Joop van Dormolen, the Netherlands, talks about the values of texts for learning mathematics for real life by showing examples of written materials and by reporting on how the values influenced the way the texts became what they are.

Global Perspectives in the Mathematics Classroom

Brian Hudson

This contribution to ICME 6 is partially the result of responses as a teacher of mathematics to a number of national and international reports and initiatives. Central amongst these, is the Recommendation Concerning Education for International Understanding, Co-operation and Peace issued by UNESCO at its General Conference in Paris in 1976. This listed the "major problems of mankind" that all students should study as:

- equality of rights of people;
- maintenance of peace; types of war; disarmament;
- action to ensure the observance of human rights;
- economic growth and social development;
- conservation of natural resources;
- preservation of the cultural heritage of mankind;
- the role of the UN in solving such problems.

In the UK the Department of Education and Science on behalf of the Government responded to this with a circular to Local Education Authorities expressing confidence that "the response from this country will be shown to be both sympathetic and constructive."

In 1982 the Committee of Inquiry into the Teaching of Mathematics under the chairmanship of Dr. W. H. Cockcroft noted that "mathematics lessons in secondary schools are very often not about anything" and urged that content of lessons be related to the "real world." In 1985 the Committee of Inquiry into the Education of Ethnic Minority Groups (the Swann Committee) reported its findings and called for the development of syllabuses that are both multicultural in their content and global in their perspective.

As a teacher of mathematics, it is possible to meet the twin aims of developing mathematical skills, processes, and understanding whilst at the same time developing an awareness of key global issues. Indeed in order to understand the complexity and interdependence of the world in the late 20th century, it is necessary to apply a great deal of mathematical skill and understanding.

During 1984/85 I was seconded to the Centre for Global Education at the University of York, in order to develop the curriculum in response to such issues. The aim of the project was to develop materials and approaches for the mathematics classroom from a global perspective. Hence, issues related to world development, military technology/the arms race, and human rights, amongst others, were considered. A computer data base was developed containing data on 127 countries including life expectancy, infant mortality rates, levels of military expenditure, etc. (20 items of data on each country). An accompanying series of problems was developed for students to work on alongside the data base, focusing upon particular issues such as energy, population growth, the arms trade, and others.

The materials were developed and trialed in five schools in the UK which included a multi-ethnic inner city comprehensive school. The materials were subsequently refined and made more widely available through the University of York. As a result, schools throughout the UK and also some schools in Europe and Australia have been sent copies of the package. The process of refinement has become an ongoing one, in an attempt to update the statistics contained on the data base in particular as far as possible.

These issues are primarily global in nature, but are also very much a part of the society of the UK in which our young people are growing up. The advances in technology and in telecommunications in particular have brought the concept of the "global village" to reality and directly into the living rooms of the citizens of the UK. Hence, horrifying images of war, famine, and disasters are a part of the experience of most young people on a tragically frequent basis. The work of the relief agencies and other pressure groups has added to the awareness of many young people and this is reflected in the concerns that many of them have about the threat of nuclear war and for the victims of war, famine, and the abuse of human rights in particular. These concerns have manifested themselves in a popular way through the groundswell of support following the "Live Aid" concert and the more recent concert, organized to commemorate the birthday of Nelson Mandela and to call for his release, which was organized by the Anti-Apartheid movement.

The classroom examples which follow are drawn from the package of materials which I first began to develop in 1984/85. The responses from teachers and pupils are drawn from the classroom trials which were carried out at that time.

It is possible to select any number of fields to be printed out from the data base and also to search through the data with parameters set on one or more particular fields. For example the tables which follow provide information on the life expectancy (LIFEXPEC) and the percentage of the population with reasonable access to safe drinking water (WATER). Table 1 provides this information on countries with life expectancy rates of less than 50, and Table 2 refers to those with life expectancy rates greater than 70.

The issues raised merely by the presentation of the data in this form are very far reaching and the scope for discussion of these issues wide ranging. It is not that any particular answers are suggested but that an awareness of the scale of the inequality is appreciated and that important questions are raised by young people who are the future citizens and voters in our democratic society. However, it is possible to analyze these figures more systematically and to seek out possible correlations. Therefore, an obvious development in this analysis is to ask the students to plot a scatter graph of these figures and also to interrogate the data base for further information. Hence, the mathematical understanding can be developed and contribute to the development of a greater understanding of some of these issues and even begin to point in the direction of some possible solutions.

Table 1

Country	LIFEXPEC	WATER
Oman	49	52 %
Yemen, Arab Republic	43	4 %
Yemen, People's Democratic Republic of	46	37 %
Afghanistan	37	10 %
Bangladesh	48	68 %
Nepal	45	11 %
Laos	43	48 %
Angola	42	17 %
Burundi	45	
Central African Republic	43	18 %
Chad	43	26 %
Congo	43	26 %
Ethiopia	46	13 %
Guinea	43	10 %
Ivory Coast	47	14 %
Madagascar	48	26 %
Malawi	44	44 %
Mali	45	23 %
Mauritania	44	17 %
Mozambique	49	7 %
Niger	45	49 %
Nigeria	49	28 %
Rwanda	46	38 %
Senegal	44	35 %
Sierra Leone	47	9 %
Somalia	39	38 %
Sudan	47	46 %
Togo	48	11 %
Uganda	48	16 %
Upper Volta	44	14 %
Kampuchea	39	45 %

Table 2

Country	LIFEXPEC	WATER
United States	75	99 %
Canada	75	99 %
Argentina	71	60 %
Cuba	73	62 %
Jamaica	71	82 %
Panama	71	83 %
Trinidad & Tobago	72	89 %
Uruguay	71	78 %
Belgium	73	89 %
Denmark	75	99 %
France	76	97 %
West Germany	73	99 %
Greece	74	97 %
Italy	74	86 %
the Netherlands	76	97 %
Norway	76	98 %
Portugal	72	92 %
United Kingdom	74	99 %
Bulgaria	73	
Czechoslovakia	72	78 %
East Germany	73	82 %
Hungary	71	44 %
Poland	73	55 %
Rumania	71	
USSR	72	
Austria	73	88 %
Finland	75	84 %
Spain	74	78 %
Ireland	73	73 %
Sweden	77	99 %
Switzerland	76	96 %
Yugoslavia	71	58 %
Israel	73	99 %
Japan	77	98 %
Singapore	72	100 %
Australia	74	97 %
New Zealand	74	93 %
Costa Rica	73	81 %

An associated issue and also an illustration of how such global issues are reflected within the UK is that of "Overseas Aid" by the British Government. For example, Table 3 gives the contributions by the British Government to the United Nations Development Programme which exists to promote technical cooperation in developing countries.

Table 3

Year	Amount (£ millions)
1978	25.0
1979	28.5
1980	15.0
1981	17.5
1982	18.5

This information could be represented in a variety of graphical forms by the students and associated calculations carried out. For instance, given that inflation averaged 12% in the UK during this period, the corre-

sponding figures could be calculated in order that the amount remained the same in real terms as the 1978 figure. The shortfall in these figures could be calculated in percentage terms and the possible reasons for the reduction and also the probable consequences could be discussed.

There will undoubtedly be resistance on the part of many mathematics teachers to the handling of controversial issues of such a wide-ranging nature. A typical view is that "if it is not abstract then it can not be mathematics!" However my own experience of such an approach during classroom trials has been of very positive responses from both teachers and students. The following comment was from a teacher in an inner city multi-racial comprehensive school:

> It offered an original and interesting approach to statistics and aided the pupils' personal development in their awareness of world problems. I received many comments from them regarding their astonishment at the differences between rich and poor.

Robert, a member of the class, made a comment which reflected as much on his own previous educational experiences as it did upon his response to this work: "I found the disc easy to work with, enjoyable, and interesting. It tells you things you thought you would never know!"

Many questions are raised in seeking to use society as a source of real problems and issues that are worth addressing. Not least amongst these is the question of how to make such an approach generally more acceptable. My own view is that there is a need for resources, courses, and also for associated modes of assessment to be developed in order to encourage such an approach. There are encouraging signs that publishers in the UK are responding, as evidenced by a number of projects, still at an early stage, with the aim of developing materials for the mathematics classroom based upon real contexts and real problems.

Mathematical Thoughts Being Latent in Various Artistic Activities

Tadasu Kawaguchi

Introduction

In this presentation, I would like to demonstrate that there are many mathematical thoughts being unexpectedly latent in such creative activities as artistic works of literature, picture, drama, and movie. They may hitherto be regarded to be irrelevant to mathematics.

It proves that mathematical thoughts themselves are not monopolistic possessions of a limited group of persons such as mathematicians and scientists, but they are universally latent as essential categories of cognitive ways originating in human activities, although their possessors may not always be conscious of them.

Such fundamental characteristics of mathematical thoughts make it possible that the usefulness of mathematics is expanded in very broad areas, not only of natural sciences, but also of almost all human behavioral sciences. And these tendencies must, I am sure, be promoted more and more hereafter.

I think that such cognitive patterns of mathematical thoughts originating in human activities should be made the bases for the organization of curricula of school mathematics education. Under these points of view, I proposed the idea of "Matheracy" in my presentation titled "Fundamental Philosophy on Curriculum Making of Mathematical Education"[1] at the ICMI-JSME Regional Conference on Mathematical Education, 10–14 October 1983, Tokyo, Japan.

Then let me go to my main presentation!

On Some Mathematical Aspects in the Literary Works by Shūgorō Yamamoto Which are Similar to the Works by Henri Poincaré

Shūgorō Yamamoto was a famous novelist in Japan who produced many masterpieces describing the humanistic world of the masses. He left this world in 1967 at age 64. Therefore his active days were much later than those of Poincaré. Furthermore, the former was a novelist and the latter was a scientist. And so there could not be any communication between them. But I was greatly surprised to find extremely similar descriptions in both the works by Poincaré *Science and method* and by Shūgorō *Kokū Henreki*.

The exciting expressions by Poincaré describing his psychological state of catching the idea of Fuchsian functions in *Science and method* (pp. 52–53, English version translated by Francis Maitland) are so famous that it may be unnecessary to introduce them here.

And so I would want to go straight to the *Kokū Henreki* which is esteemed to be one of the most important novels written by Shūgorō.

The hero in this novel was a 'samurai' (warrior) named Chūya Nakatō. His hobby was composing ditties, but would not satisfy himself with them. And then, making up his mind to be a regular composer of 'joruri' (Japanese classical music), he gave up his social position of 'samurai,' and went on a wandering journey in order to train himself. But he could not succeed in being a satisfactory composer in spite of exerting himself to very hard work and dying of sickness finally in the course of pursuing literature.

Now let me go to my main subject! The following shows one section in the first volume of *Kokū*

[1] Kawaguchi, T. (1983). Fundamental philosophy on curriculum making of mathematical education. *Proceedings of ICMI-JSME Regional Conference on Mathematical Education* (pp. 3–9). Tokyo: Maruzen & Co.

Henreki (pp. 159–160, Shinchō pocketbook series). It describes the scene in which Mr. Chūya seized a turning point of initiating himself into the depth of inspiration to musical composition during his stay at an inn in the mountain district of Hakone.[2]

In the following afternoon, it happened that he fell asleep while throwing himself down on a parlor of the inn without taking a bath. And he was composing in the dream but failed after all and threw out his 'samisen' (a sort of Japanese three-stringed guitar) and broke its neck underfoot.

At that moment he woke up, but a melody which was composed inside the dream continued to sound on in his head and was felt to be so fresh as if it began to move vividly. Chūya got up and, taking a 'samisen,' put it in tune. It was raining outside. The murmurs of water were running in a pond and patterings of raindrops were heard secretly. But it may be that they could not be heard by Chūya . After finishing to put the 'samisen' in tune, he closed his eyes for a little while, and then he began to follow the melodies by playing the 'samisen' quietly with his fingers. After following five melodies or so, he went back and then began from the first. Next, after trying the sixth melody, he followed over again from the first. But his fingers stopped suddenly with stiffness when they got to the sixth melody from the fifth one.

"Hell!" He closed his eyes and whispered "fail again?" with his face turned upward. At that moment, a lip music like the way of playing 'samisen' was heard outside.

They were melodies with two pitches upward and three pitches downward and seemed to be heard to connect directly with the melodies being just followed by Chūya . He made his whole body be stiff and listened attentively to the lip music coming from the outside. *He felt that the doors opened in his own inside and a free space stretching to a wide world seemed to be seen.* He carefully examined those melodies in his mind and tried them on his 'samisen.'

"That's just it!" he thought in excitement. "This! This! These are just melodies I am seeking for. They must develop without fail!" Setting the 'samisen' down, Chūya stood up and opened a 'shōji' by the window.

I was intensely excited when I reached this scene in the course of reading *Kokū Henreki*. It was an outburst of excitement owing to the surprise of finding the scene written in italics of Chūya's seizing a turning point towards the seeked melodies closely resembling the descriptions of Poincaré's introspection into the mathematical elucidation.

Such close resemblance between the two works suggests that artistic creative activities have essentially common characteristics with the scientific creation.

Inner Process of Pictorial Creation by the Artist Kaii Higashiyama

Kaii Higashiyama is one of the foremost artists of our time. Several years ago, I had an opportunity to hear his lecture on the title "Encounter with Beauty." It was a very splendid lecture, as I had expected. He is a so-called landscape painter who not only visited the interior of Japan but also various places in Northern Europe and China in order to paint their landscapes. He said in his lecture that he did not paint the landscapes as they were, because the realities of them had disorderly objects such as buildings, telegraph poles and wires, and other miscellaneous obstacles.

By removing all hindrances to his artistic sensibility, he represented his pure image stimulated by the landscapes into concrete forms. That is to say, the landscapes are, of course, very important subject matters for painting, but they should not be painted as they are, and be realized into artistic forms through abstraction of purity by his own artistic sensibility.

When I considered his pictorial process of creation, I felt that it was closely similar to the mental process of a mathematician organizing a beautiful mathematical theory.

Listening to the vivid story of his own experiences as to the encounter with beauty and appreciating the slides of his artistic works in the lecture hall, I had the feeling as if I was hearing the lecture of creative activities of mathematics itself.

Taking advantage of the chance to hear his lecture, I could read several books written by him on his artistic works. In the following, let me refer to some writings in his book titled *Searching for Japanese beauty*.[3]

I found my way in a hurry to the park Maruyama, Kyōto, and I breathed a sigh of relief when I got there.

As there was a mountain close by, the moon fortunately was not visible yet. Against the dark blue profile of Mt. Higashiyama, a drooping cherry tree wore gorgeous pale-rose attire on its branches and seemed to gather up all of the beauties of spring in Kyōto. Besides, there was not even one fallen blossom on the ground.

The summit of the mountain grew light. The moon appeared slightly and began to rise up quietly in the evening sky tinged with purple.

Now the cherry blossoms looked up at the moon and the moon watched the blossoms, too. At that moment, all of the lights on 'bonbori' (Japanese hand lamp) and the crowd of people disappeared and there remained a pure and beautiful world with moon and flowers only.

The writings shown above describe the inner process of pictorial creation by the artist Kaii Higashiyama when he got the motif of the work 'Hana Akari,' one of his masterpieces. The word 'Hana Akari' refers to such a motif as the scene in ecstasy and stillness with twilight atmosphere produced by the interplay of cherry blossoms and the full moon ('Hana Akari,' 1986, 130 x 97 cm).

Here, too, I could dare say that we feel a very strong similarity between both processes of encounter with beauty in artistic and scientific creative activities.

Some Aspects of Mathematical Structure in Form and Beauty of Plays of Noh and Kabuki (Japanese Classical Plays)

It seems to me that the theatrical structure of Noh play is very similar to mathematical construction because they are both abstract, formal, and promissory. An explanation as to the Noh can be found in the World Encyclopedia.[4]

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[2]Yamamoto, S. (1981). *Kokū Henreki*, Vol. 1 (pp. 159–160). Shinchōsha & Co.

[3]Higashiyama, K. (1976). *Searching for Japanese beauty* (pp. 26–27). Kōdansha & Co.

[4]World Encyclopedia, Vol. 17, pp. 543–545. Heibonsha & Co.

In the Noh play, there are no fitted stage sets. Nothing of the set could, on the contrary, permit and yield fantastic imaginations of the audience by the sympathy with both music 'yokyoku' (Noh vocal music) and movements of actors. The performances of actors are shown on the main stage and 'hashigakari' (a corridor looking like a bridge and leading to the main stage from the green room). The main stage and 'hashigakari' are used properly in various meanings by setting promises. For example, the main stage is looked upon as this world and 'hashigakari' as that world, heaven. Or, in another example, the situation of drama on the main stage has been turned into the Lake Biwa from Kyoto while an actor goes to the 'hashigakari' and comes back to the main stage.

As to the masks used and actings in the Noh play, the following remark may also be found in the encyclopedia referred to above:[5]

The facial expression of a good mask can variedly be changed by slightly turning upward or downward besides its own proper expression. Of course, it is impossible for a mask to change its expression by itself. In spite of this, we feel a wonderful change of its facial expression in the appreciation of Noh play. That is to say, an actor's feeling or will can be expressed by his whole body without any muscular variations of face in Noh play. Namely, this type of acting may be said to have the entirely same effect, as the mask has its own facial expressions.

As to this affair, the acting in Kabuki is rather real and weaker in abstract than Noh play and apt to reveal living expressions of humans themselves. From this point of view, it seems to me that the Noh play has extraordinary beauty owing to more purity and formalization of its constitution than Kabuki. As to this matter, I have written an essay for a journal as follows:

The beauty of Noh art is originated in its formal constitution which is, I am sure, very similar to the beauty yielded by the system of mathematics. All acting is done by 'shite' (star player, 'waki' (co-actor for a star player), and 'tsure' (companion for a star player or a co-actor) in a small stage called 'Noh butai' with almost no set are composed of small units of action which are to be played according to some definite modes.

And, only one circuit by an actor within the 'Noh butai' represents such a feeling effect as making a long journey far off away. Furthermore, despite the fact that there are no facial expressions in the actor's mask, we may feel his deep sorrow or furious anger by some slight actions of his shoulder or limbs. All of this is evidence to show that the formal constitutions have deep meanings and functions inside themselves and may be said to have the same functions and beauty as those of the abstract form constituting mathematical contents.

The stage space in Kabuki play has, so to speak, the same function as that of a universal set in mathematics. In the former, a stage 'hanamichi' is ordinarily used for a long passage leading to the main stage from the green room. A unique Kabukian act called 'hikkomi' is performed at the end of the drama on the 'hanamichi' in order to attract the audience's attention to it by separation of space from the main stage through skillful operation of 'donchō' (drop) or 'hikimaku' (drawing curtain). This situation is surely very similar to the idea of transformation from one universal set to another in order to change the objectives to be attained.

Conclusion

In spite of the fact that I have not referred to movies, it seems sufficient to show that mathematical thoughts are surely latent in various artistic activities.

But, despite such assertions, I could not erase my natural feeling that there should be some distinct differences between artistic and mathematical activities. Although it is difficult for me to describe them clearly, I dare say that the emotional aspect is more stimulated in the former, but in the latter, the intellectual aspect is characterized more.

I do perceive the differences between them, and yet their sensibilities may not be confronted with each other or not be separately coexisting in parallel, but must be developed harmoniously as a whole in both cases of mathematical and artistic creative activities. Although the mathematical contents have intellectual qualities of cool, the creative or cognitive activities in mathematics are derived by an emotional character having strong feelings toward the beauty as in the case of artistic creation.

PRIMITE—Projects in Real-Life Integrated Mathematics in Teacher Education

Diana Schultz

At Footscray Institute of Technology, Melbourne, Australia, a program has been developed as part of the Bachelor in Education (Primary) course, which attempts to create innovative solutions which are sensitive to community-based mathematics, to the elimination of sexism, and to the needs of a multicultural society; a program which directly models the pedagogic strategies which students themselves will use as teachers of real-life mathematics in schools.

This paper describes the theoretical framework and practical strategies for developing and implementing a mathematics education program designed to make mathematics more meaningful to a larger majority of the school population. It explains the theoretical concepts which underpin the course, that is, how the research on metacognition, modeling, real-life mathematics, integration, and cooperative learning is applicable and contributes to developing a real-life project style mathematics, called PRIMITE. The program develops the student teacher's mathematical skills and concepts, at the same time as they learn how to teach primary mathematics curriculum topics.

PRIMITE—Real-Life Mathematics

The fundamental concept of PRIMITE is to take normal everyday activities from the society, and more particularly the community, in which the students live and actually carry out those activities using and teaching the math skills inherent in them.

[5]World Encyclopedia, Vol. 17, p. 544. Heibonsha & Co.

PRIMITE is basically an overlay on existing methods, so that students can relate math to real life and enjoy discovering math concepts for themselves rather than having the concepts imposed upon them by teachers.

In a PRIMITE course, students, working in small groups, carry out relevant real-life projects, the successful completion of which requires that certain math skills and concepts be used.

Typical projects are: designing a house, planning and preparing a birthday party, or planning, preparing, and cooking a meal. These particular projects are taken from Western society and actual activities would need to be selected from the society in which the students live.

As the students reach parts of the project which require that a particular math skill is used, the skill is taught either within the project or through use of effective traditional methods, particularly concrete materials. In this way, students learn the skill when they need to learn it, not through an artificially imposed timetable, and they are able to see the relevance of that skill to everyday life. In some cases, students are able to discover the skill for themselves with little or no guidance from the teacher. This provides additional satisfaction for the students and they are more likely to remember something they have discovered for themselves.

According to Dance and Higginson (1979) it is preferable to present a unified math curriculum, e.g., project work, as opposed to segmented sequenced mathematics. This enables a student to transfer skills and knowledge from one situation to another.

Because of their traditional training and background, most teachers tend to develop contrived 'real-life' exercises which have little relevance for students, e.g., 'If Farmer Brown has three apples ...?' Such exercises are rarely relevant to students' lives. Projects must be of the type which students can see as relevant and should be actual real-life projects, not just theoretical exercises. Planning an actual birthday party for a particular student is much more exciting than simply doing the exercise in theory. It also brings in real-life problems which may be overlooked in a theoretical exercise. In designing or guiding any project, teachers must always ask themselves "Is this what I would do if I was actually doing this project?" Students should therefore engage in 'real' real-life mathematics as opposed to 'pseudo' real-life mathematics.

Integration

The PRIMITE concept is very much that of a math course which is fully integrated with activities that are part of the community in which the students live. Furthermore, projects often incorporate several mathematics curriculum topics. Such an approach also leaves the way open for the integration of other subjects within the projects.

One danger with an integrated course is that students can complete a project with certain topics and skills not covered or only covered superficially. Another danger is that topics can be presented in a haphazard and disorganized fashion. These problems tend to be particularly noticeable in the math area, especially where teachers themselves are weak in math.

It is vitally important that teachers are shown ways to select mathematically rich, real-life projects. 'Witches and Goblins' may be a worthwhile theme for language and art work, but it is not suitable from a mathematical perspective. Through PRIMITE, mathematics can be studied in the required depth and in a logical sequence at a time relevant to students' work.

Whilst some sequencing is essential, it is not clear that all children learn concepts in the same order (Labinowicz, 1985). In other words, teachers may teach in a mathematically logical order, but students do not necessarily learn concepts in that order.

Metacognition

In the PRIMITE course, the problem of superficial coverage is handled in a number of ways. Foremost among these is metacognition and related techniques.

Flavell (1976), one of the first and most prominent writers on metacognition, defines metacognition as "one's knowledge concerning one's cognitive processes and products" ... [and] refers to the active monitoring and consequential regulation of these processes (p. 232).

The metacognitive approach operates at two levels. For both school students and student teachers, it involves becoming aware of the math and other (e.g., problem solving) skills they are learning and how they are learning them.

However, student teachers must not only be aware of their own learning methods, but also of the learning strategies of others. Through cooperative learning, they need to become familiar with the metacognitive strategies their peers use, which will develop their sensitivity to the ways children may learn in their classes.

Metacognition and cooperative sharing of information has been very valuable in this course. Many student teachers were not even aware that they had learned a particular skill. It was only when one student pointed out that they had learned some skill, that others realized they too had learned the same skill.

At the end of each PRIMITE class a period is set aside for students to evaluate what they have learned and how they have learned it.

Metacognitive development is monitored and guided in two ways: contracts and journals. At FIT, the writing of the journals is also used to introduce students to computers.

Contracts

At the start of each project, students negotiate a written contract with the teacher which details: the project, topics to be covered, and other items such as the mode of final presentation of the work and timetables. Stu-

dents, particularly at higher levels, are able to keep a check on their own progress, guide, and evaluate their own development. This contract, together with the fact that the projects are relevant and interesting, provides a significant reduction in discipline problems in school classrooms.

Teachers much also be aware of the curriculum topics and math skills which are being covered by each student and ensure that, over the year, students do projects which enable them to learn all the required skills. Teachers should keep a record of the skills learned by each student through a method such as contracts or check lists.

Journals

Students are required to keep a journal detailing what they have learned and how they learned it. The journal is completed by each student during the metacognition period in each class and is the major aid to students in evaluating their own development and progress. It also assists the teacher in assessing the progress of students.

Keeping a journal is particularly important for student teachers. Student teachers, doing projects as part of teacher training, must pay particular attention to their own learning processes, so they can understand them and thus have a clearer understanding of how their students learn. By comparing journals, student teachers also come to realize that the particular way they learn is not necessarily the way everybody learns. The journals also provide a valuable reference for students later in their course when they can refer back to them to clarify learning processes.

Computers

At FIT, student teachers are required to write their journals using word-processing software on laptop microcomputers. While this is not a critical part of PRIMITE it is a useful way to ensure that students come to grips with, and learn to use computers as productive tools. Writing journals on a laptop is an effective antidote to the computer phobia of many students entering the course. This familiarity can then be transmitted to their primary students to better equip them for the real world in which they will almost certainly encounter computers. The approach ensures that all teachers are familiar with computers, and that computers in schools are not relegated to a computer lab under the control of a specialist computer teacher, but rather are available to all students. Laptops have proved effective because they are easily transported as portable tools and their size is less threatening.

Teacher Role and the Development of Cooperative Learning

Because many groups of students work at different speeds and often on different projects, teachers do not have time to carry out the traditional role of providing most or all the information students need to learn. Wherever possible, students are required to learn in the same way adults do in real life: through reading, trial and error, research, discussion, and pooling of knowledge. This cooperative group learning is very effective and has a number of advantages:

- Students develop learning skills which are useful throughout their lives.
- Peer teaching and cooperative learning "... may be an example of what every beginning teacher discovers: that one must relearn one's subject in order to teach it, that one has to reanalyze what he knows and how he learns in order to promote similar knowledge and learning in others" (Cloward, 1976, p. 227).
- Students are able to discover and learn information which the teacher may not have, that is, their learning is not limited by what the classroom teacher knows.
- Students are able to experience the joy of discovery and tend to be much more enthusiastic, because they feel they have more control over their activities and are doing projects which interest them. Math becomes fun.
- Discipline problems are minimized because students are enthusiastic about their work.

Experience and research (Schultz, 1985) show that student teachers have enormously disparate levels of math skills. It is extremely difficult for the teacher educator to teach math skills at the variety of levels required by the student teachers. By learning through PRIMITE, students develop their own math skills, and often discover deficiencies which the teacher educator may not even have considered. Cooperative learning is particularly valuable in this setting because the more highly skilled members of the group can assist the weaker ones.

The primary role of the teacher in this type of learning is to guide students in their learning to provide information only when this is the most effective way for the students to learn.

This role is quite a radical change for most teachers, and many teachers are likely to have difficulty in relinquishing their role as sole repository of knowledge.

Modeling

Student teachers usually expect to be shown the pedagogy they themselves experienced as children, that is, their concept of teaching is modeled on that of their past teachers.

The biggest difficulty in using PRIMITE with student teachers has been the resistance of many students to change. Students have preconceived notions of what teaching is, and often take some time (weeks or months) to recognize the value of PRIMITE. The realization of what PRIMITE is about is usually accompanied by a lot of excitement—the joy of discovery.

Modeling is a very powerful teaching method. Many writers, including Bandura (1965), Fromberg (1977), Zimmerman and Ghozeil (1974) assert that modeling is a very potent form of learning.

In order for student teachers to be able to teach the use of PRIMITE, it is necessary for them to learn the use of PRIMITE. They discover the advantages, joys, and difficulties of learning to use PRIMITE. They are also able to model their PRIMITE teaching of children on what they have experienced in classes at college.

Student teachers discover the enjoyment of learning through this method, and through metacognition, cooperative discussions, and their journals, become aware of what they have learned and the way they and others have learned it. They can develop methods and ideas for teaching their own students.

Summary

PRIMITE is a radical and effective method of improving math learning, problem-solving skills, and learning skills among students. None of the individual concepts used within PRIMITE is new, and all have been well researched. What is new, however, is the way in which these concepts have been combined.

There are a number of critical factors in the course:
- real-life (not contrived) projects taken from the society in which the students live;
- careful analysis of projects and evaluation by the teacher to ensure that all required skills are covered;
- cooperative learning;
- cognitive and metacognitive thinking (journals and contracts);
- modeling of teaching methods.

In order for PRIMITE to be successful, it is important that each of these critical factors be considered and implemented. Failure to consider each factor independently will probably lead to failure of the method, as has happened with other new teaching techniques. For example, cuisenaire has fallen into disrepute because many teachers used the material incorrectly, without full understanding of the concepts behind it.

For PRIMITE to be successful, it must be properly modeled by teacher educators for student teachers who can implement it in schools appropriately.

One last common sense test of the value of PRIMITE (or any teaching method) is to compare it with the way in which most people learn in the real world. PRIMITE models very closely, not only in the activities in today's society, but also the learning and problem-solving techniques of effective adults. It provides children with research and problem-solving skills as well as the skills and knowledge of mathematics.

Whereas traditional math teaching techniques fail the vast majority of the population, PRIMITE attempts to develop an innovative, comprehensive and *effective* approach for all students!

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Values of Texts for Learning Mathematics for Real Life

Joop van Dormolen

Introduction

The social environment of pupils is a great source for teaching mathematics, not only for practical reasons because one can expect the pupils to understand better and to be more interested in learning the subject matter involved, but also for ethical and idealistic reasons. The teacher's and the textbook author's set of values play a fundamental role in the choice of goals, contents, and teaching methods.

In my talk at ICME, of which this article is a short abstract, I showed many examples from a textbook for grade 7, called *Wiskunde Exact* ("Wiskunde" is the Dutch word for mathematics). It is a volume from a course for grades 7–12, published by Meulenhoff Educatief, Amsterdam, in which reality is chosen as a primary source for ideas and problems.

Goals, contents, and teaching methods are values because they depend on subjective choices. Statements, like definitions, or theorems, or methodological rules are objective, but by calling them theory, or goals, or true, or genuine mathematics we give them a value.

Values—A Categorization

Here is a categorization of the values that we use explicitly in our work as textbook authors:

(1) Values of objectives
  - Subject matter
  - General goals
  - Mathematical goals

(2) Mathematical values
  - Correctness
  - Aspects of mathematics
  - Genuineness

(3) Values of sequencing
  - Tracks of learning content
  - Tracks of personal development

(4) Values of adaptation to students
  - Instructional activities
  - Short-term sequencing
  - Language
  - Text management by pupils

(5) Framing conditions
  - Teachability
  - Implementation

Values of Objectives

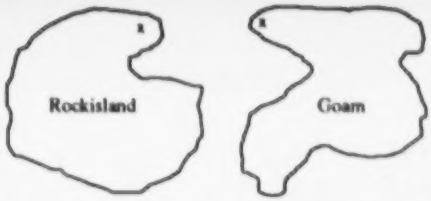
There are many exercises in which we strive for general goals. For example: Reading complex tables and realizing what all the numbers in them mean, is one means among many others to help pupils to grow up to be independent and critical members of society. Annex to this, they learn to value mathematics as a means for being such a person.

Another example: One sees many numbers around one, but not all of them are used for ordering, or counting, or measuring. The number on a football player's shirt, or the number on a bus are just labels and have nothing to do with order or amount. Realizing this helps one to fight dishonest use of numbers.

Mathematical Values

In *mathematical values* we have chosen three kinds. *Correctness* is one of them. We often decide that we cannot be as precise as we might want to be from a mathematical point of view, because we expect that pupils will not understand us if we are.

Here is an example from the textbook.



This problem is one of the first leading to the concept of distance. We assume that the pupils understand (intuitively) what distance between two points is. We want the pupils to learn about the distance between two figures and then, as a special case, about the distance from a point to a straight line. Is it correct to talk about distance between two figures without explaining what a figure is and without defining what the distance between two points is? This point also has to do with long-term sequencing: Maybe we can teach the 'correct' notion of distance later.

We recognize five *aspects of mathematics*. One of them is *theory* (we prefer to call it *structure*). *Algorithms* is another one.

The counterpart of structure is *logic*. To state definitions and theorems and to understand them is a theoretical aspect, while using them to prove a theorem is an aspect of logic.

The counterpart of algorithms is *methods*. There is not always an algorithmic way to find an answer to a problem. In that case we have to do it in a heuristic way. We call that knowledge of and ability to use methods.

The fifth aspect is one of *conventions*.

One example: Pupils see two photographs of the same tree; one was taken in summer and one in winter. They are asked to discuss similarities and differences. This exercise has several mathematical aspects. There is a logical aspect because we want pupils to learn to separate the relevant from the irrelevant arguments. There is also the aspect of *methods* because there is no algorithmic way to find an acceptable answer to the problem. For example, one can realize that there are several different ways to talk about differences and similarities: form, color, environment, measurement, life form, etc. This problem was also included because we want pupils to realize that mathematics, as all other subjects, has a strong *conventional* aspect. When one reflects and discusses several answers to the problems, one discovers that some differences are not considered as such by others. The only way to solve this is to come to a conventional agreement.

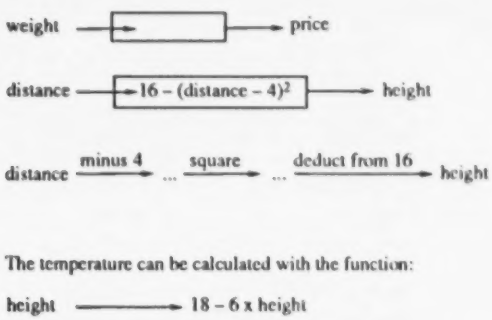
Genuineness of mathematics has a specia¹ meaning in our group. We call mathematics genuine when there is a dynamic activity in which *kernels* and *problem situations* complement each other. Kernels are statements from the barren mathematics, like definitions, theorems, algorithmic rules, conventions, etc. Problem situations give us reasons to make such statements.

In the exercise about the two islands there is a problem situation. We want the pupils to realize that there is a reason to talk about distance between two figures. We did not want just to tell them what distance is, either by definition or by example.

There are two other reasons why we think it is important to have problem situations, both as starting points to learn new kernels and as applications afterwards. Both reasons have to do with motivation. The first reason is a tactical one. Psychologically one is more motivated to learn and to make an effort to learn more when one recognizes the problem as such. The second reason is an ethical and idealistic one. In dealing with persons respectfully, one is obliged to make them aware of why you want to teach them anything instead of telling them just to do what you say.

Values of Sequencing

Sequencing has to do with long-term planning. One can look at this planning in two ways. One way is to look at the *tracks of learning content*. It is simply the planning of which subject must be learned before another subject. For example, we want pupils to understand the concept of variables before they learn any algorithm for solving equations. Another way is to look at *tracks of personal development*. This has to do with the way people learn concepts. Generally speaking, we cannot teach explicit kernels (definitions, rules, theorems, and the like) unless our students are ready for it. In building up the tracks for our textbooks, we rely very much on Vygotskii's idea of the zone of proximate development. The following examples explain how.



This diagram shows the way we treat variables and functions. It contains examples from several pages in the book, the first is from page 64, the last from page 177. I mention this difference because I want to stress that there is a gradual development throughout the book.

Why do we think this is important? We are convinced, that the most important reason for not understanding mathematics is bad preparation on the concept of variable. By slowly and gradually letting the pupils get acquainted with this phenomenon of variability, closely connected to phenomena that they know in reality, we put the idea in their heads, so to speak. Not before page 165 do we help them to realize this and to make it explicit.

This way of using the theory about the zone of proximate development we call implicit gradual development.

Here is an example of implicit gradual development of the concept of variable. Pupils see a picture of two appartment buildings behind each other, both facing south. In order that each floor of both buildings gets sun at noon the distance is given by the formula: *distance* = 2 x *height*. Pupils are asked to do some calculation with this formula without knowing that this is called a formula and that *distance* and *height* are called variables. Only much later, after having learned these notions implicitly, they learn the terminology.

The same principle goes for the concept and the symbolic form of function. Here we have a complication. The traditional symbolic form, like the last example in the diagram above, is an unfortunate one. There is no clear distinction between the output and the function prescription. We deplore this, because this also must be one of the reasons why so many pupils do not understand the concept. In the beginning of our text functions are called 'calculator boxes' because of the close connection to the calculators, automatic weighing machines, petrol pumps, etc. that the pupils know very well. In the symbolic form one sees a clear distinction between input, output, and prescription. At first the box is a black box. Later one knows what the box does. Then there is a transition to a more abstract symbolic form. Unfortunately, teachers have to teach the conventional form, too.

In the example of the two trees the pupils implicitly learn to realize that it is important to agree on which way two things are different and in which way they are similar. This goes for many things, such as congruence and similarity in geometry, but also for equivalence of fractions, the equivalence of equations, the way we draw 2-dimensional pictures of 3-dimensional bodies, the "equalness" in expressions like

$$\underline{a} * \underline{a} * \underline{a} * \underline{a} * \underline{a} * \underline{a} + 1 = \underline{a}^6 + 1$$

on the one hand and

$$\underline{a} * \underline{a} * \underline{a} * \underline{a} * \underline{a} + 1 = \underline{a}^5 + 1$$

on the other hand, etc. Thinking about differences and similarities is an important methodological aspect of mathematics.

Values of Adaptation to Students

It takes a decision how and when we want to adapt our teaching to the pupils' abilities and interests. So there also is a set of values involved here. The last category was concerned with long-term planning. In this one, we consider short-term planning the subject matter.

One item in this category is *short-term sequencing*. The example of Hans and Mieke was taken out of a sequence of exercises in which the pupils learn about distance. Starting from getting acquainted with a problem situation, the pupils have to sort out different ways to talk about distance and to, finally, arrive at an

explicit description of the distance from a point to a straight line. Similar remarks can be made on the example of numerals and numbers.

These examples, like all others, have also to do with decisions about the instructional activities of the pupils, like examining, representing, transforming, proving, applying, measuring, etc.

Another important value is the way authors and teachers use *language*. Here we have to answer questions like: Do we, and if so in what situations, use complex sentences? How much information and how much redundancy shall we put in our texts? Do we use verbs that imply an activity, or verbs that describe existence?

Another text problem is where and when to use metaphors. The calculator box is clearly a metaphor for function. In what situations do people learn something from metaphors?

The last item on this list is *text management by students*. This has to do with the implicit or explicit didactical contract between the author (or the teacher) and the pupil. Most often such contracts are implicit. Do we expect pupils to answer just "yes" or "no" to a question, or do we want them to explain? If we make such a choice, should we not tell the pupils about it? Or do we want them to decide for themselves? And if so, how do they know that we want them to decide for themselves?

Framing Conditions

The last set of values are concerned with the framework of external conditions, such as *teachability* and *implementation*.

These values have to do with the necessity that teachers understand and accept the underlying principles, goals, values, didactical methods, mathematical content, etc., with the possibility to finish a book in time, with being acquainted with the existence of the book, with the price, etc.

A Final Remark

A person's set of personal experiences is a good starting point for learning. It must be understood, that this set does not only consist of experiences with the outer world, which is usually designated by 'daily life.' One's personal world also consists of imaginary experiences, or experiences that easily can be imagined. In other words, subjective reality is not congruent with authenticity. This was a great help in our author group because it took away the necessity of spasmodically trying to find things that really happened.

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How Autonomous is the Mathematics Teacher?

In any society the teacher is socially constrained and some of the consequences for mathematics educators are considered in this topic. Dianne E. Siemon, Australia, describes in general the social constraints surrounding teacher's work and pleas for investigation of the belief systems underlying the teaching and learning of mathematics. Paul Ernest, United Kingdom, asserts that all reforms depend on the teacher's system of beliefs, e.g., conceptions of the nature of mathematics and models of teaching and learning mathematics, and presents a model of mathematics teacher's knowledge, beliefs, and attitudes which elucidates the theoretical relationships between the teacher's belief system, practice of teaching mathematics, and the learner's belief. Kurt Kreith, USA, reports about a teaching credential program for the recruitment and training of master teachers which has developed training and certification procedures that correspond, in appropriate ways, to those employed in other professions, such as medicine and law. John Suffolk, Zambia, speaks about the specific difficulties constraining mathematics teachers in developing countries.

How Autonomous is the Teacher of Mathematics?

Dianne E. Siemon

There have been a number of exemplary attempts to change the face of mathematics education in Australia over recent years. Government agencies at both the national and state level have been prepared to support innovative programs, run very largely by teachers for teachers with little interference or direct control. Teacher organizations such as the Australian Association of Mathematics Teachers and the Mathematics Association of Victoria continue to provide a range of practical materials and ideas which support change imperatives. Recently, these organizations have begun to adopt a higher public profile with respect to position statements and policy documents on important issues affecting mathematics education.

This trend is reflected in other countries as well, notably England and America. But for many of us who visit schools regularly, the actuality has fallen far short of the promise. It would seem that it is not enough to address the priorities, identified by reports such as the *Agenda for action* (NCTM, 1981) and the *Cockcroft Report* (1982), by advocating changes to classroom practice alone. Providing interesting learn-

ing materials to support process learning, problem solving, group-work, and a wider range of assessment options does not appear to have reduced the reliance on 'traditional' textbooks, nor does it appear to have made am impact on the type of mathematics being taught. Even well-conceived, longer-term professional development activities designed to expand the repertoire of teaching strategies to include investigations, discussion, and technological applications, seem to have had little effect on what the majority of teachers do in classrooms (Siemon, 1987). This is not to say that these initiatives are misguided or inappropriate, indeed they are invariably exemplary and are *absolutely necessary* to the overall success of the change effort, but rather, that they are simply not *sufficient* in themselves to bring about effective long-term change. Something more is needed.

I believe that the quality of any mathematics education program is a function of three variables:

(a) what we teach,
(b) how we teach it, and
(c) why we do what we do.

It is the first two of these that have been the focus of the recent change initiatives. The third, *why* we do what we do, is only just beginning to be recognized as an important component affecting change (see Britzman, 1986). Why we do what we do in classrooms is very much dependent on what we know and believe about mathematics, about the teaching and learning of mathematics, and about the nature of our particular task as mathematics educators. This very idiosyncratic grab bag of knowledge and beliefs can be referred to as our *personal theory of mathematics education*, a term derived from George Kelly's Personal Construct Theory. Each one of us has some sort of personal theory of mathematics education whether we like it or not. We may not be aware of its entirety or fully appreciate its impact on our decision making, but it is there nonetheless, and whoever we are, our personal theory of mathematics education needs to be made conscious and critically examined. Many would argue: "But of course, that's what I do all the time." But do we? I would suggest that we only examine our *public* theories of mathematics education, what we think we should know and believe about mathematics, about the teaching and learning of mathematics, and about our task as mathematics educators. I believe that the demise of the 'New Mathematics' in the 1970s and the recognition that there is no such things as 'teacher-proof' materials, testify to the fact that our personal theories of mathematics education are alive and well and functioning very powerfully behind the scenes to temper and qualify what our public theories should otherwise achieve. Recognizing, understanding, and challenging our personal theories of mathematics education holds the real key to effective and meaningful change. What influences and shapes those theories; whether it is our biography, unrealistic curriculum objectives (hidden or real), inappropriate performance criteria and/or the perceived demands of peers, school authorities, parents, and employers; deserves the same sort of scrutiny (public funding, research, and development) that the first two issues have received. Until these underlying beliefs, attitudes, and knowledge bases are meaningfully challenged, the change effort is in real danger of becoming yet another 'bandwagon.'

Teachers are not autonomous, nor can they act in a vacuum independent of societal values and cultural traditions. Many factors determine what society values with respect to mathematics education and therefore what assumptions and decisions are made about:

(a) the type of mathematics that is taught,
(b) when it is taught and the amount of time provided,
(c) the environment in which it occurs,
(d) the nature and extent of teacher training,
(e) the level of support it attracts, and
(f) how it is evaluated.

These assumptions and decisions influence or determine the sort of teaching and learning strategies adopted. Very often, they are so robust that they prevail, despite what we might know and believe about the nature of children's learning and the value of certain approaches. Speaking about the pre-eminence of written algorithms in her classroom, despite what she has observed when her children went to the Victoria Market to buy something and to work out the change, a young teacher states that:

> ... for the maths the kids use when they go to the football or milkbar or whatever ... they hardly ever use recording ... I only do it because it's in the program ... there's only a handful of activities that they have really enjoyed and gained a lot from ... the rest has just been from the program.

At a recent workshop an experienced primary school principal made the following observation:

> I think activities and understanding are fine for the average child but some kids will never understand it no matter how long you spend with them or what you do ... you have to show them a rule, otherwise they'd never get anywhere.

What these statements reveal is the dichotomy between our public and personal theories of education. The first teacher acknowledges the real value of alternative teaching and learning strategies and makes an effort to include these, yet she still perceives a need to adhere to a rigid program which she herself has devised. The principal, while openly supportive of change efforts, reveals, when challenged, that he is uncomfortable with some of the assumptions and values which underpin change, particularly if they mean that curriculum expectations may not be met in the time provided.

In conclusion, it is simply not realistic, fair, or reasonable to assume that changing *what* we teach and *how* we teach it is something that individual teachers in isolated classrooms have any real control over. Once again, this is amply illustrated in discussions with practising teachers. Having expressed the opinion that their students "enjoyed the maths activities and really got a lot out of doing them," a group of teachers involved in a post-initial training course in mathematics education were confronted with the question: "If that's the case, why don't you use such activities more often?" The unanimous and almost instanta-

neous reply was "there isn't time." What this is acknowledging is the belief that there is a body of knowledge 'out there' which *has to be* covered. One teacher stated this explicitly:

... you've got to get through that program and have the children up to a certain level by the end of the year.

Other reasons given for not adopting change initiatives were:

It's not school policy.

I wouldn't feel confident that I was covering everything in the school program.

Parents and children don't see it as mathematics.

The principal would ask questions if I did that all the time.

I just don't feel confident that I've learnt enough ... that I know enough to be justified doing something else.

A lot of people don't tend to think that if you've got your children outside you're actually teaching them ... they think you are just playing games outside ...

... you can't just be the one to break the system ... you've got to work within the system as such.

to do something you haven't met with before ... to do it by yourself ... you just can't expect ... you need time ... other people to help you ...

Acting together proactively, to inform and change societal values, beliefs, and understandings; and cooperatively, to refine and extend teachers' knowledge, skills, and beliefs; as two important projects in the United States have begun to do (see Romberg, 1987; Fennema et al., 1987, respectively), provide a real opportunity to redress the imbalance in the current change initiatives. But this is the second step, the first step is to recognize that teachers are not autonomous by asking the questions: "Who or what *really* determines *what* we do in classrooms?" "Where do our decisions come from?" "Why?" "What are they informed by?" "Who has the responsibility and/or authority for creating and exercising these subtle forms of control over our practice?"

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The Impact of Beliefs on the Teaching of Mathematics

Paul Ernest

Official reports such as NCTM *Agenda for action*, and the *Cockcroft Report* recommend the adoption of a problem-solving approach to the teaching of mathematics. Such reforms depend to a large extent on institutional reform: changes in the overall mathematics curriculum. They depend even more essentially on individual teachers changing their approaches to the teaching of mathematics. Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change. Furthermore, these changes in beliefs are associated with increased reflection and autonomy on the part of the mathematics teacher. Thus, the practice of teaching mathematics depends on a number of key elements, most notably:

- The teacher's mental contents or schemas, particularly the system of beliefs concerning mathematics and its teaching and learning;
- the social context of the teaching situation;
- the teacher's level of thought processes and reflection.

These factors are therefore those which determine the autonomy of the mathematics teacher, and hence also the outcome of teaching innovations—like problem solving—which depend on teacher autonomy for their successful implementation.

The mathematics teacher's mental contents or schemas includes knowledge of mathematics, beliefs concerning mathematics and its teaching and learning, and other factors. Knowledge is important, but it alone is not enough to account for the differences between mathematics teachers. Two teachers can have similar knowledge, but while one teaches mathematics with a problem-solving orientation, the other has a more didactic approach. For this reason, the emphasis below is placed on beliefs. The key belief components of the mathematics teacher are the teacher's:

- view or conception of the nature of mathematics,
- model of the nature of mathematics teaching,
- model of the process of learning mathematics.

The teacher's conception of the nature of mathematics is his or her belief system concerning the nature of mathematics as a whole. Such views form the basis of the philosophy of mathematics, although they may be implicitly held philosophies. The importance for teaching of such views of subject matter has been noted both across a range of subjects, and for mathematics in particular (Thom, 1973). Three philosophies are distinguished here because of their observed occurrence in the teaching of mathematics (Thompson, 1984), as well as in the philosophy of mathematics and science.

First of all, there is the instrumentalist view that mathematics is a set of unrelated but utilitarian rules and facts.

Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created.

Thirdly, there is the problem-solving view of mathematics as a dynamic, continually expanding field of human creation, a cultural product. Mathematics is a process of inquiry and its results remain open to revision.

The model of teaching mathematics is the teacher's conception of the type and range of teaching roles, actions, and classroom activities in mathematics. Many contributing constructs can be specified, including unique versus multiple approaches to tasks, and individual versus cooperative teaching approaches. Three different models which can be specified through the teacher's role and intended outcome of instruction are:

Teacher's Role	Intended Outcome
(1) Instructor:	Skills mastery with correct performance
(2) Explainer:	Conceptual understanding with unified knowledge
(3) Facilitator:	Confident problem posing and solving

The use of curricular materials in mathematics is also of central importance in a model of teaching. Three patterns of use are:

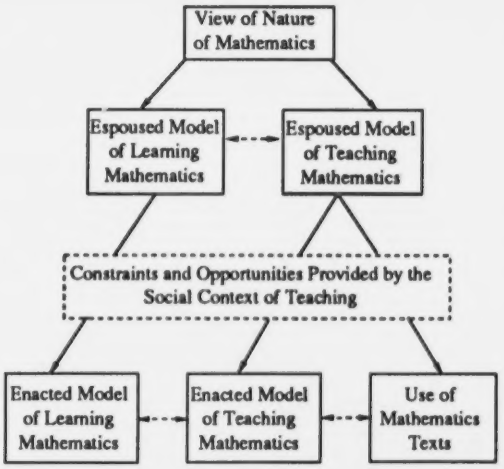
(1) the strict following of a text or scheme;
(2) modification of the textbook approach, enriched with additional problems and activities;
(3) teacher or school construction of the mathematics curriculum.

Closely associated with the above is the teacher's mental model of the learning of mathematics. This consists of the teacher's view of the process of learning mathematics, what behaviors and mental activities are involved on the part of the learner, and what constitutes appropriate and prototypical learning activities. Two of the key constructs for these models are: learning as active construction, as opposed to the passive reception of knowledge; the development of autonomy and child interests in mathematics, versus a view of the learner as submissive and compliant. Using these key constructs the following simplified models can be sketched, based on the child's:

(1) compliant behavior and mastery of skills model,
(2) reception of knowledge model,
(3) active construction of understanding model,
(4) exploration and autonomous pursuit of own interests model.

Relationships Between Beliefs and Their Impact on Practice

The relationships between teachers' views and models are illustrated in the following diagram.


View of Nature of Mathematics
Espoused Model of Learning Mathematics
Espoused Model of Teaching Mathematics
Constraints and Opportunities Provided by the Social Context of Teaching
Enacted Model of Learning Mathematics
Enacted Model of Teaching Mathematics
Use of Mathematics Texts

It shows how teachers' views of the nature of mathematics provide a basis for the teachers' mental models of the teaching and learning of mathematics, as indicated by the downward arrows. Thus, for example, the instrumental view of mathematics is likely to be associated with the instructor model of teaching, and with the strict following of a text or scheme. It is also likely to be associated with the child's compliant behavior and mastery of skills model of learning. Similar links can be made between other views and models, for example:

- mathematics as a Platonist unified body of knowledge—the teacher as explainer—learning as the reception of knowledge;
- mathematics as problem solving—the teacher as facilitator—learning as the active construction of understanding, possibly even as autonomous problem posing and solving.

These examples show the links between the teacher's mental models, represented by horizontal arrows in the diagram.

The teacher's espoused models of teaching and learning mathematics, subject to the constraints and contingencies of the school context, are transformed into classroom practices. These are the enacted model of teaching mathematics, the use of mathematics texts or materials, and the enacted model of learning mathematics. The espoused-enacted distinction is necessary, because case studies have shown that there can be a great disparity between a teacher's espoused and enacted models of teaching and learning mathematics (for example, Cooney, 1985). Two key causes for the mismatch between beliefs and practices are as follows.

First of all, there is the powerful influence of the social context. This results from the expectations of others including students, parents, peers (fellow teachers), and superiors. It also results from the institutionalized curriculum: the adopted text or curricular scheme, the system of assessment, and the overall national system of schooling. These sources lead the teacher to internalize a powerful set of constraints af

fecting the enactment of the models of teaching and learning mathematics. The socialization effect of the context is so powerful that despite having differing beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices.

Secondly, there is the teacher's level of consciousness of his or her own beliefs, and the extent to which the teacher reflects on his or her practice of teaching mathematics. Some of the key elements in the teacher's thinking—and its relationship to practice—are the following:

- awareness of having adopted specific views and assumptions as to the nature of mathematics and its teaching and learning;
- the ability to justify these views and assumptions;
- awareness of the existence of viable alternatives;
- context-sensitivity in choosing and implementing situationally appropriate teaching and learning strategies in accordance with his or her views;
- reflexivity: being concerned to reconcile and integrate classroom practices with beliefs; and to reconcile conflicting beliefs themselves.

These elements of teacher's thinking are likely to be associated with some of the beliefs outlined above, at least in part. Thus, for example, the adoption of the role of facilitator in a problem-solving classroom requires reflection on the roles of the teacher and learner, on the context suitability of the model, and probably also on the match between beliefs and practices. The instrumental view and the associated models of teaching and learning, on the other hand, requires little self-consciousness and reflection, or awareness of the existence of viable alternatives.

I have argued that mathematics teachers' beliefs have a powerful impact on the practice of teaching. During their transformation into practice, two factors affect these beliefs: the constraints and opportunities of the social context of teaching, and the level of the teacher's thought. Higher level thought enables the teacher to reflect on the gap between beliefs and practice, and to narrow it. The social context clearly constrains the teacher's freedom of choice and action. Only by considering all three factors can we begin to do justice to the complex notion of the autonomous mathematics teacher.

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The Recruitment and Training of Master Teachers

Kurt Kreith

Programs aimed at improving mathematics education in the United States tend to be closely related to larger efforts at educational reform. They reflect a growing recognition of the fact that there is no "quick fix" solution to the problems we face in the teaching of mathematics, but that changes are required in a variety of areas affecting our educational system. In particular, while there is widespread interest in developing new curricula and methods of instruction, it is also recognized that the successful implementation of such innovations will have to be accompanied by fundamental changes in the profession of teaching.

This point of view has been reinforced by the reports of several national commissions which have studied the problems facing education in the United States. Among the most highly regarded of these is *The report of the task force on teaching as a profession*, issued in 1986 by the Carnegie Forum on Education and the Economy, and reflecting the consensus views of a distinguished panel of leaders from industry, government, academia, and the teaching profession. Of special relevance to the question "How autonomous is the mathematics teacher?" is the fact that the first recommendation of this commission states:

> Teachers should be provided with the discretion and autonomy that are the hallmarks of professional work. State and local governments should set clear goals for schools and greatly reduce bureaucratic regulation of school processes. Teachers should participate in the setting of goals for their school and be accountable for achieving agreed upon standards of performance.

In elaborating on this recommendation, the report observes that the granting of such autonomy must be accompanied by a sense of confidence in the professional competence of those who are to exercise it, and that the development of such confidence in our teachers will require a variety of changes in teacher training and accreditation procedures. On this point the report allies itself with the recommendations of another commission, the Holmes Group, in emphasizing the importance of academic depth in the discipline being taught and an increased reliance on professional achievement rather than seniority in determining salary and professional standing. It also goes on to acknowledge the circular nature of such a process:

> Raising the quality of teacher preparation will not work by itself, since few people will go to the added expense and effort of a longer period of teacher preparation unless the career that is offered is at least as attractive as other professions requiring demanding preparation ... Giving teachers a greater voice in the decisions that affect the school will make teaching more attractive ... to people considering teaching as a career.

In other words simultaneous steps to enhance the profession of teaching, in terms of autonomy, salaries, and professional standards, is crucial to success in achieving any one of them.

At the University of California at Davis we have confronted some of these issues in connection with a unique program for training "master teachers" of

mathematics. For while we believe we have succeeded in designing a Master of Arts in Teaching (MAT) degree program which reflects the professional standards which the Carnegie Forum and others seek to achieve, such programs face fundamental difficulties in recruiting qualified students in numbers which would enable them to change the nature of mathematics teaching in the United States.

A description of our MAT program should begin by comparing it with the standard program for credentialing secondary school teachers of mathematics: In contrast to the usual one year of post-baccalaureate training, we require two years; we also require admission to graduate standing in mathematics as well as admission to a teacher credential program. Much of what we do with this extra time and student talent is to establish historical and structural connections between "higher mathematics" and the school curriculum. Or, borrowing a phrase from Felix Klein, much of our curriculum is based on "elementary mathematics from an advanced [and historical] point of view."

By way of more specific examples, the mathematics curriculum for this degree consists of six courses. Two of these deal with geometry (Euclidean, non-Euclidean, and transformational), and one explicit goal of the geometry sequence is to provide a basis for understanding the compromises between Euclid and Hilbert which are central to secondary school geometry curricula and texts. While our candidates tend to be well-versed in calculus, few know much about its roots in the work of Archimedes or its 17th century developers, topics which are covered in a course on the history of calculus. Two courses in the general area of algebra enable us to make contact with topics from computer science as well as discussing the historical accidents which have served to establish artificial barriers between algebra and geometry. Finally, there also exists a course dealing with probability in a way which exposes, for example, the delicacy of concepts such as "independence" which secondary (and many advanced) texts tend to obscure. We are perhaps uniquely fortunate in having in our mathematics department a number of faculty whose interests coincide with such courses.

While pleased with our curriculum and activities, we have been painfully aware of the limited impact of our program. During the years 1983 to 1986 we graduated eight MATs, and an increase in enrollment was clearly called for if continuation of such a program was to be justified in practical terms. It was at this point that we began to reflect more seriously on the decision processes of students, especially those whom we would like to attract to the profession of teaching through our MAT degree program.

Rather obviously in retrospect, it makes little sense to wait until students graduate so as to see what their professional interest might be. Instead, we decided to try to reach our own mathematics majors, those who are enrolled in mathematics courses because they like the subject and do well in it, and to give them a realistic exposure to the profession of teaching well before their graduate school commitments are made. This was to be done by means of "undergraduate teaching internships" which provided highly qualified undergraduates with modest stipends in return for spending one hour per day in a nearby school classroom. In the course of a 10-week quarter, the intern progressed from the role of observer, to tutor and helper, and finally to the role of actually teaching lessons to the class. These activities were accompanied by a weekly seminar in which we shared the week's experiences, discussed pedagogy and curriculum, and, most important, met with practicing teachers and principals who could provide meaningful insights into the profession of teaching. All interns maintained "diaries" which were handed in each week and commented upon by program staff. The program began on an experimental basis during the 1985–86 academic year with 10 interns. During each of the subsequent two years 40 undergraduates were placed in nearby classrooms.

A crucial element in this program was the selection of the "resident teachers" in whose classes our interns were to be placed and who, in turn, served as mentors to our interns. We soon came to regard teachers who are capable of serving as role models to our interns as unique resources, ones which should be recognized for the important role which they can play in renewing their profession. We were also fortunate in having a uniquely capable ex-teacher serve as coordinator of the program. He has since become the Supervisor of Teacher Training at our campus, and as a result our teacher training program has become melded with such undergraduate internships.

Determining the precise effect of such internships on career decisions is, of course, a difficult task. Perhaps the most compelling evidence was the intensity of the involvement which the interns demonstrated in their journals and seminar participation. Playing a leadership role in the classroom affected all of them profoundly, and most were surprised by both the difficulties and rewards inherent in the teaching experience.

During 1986–87 we also worked with a sociologist who interviewed the interns and their resident teachers, read the interns' journals, attended our seminars and analyzed questionnaires completed by the participating interns. Her conclusions supported our impressions in a quantitative format.

While we have enjoyed a significant increase in applications to our MAT program, much of it from students who participated in our internship program, there are ambiguities in assessing the role our recruiting efforts played in this regard. Many of our interns had an interest in teaching before their participation and not all such decisions were precisely tracable. Furthermore, there has been an overall increase of interest in teaching as a profession, as evidenced in a recent report of the Higher Education Research Institute: whereas in 1982 only 5% of the U.S. entering freshmen intended to become teachers, this figure had risen to about 8% in 1987. Also, we came to regard students who thought they wanted to become teachers

but, as a result of the internship, realized otherwise, as having been served by the program in a useful manner.

Rather than concluding with numerical data of uncertain significance, quotes from participating students may give a more incisive picture of the effects such internships can have. The following were anonymous responses to the question, "In what ways was your internship a successful and meaningful experience?"

It would be very hard for me at this stage to idealize teaching, as I believe many young people do ... On the other hand, I have seen real teachers derive real satisfaction from their work, and have felt the warmth that can flow between a good teacher and his/her students.

The program totally changed my views on teaching. I never knew how hard the job can be, nor how rewarding.

I now have confidence in my ability to control a class and gain their respect.

The program showed me how much I enjoy teaching and working with kids.

It made me realize how difficult and rewarding teaching can be ... I will never again think of teaching as simply presenting some information to a class.

I discovered that teaching is really difficult. It is a profession of disciplining and teaching. However, it is also very rewarding if you do a good job.

My view of teaching as a profession hasn't really changed, but been made clearer. I've always had an ideal picture of teaching ... but now I know a little more.

I come out of this program with a positive outlook on teaching. It appears to be something I could enjoy doing very much.

While not conclusive, such comments reinforced our belief in the value of internships as a means of encouraging our most capable students to consider teaching as a profession.

The Role of the Mathematics Teacher in Developing Countries

John Suffolk

Despite its unpopularity with pupils, mathematics has had in the last century a secure place in the school curriculum all over the world. It is accepted that mathematical fluency is important to the economic development of any country and, when it is believed that standards have fallen, politicians and employers are quick to express their concern. A lot of effort is made to ensure that school leavers have a good working knowledge of mathematics.

In developing countries, aid agencies have been keen to assist in the development of mathematics learning. In Zambia, for example, there are several projects aimed at raising the standard of mathematics and science teaching. The project I am part of, the Zambia Mathematics and Science Teacher Education Project, which is largely funded by the European Community, is an attempt over three years to raise teaching standards with a team of ten supporting an upgrading course for junior secondary teachers, running inservice workshops for primary and secondary teachers, and assisting in curricular and materials development. It is significant that the first book produced by the Zambia Educational Materials Project, a Finnish funded project, is a new mathematics textbook.

The place of mathematics in the curriculum is not justified merely in terms of a body of knowledge to be acquired. The Papua New Guinea National High School Syllabus of 1978 aims

to enable students to see mathematics as a living and relevant subject which will help them to fit better into their own environment, develop their ability to analyze problems and to apply logical thinking to their solution ... The course should enable them to benefit from the logic and thinking strategies of the subject as they solve their day-to-day practical problems. (PNG Department of Education, 1978)

In Zambia the grades 1 to 9 Basic Education Syllabus lists five aims—to equip the child to live effectively in a technological world, to stimulate creativity and problem solving, to develop the child's mathematical abilities, to help the child to use mathematics to understand its environment, and to develop an appreciation of mathematics (Zambia Ministry of General Education and Culture, 1983).

The Papua New Guinea Syllabus states that it "does not stipulate any particular teaching method." But, at the very least it does seem to rule out some methods, for, as with other syllabuses, many of its aims are couched in terms of what happens to the pupils. It is their mental development that is being sought, and the methods of teaching used must reflect that. Mathematics should not be taught as an uninteresting presentation of a series of tricks, methods, and formulas whose justification is neither taught nor felt to be necessary. Mathematics teaching should not become what Griffiths (1983) aptly describes as a "display of jewellery."

If the curriculum aims are to be achieved the teaching has to involve the pupils actively. In other words, the quality of the education given determines whether or not the curriculum aims are being met. Over 20 years ago C. E. Beeby (1966) delineated stages in the growth of a primary school system. He saw an education system improving as the teaching moved from highly formal low level teaching with a stress on memorization with ill-educated teachers through to informal high level teaching with emphasis on activity methods, creativity and problem solving and well-educated and well-trained teachers. He called his stages Dame School, Formalism, Transition, and Meaning and he argued that movement through the stages was dependent on the ability of teachers to change, which was itself dependent on their confidence. Confidence came from education. His model has been criticized (Guthrie, 1980) for equating Western education with good education but his basic insight, that the teacher is the one who has to carry out educational innovation, and such innovation can only be carried out gradually with his willing involvement, remains valid.

In developing countries the level of education of those studying to teach mathematics has been steadily rising for many years. In Zambia a full "O" level Certificate (grade 12) is now required for entry into

Primary Teachers College, whereas 15 years ago grade 9 or 10 education was sufficient. The quality of intake, as judged by level of certificate, of Secondary Teachers College entrants has also risen. One would think, therefore, that the standard of education in the schools has risen, and that the teacher would be able to teach for meaning, to satisfy curiosity, to encourage problem solving, to stimulate critical thinking, and so on.

But the teachers of mathematics in many developing countries have faced many difficulties that have worked to lower the quality of education given. The growth in educational provision has been most impressive; between 1960 and 1983 sub-Saharan Africa primary enrollment rose from 36 to 75% and secondary enrollment rose from 3 to 20% (Moock & Jamison, 1988). This has involved the allocation of tremendous resources to education. But most of the money goes on teachers' salaries and buildings, leaving little money for materials. The money available for books, chalk, maps, chairs, etc. has in many countries been very small indeed. Whereas in 1980 the USA was spending $220 per pupil on such items, in Malawi in 1978 the figure was $1.24 (Heyneman, 1984). With such a small amount of money available, the provision of any education materials becomes impossible. Teachers of mathematics are teaching pupils without textbooks, graph paper, mathematical instruments, or teaching aids. Yet, it is difficult to see how anyone can advance through Beeby's stages without such things. Indeed Moock and Jamison (1988) go so far as to assert that

> the provision of instructional materials, particularly textbooks, is the most cost-effective way of improving the quality of primary education.

Moock and Jamison further argue that studies suggest that reducing class size is expensive but has no noticeable effect on school quality. But when, as is common in Zambia, secondary school mathematics teachers are facing classes of 60 pupils, the teacher's morale suffers, individual pupil's needs are neglected, and the standard of education must fall.

Other pressures exist to prevent the mathematics teacher from having the stimulating effect he should have. In developing countries, success at school opens the way to further education and to very much better employment prospects. That is what education is seen as being for. In the Trobriand Islands, Papua New Guinean parents withdraw their children from school if they feel their children have no prospects for further education (Montague, 1982). Examinations are the measure of educational success, and advancement is almost totally dependent on them alone. The pressure on the teacher to teach for the examinations is enormous—he is judged by the results of his pupils. Bell (1980) describes Brownell as having showed that "drill increases fluency but not comprehension; its effects fade quickly." But if the effects survive up to the examination, and the pupil passes, the fading becomes the problem of the next stage of education. The teacher is a success if his pupils do well in the examination no matter what method and the whole system (parents, headmasters, pupils, education officers) will praise him.

All this is reinforced by the teacher's past experience. He is a product of such a system so will not react with horror to such methods. What was good enough for him will be good enough for his pupils. He also was taught not for Meaning but for examinations. He is used to applying formulas without knowing their origin (he did it all through school and college). Moreover, in many developing countries, he is expected to have a formal relationship with his pupils, to be respected because of his age and position. Pupils are expected to be quiet, respectful, and submissive. Such expectations work against teaching for stimulation, for curiosity and towards teaching by prescription.

At College, teachers learnt of Piaget and Bruner and of the need not to treat children as little adults. They will have learnt that children learn more effectively and enjoyable from practical activities than from lectures. They may even have had the opportunity to try out such methods on Teaching Practice. But when they get into schools as teachers and are faced with large classes and with no materials, it is expecting a lot to expect them to do anything other than "chalk and talk."

At college they will also have learnt about Nyerere and the need for self-reliance. Normally this finds expression in "Production Units" that help to keep the cost of schooling down. A significant way teachers of mathematics can follow his exhortations is by learning to look to their surroundings and their ingenuity for freely available materials that can make even the teaching of a class of 60 more practically meaningful. Teacher educators must prepare them for this.

Piaget brought to education the realization that children pass through stages of development and that some concepts, for example, those of ratio, are not grasped by children until they are quite old. The fact that primary education has been terminal for many has led curriculum developers to try and cram into primary education all the mathematics needed for life. As junior secondary education becomes more widespread the pressure for this will ease. Curriculum developers will be able to consider the implications of what is included in the syllabus for the cognitive demands placed on pupils and teachers. They will also be able to consider the implications for how the syllabus might be taught.

Conclusion

Mathematics teachers in developing countries are officially expected to teach mathematics so as to stimulate pupils to use mathematics competently in later life. They labor in circumstances many would find unacceptable, with many pupils and few materials, and are judged according to their pupils' examination results. Many retain a level of dedication and enthusiasm which is remarkable, but until conditions become more favorable, is it realistic to expect them to fulfill

the role that they should have in the education of young people?

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Ethnomathematics and Schools

Mathematical knowledge of a different kind from that which is usually dealt within the school curriculum was considered in this topic. Sources for ethnomathematical ideas and their significances in schools are shown. Gloria F. Gilmer, USA, gives a survey report of research activities in ethnomathematics as reported to the newsletter of the International Study Group of Ethnomathematics. Randall Souviney, USA, discusses the role of Indigenous Mathematics Project in Papua New Guinea. Eduardo Sebastiani Ferreira, Brazil, shows by a lot of examples from history of mathematics and ethnomathematics that both the generic principle and ethnomathematical methods are linked and can be part of the same method of the teaching mathematics in school.

World-wide Developments in Ethnomathematics

Gloria F. Gilmer

The International Study Group on Ethnomathematics publishes a newsletter semi-annually with a distribution of approximately 400 educators in 47 countries. These educators are identifying mathematical concepts and practices that are embedded in the traditional and contemporary life-styles of indigenous learners and translating their learnings into practice. This presentation is a report of a variety of sources of ethnomathematical ideas and their significance in schools.

Ethnomathematics lies at the confluence of mathematics and cultural anthropology. At one level, it might be called "math in the environment" or "math in the community." At another related level, it is the way that specific cultural groups go about the tasks of ciphering and counting, measuring, classifying, ordering, inferring, and modeling.

Ethno encompasses identifiable cultural groups such as national-tribal societies, labor groups, children of certain ages, professional classes, etc. and includes their jargon, codes, symbols, myths, and even specific ways of reasoning and inferring.

Ethnomathematics is a new subject. As such, it is recognized as a set of mathematical practices which are known to culturally differentiated groups. D'Ambrosio, who might be regarded as the father of ethnomathematics, has focused much of his writing on what is required to translate what is known in this area into scientific theories. D'Ambrosio particularly notes that the mathematical competences learned at home and which are lost in the first years of schooling are essential for everyday life and labor. The former spontaneous abilities have been downgraded, repressed, and forgotten, while the learned ones have not been assimilated. Thus, early education instills a sense of failure and dependency. Also, the fact that practical mathematics is largely absent from the secondary school curriculum causes early exits from mathematics courses for many high school students. These facts alone suggest the need for school mathematics of a different kind.

Thus, the study of ethnomathematics treats the following questions:

(1) How are ad hoc practices and problem solutions developed into methods?

(2) How are methods developed into theories? And, how are theories developed into scientific invention?

At present, we are collecting examples and data on these practices. We wish to link these practices into a pattern of reasoning and trace their origins. In this way, we may be able to organize our findings into a structured body of knowledge which addresses the issue of how to develop well more people in mathematics.

Sources for Ethnomathematical Ideas

Many scholars use games as rich sources for motivating math learning. For example, Marcia Ascher has analyzed the game Mu Torere which has been popular among the Maori people of New Zealand for more than two centuries. Mu Torere is a game of strategy that is played on an 8-pointed star design which may be drawn in the sand.

There are two players and each has four markers which are placed beside adjacent star points. The markers must be distinguishable by players. Players move one marker per turn, alternately, to an adjacent star point or the center, if it contains no marker. The game is won when the opponent is blocked so that he cannot move. On the first two moves, players may only move their outer markers so that no player is quickly blocked. Ascher found 92 configurations of the game but noted that only 86 were attainable. Of the 86, each player had three winning configurations. By varying the number of points on the star and the corresponding markers in the game, Ascher concluded that the 8-pointed star resulted in the most enjoyable game.

Paulus Gerdes used Angolan sand drawings to discover diverse arithmetical relationships. For example, in order to represent a certain motif, such as a tortoise, traditionally Tschokwe people of northwest Angola memorized the number of rows and columns in an orthogonal net of equidistant points, then the drawing of the tortoise, for example, redistributed the points of the reference net so that the student would discover some arithmetical relationship. In this way, Gerdes feels that the art of sand writing might be revived and the revival may contribute to a more productive and creative mathematics education for the Tschoke people.

Devices for learning or organizing mathematics are also good sources for ethnomathematical ideas. Diego Pareja Heredia studied numbers and their uses by Maya and Inca civilizations in Central and South America prior to the arrival of the Spanish in the late 15th century. The quipu was the closest approximation to writing which pre-Columbian South Americans had. The quipu is a bundle of knotted cords of varying lengths and colors, resembling a mop. It was used by the Incas for keeping various kinds of records, such as the number subdivisions of each nucleur farm in the empire, the number of soldiers in each garrison, or the tribute owed to the emperor. Quipus involved organizational principles such as hierarchical placement of chords and the systematic use of color. Donald Crowe of the USA described the quipu as a kind of spreadsheet which can represent elaborate collections of numerical data of various kinds.

Many educators have expressed interest in learning appropriate ethnomathematical techniques. Some of us are developing a different kind of teacher education program to respond to this need.

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The Indigenous Mathematics Project: Mathematics Instruction in Papua New Guinea

Randall Souviney

The Indigenous Mathematics Project (IMP) was initiated in 1976 by the government of Papua New Guinea (PNG) and UNESCO in response to a historical inequity between the coastal and highland provinces in the distribution tertiary enrollments and wage-employment (Kelly, 1977). Paralleling education reform movements in many other countries, this issue has been gradually subsumed by an overall concern with the education standards and plans to achieve universal primary education (UPE) by the early 1990s.

IMP initially conducted extensive studies of cognitive development and cataloged numerous indigenous counting and measurement systems (Lancy, 1978). This was followed by a curriculum development and evaluation study of the *Mathematics for community schools* (MACS) texts in use in PNG primary schools (Souviney, 1981; 1983). These pilot materials were designed to enhance the teaching of number operations, measurement, and logical thinking. A fundamental assumption guiding the curriculum study was that childrens' knowledge of traditional number, measurement, and geometry concepts should be employed in the service of formal mathematical development. Adapting and extending traditional knowledge helps children define concepts that are not readily expressible in local terms. To be effective, this principle must be embodied in the curriculum materials and methods used in the schools.

Utilizing Traditional Knowledge

As an example of this approach, Saxe (1982) investigated the use of the Aksapmin body-counting system among children learning school mathematics and men operating trade stores. Literally hundreds of body-counting systems are in use throughout Papua New Guinea. Each system defines a one-to-one correspondence between the counting numbers and specific points on the body. The maximum number that can be counted in one round is limited by the number of body parts is specified. The Aksapmin system contains a cycle of 27 points. With the introduction of a money economy and decimal currency, trade in the marketplace requires more frequent application of counting and operations on large numbers. Such environmental demands encourage enterprising adaptations of the traditional counting system. Saxe reported that unschooled trade store operators invented a complex procedure for doing subtraction which involved using their body-count system as a number line to store the minued and invoking a "stop rule" for subtracting the subtrahend. A similar procedure was employed by children in the community school. When understood by the teacher, such invented procedures rooted in the child's experience, offering relevant benchmarks for symbolic algorithms.

Mathematics for Community Schools

Widespread public education has been in existence in Papua New Guinea for less than 30 years. Attempts at developing a modern mathematics curriculum for use in primary schools was initiated in the mid 1960s using materials inspired by Zoltan Dienes' work with multi-base materials. Since then, the Territories of Papua and New Guinea have become a unified nation. The new government adopted the goal of instituting UPE and the localization of the teaching force. In 1979, approximately 9,000 PNG primary teachers taught 280,000 students in 2,000 schools. If UPE becomes a reality, by 1992 a projected 14,000 teachers will be responsible for the education of nearly 450,000 students in over 2,500 schools.

Curriculum Evaluation Study Design

It was with a view towards these concerns that IMP turned its attention from documenting variations in cross-cultural cognitive development (Lancy, 1978; Lancy, Souviney, & Kada, 1981) to analyzing the effects of curriculum innovation on teacher behavior and student achievement.

Pilot Curriculum Materials

The IMP staff developed over 30 pilot lessons for grades 4 and 6, and 60 lessons for grade 2. Each school scheduled five periods of IMP mathematics per week. Lessons were 30 minutes in length for a total of 210 minutes of instruction per week. One teacher taught mathematics for the three classes at each site.

Physical materials designed to offer optimal perceptual input for the novice learner were considered vital in the multilingual community school environment. It was envisioned that teachers who had difficulty communicating with children from different language groups could use structured teaching aids to help present the mathematical concepts and processes rather than relying exclusively on symbolic examples and chalkboard drills. Three primary areas of mathematics content were addressed using various concrete aids:

(1) Number and Operations (base-10 materials, coins, squared paper): place-value, addition, subtraction, multiplication, division, and problem solving;
(2) Measurement (meter sticks, balances, trundle wheels): estimation, comparison, length, area, mass, volume, time, velocity, metric system, and problem solving;
(3) Classification and Logical Thinking (Kina Cards): likeness and difference relations (attributes), logical connectives (and/or), negation (not), union, intersection, class inclusion, and problem solving.

To interpret variations in achievement, a trained expatriate research associate (RA) was placed in each school to systematically document classroom interactions. A detailed field note-guide (Levin, Gearhart, & Souviney, 1980) was used to standardize the classroom observation procedures. Each evening, the teacher was interviewed by the RA to review the lesson plans for the next day. Specific questions asked by the teacher about the lesson or materials were noted. After lesson implementation, a formal debriefing took place, generally in conjunction with the briefing on the next day's lessons. The teacher's perception of his/her effectiveness in meeting the lesson objectives was noted. Detailed field notes were taken in approximately three lessons per week for each grade level. The other two lessons each week were devoted to observing language usage and classroom interaction, and conducting student interviews.

Achievement Results

Several measures of mathematics and language achievement were administered. Though statistically significant differences were found between schools, the pattern was inconsistent. The highland schools generally showed lower achievement levels thar those in coastal areas. Though English reading competence was highly correlated with mathematics achievement ($r = 0.76$), the site with the highest reading achievement scored near the mean in overall mathematics achievement. The school which scored the lowest on most measures, performed well on the oral dictation exercise. The two most rural sites (WSP and MP) exhibited little similarity in their achievement profiles, suggesting that proximity to provincial education ser-

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vices may not influence student achievement. Reading achievement was also higher in the coastal regions than the highland sites. English language competence seemed to have a positive effect on mathematics achievement up to a threshold, beyond which other factors become dominant. Improving language skills in the lower grades may therefore elicit a greater effect on mathematics achievement than in the upper grades where threshold language skills necessary to support mathematics learning are more likely to have been attained.

Classroom Observation Results

Significant classroom events were documented over the entire 6-week trialing period (Gearhart, 1980; Johnson, 1980; Levin, 1980; Lineberger, 1980; Rowe, 1980). A review of observers' notes (Miller-Souviney, 1980) identified several problems which seemed to detract from the quality of instruction at the majority of sites. The problems fell into the three categories of (1) instructional materials, (2) classroom management, and (3) subject matter.

Instructional Materials

All the teachers had some difficulty coordinating the various instructional components associated with the lessons. The most prevalent problem involved the preparation, distribution, and storage of student- constructed learning aids. For example, picture cards designed to introduce time-seriation proved to be too difficult for 2nd grade children to trace. The resulting lesson took far longer than was justified. Attempts to circumvent the problems by using words instead of pictures were successful at two of the sites. One teacher reproduced copies of the worksheet for each student and the lesson proceeded efficiently. Duplicating machines were available, however, at only two schools. Two teachers suggested that sequence stories could be drawn by the children during art or language class and used in mathematics for seriating timed segments. Grade 2 children also spent considerable time drawing place-value tables on squared paper in preparation for practice exercises. Though the tables proved useful in helping the children develop place value and arithmetic skills, an inordinate amount of time was required in setting out their work. Unless children can be encouraged to sketch the place-value tables more quickly (even at the expense of accuracy), the use of consumable worksheets should be considered for number work in the lower grades, especially in rural schools. Urban schools that often have duplicating facilities could duplicate the worksheets at the school site.

Classroom-produced learning aids proved to be extremely useful for teaching difficult concepts. The time and expense involved in their construction, however, must be balanced against potential utility. If the aid is to be used in only one lesson, preparation time must be reasonable or construction must take place outside of class time.

Classroom Management

Several teachers initially experienced difficulty distributing manipulative materials during the lessons. The process required an excessive amount of class time during the first two weeks. The situation improved considerably when teachers began to give students the responsibility for distributing materials while they prepared the board for the lesson introduction.

Subject Matter

Misunderstandings regarding the purpose of multibase arithmetic were frequently observed. Those teachers who possessed a good grasp of the methodology often treated each base as a mastery objective rather than as interim steps leading to base-10 operations. Results suggest development of numeration focusing almost exclusively on base-10 operations with perhaps traditional base-3 and base-5 exchange activities introduced early in the development of place value. Number and counting should be introduced using body and tally-counting systems indigenous to PNG. Structured base-10 materials (die-cut cards or locally produced alternatives), coins, and squared-paper activities are used to provide a developmental transition to symbolic arithmetic. All algorithms should be developed with consistent reference to place value and regrouping (Souviney, Keyser, & Sarver, 1978).

Conclusion

The results of the study suggest that the pilot textbooks improved mathematics achievement for children in remote areas and grade 2 children in general. These groups showed greater achievement gains than older and/or more urbanized children. It was concluded that a locally developed textbook would enhance mathematics learning, particularly in remote-rural areas where current teaching conditions and lack of materials interact with poor English skills to create an imbalance in achievement.

The teachers involved in the study were capable of effectively coordinating student activity books and learning aids. There were, however, definite limits as to the number of elements the children were capable of managing efficiently. The teaching suggestions included with each lesson were used by the teachers on a daily basis for lesson planning and as notes during lesson implementation. A series of brief criterion-referenced tests should also be included. The tests should appear in the text and be administered every week or two to encourage the teacher to use the results for planning future instruction. Topic or term tests should also be included to provide broader benchmarks of achievement.

Since the termination of IMP in 1981, locally designed textbooks containing many features recommended in the current study have been produced by the PNG Department of Education (DOE). *Community school mathematics* (grades 4 and 5) and *Mathematics our way/Secondary school mathematics* (grades 7, 8,

9, and 10) textbooks have been field-tested and are now in use in the community and provincial high schools. Work on the grade 6 textbook is currently underway. An increase in performance in the National Grade Six Mathematics Exam from 1983 to 1987 provides preliminary evidence of the effectiveness of the new primary mathematics textbooks (Department of Education, 1988). When the grade 6 book is in place, a systematic evaluation of the impact of the upper-grade program is recommended.

With the approach of UPE in the early 1990s, the PNG Department of Education is exploring the feasibility of purchasing grade 1 to 3 textbooks as an alternative to locally developed materials. As indicated in the IMP final report (Souviney, 1981), providing appropriate materials for mathematics instruction in grades 1 to 3 will likely be the most complex component of the PNG curriculum development effort. The multilingual, culturally diverse community school environment presents difficult obstacles to the implementation of an effective, unified program of study. Textbooks with a very low English language load may have the effect of reducing instruction to a series of memorized number facts. A more effective approach may be to develop a PNG teacher's resource manual like *Mathematics their way* and *Workjobs* (Baratta-Lorton, 1972; 1976) for grade 1 and perhaps grade 2. A grade 3 textbook similar to *Community school mathematics 4A & B* would complete the primary curriculum. Based on the IMP results, the time and resources required to complete the curriculum would be well spent, however, since the result would promote greater mathematics understanding and school retention of rural children throughout the country.

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The Genetic Principle and the Ethnomathematics

Eduardo Sebastiani Ferreira

Introduction

There are two important methodological postures in the learning process of mathematics; the genetic principle dating the beginning of the century and the ethnomathematics from our decade. The genetic principle has got a method already tested many times but the ethnomathematics is something extremely new which still has to be accepted in the learning process. What I want to show here is how linked the two of them are, even intermingling sometimes.

Genetic Principle

The Haeckel's fundamental biogenetic law "A ontogenia recaptula a filogenia"—was in the beginning of the century inserted in the learning process as a "genetic principle." It was supported by influencial mathematicians such as Felix Klein and Henri Poincaré. The principle, "the effective learning requires that each learner goes through the steps of the historical evolution of the matter to be studied," (Byrs, 1982, pp. 59–66) has shown efficacy in teaching mathematics when not taken restrictively.

According to Polya

> The genetic principle in teaching can be formulated in different ways; e.g., when teaching a certain scientific point (either a theory or a concept), we should let the child outline the stages of the human being intellectual development.
>
> It does not mean committing the same mistakes again and again, but just going thorough the main stages. (emphasis by E. S. F.)

or even Byrs

> ... the child's ability increases if its presentation is examined under the lights of the historical acquisition, for that knowledgement. (Byrs, 1982, pp. 59–66)

Grattan-Guinness is one, out of a few authors, who is against the principle, mainly at the first stages, referring to mathematics education:

> ... at the first stages and specially in basic education, the history is rather useless, becoming necessary at high school. (Grattan-Guinness, 1973, p. 421)

Or, according to Moises

> A rule that admits great exceptions is not a rule. In order to save the genetic principle from the absurd we need to build it in such a free way that it could seldom be used as a whole. (Moise, 1965, pp. 1–411)

The Ethnomathematics

The name "ethnomathematics" appeared in this decade and I believe that for the first time in the article *Ethno-mathematics and its place in the history and pedagogy of mathematics*, from U. D'Ambrosio (For the Learning of Mathematics—An International Journal of Mathematics Education, Canada, 1985, February, p. 44). In this article the author gives a definition of the word as

> A relation between anthropological cultures—historical and mathematical is an important step towards the recognition of different ways of thinking which direct to different mathematical forms, this is what we call "ethno-mathematics."

In the same year another author—R. P. Hunting—defined the ethnomathematics as

> ... the mathematic used by a defined cultural group in proceeding the relations with the problems and activities in context. (Hunting, 1985)
>
> We believe that to motivate people's creativity, to amplify their knowledge and aptness in mathematics it is necessary to have an integration of their mathematical traditions in the teaching programme. (Byrs, 1982, pp. 59–66)

The introduction of the ethnomathematics in the schools curriculae is an important methodological posture. The latest pedagogical posture towards a new teaching method is becoming more popular among educators who are introducing the ethnomathematics, extracted from the cultural context, in the academic mathematics. I believe it is more than a simple method, it is a philosophical posture.

The Genetic Principle Versus the Ethnomathematics

The genetic principle is the development of the mathematical knowledge in individuals following the same steps the humanity took to acquire such knowledge. When research in ethnomathematics is done, we find communities which are in certain stages of the history of the mathematics.

> The documented (official) history of the humanity reminds us of the origine of the counting, of the numbers, of the study, the forms and the ways of thinking (logical) of the first stages of any civilization,

says D'Ambrosio (1984). In the same book the author mentions that the human behavior follows a series current from reality–individual–action–reality. Then, the impact of the reality generates the individual's action. This action will then depend on his level of awareness of the reality. If his level of knowledge on mathematics refers to a stage of mathematics, his action on reality will then be the same action already practiced by people that were on the same level, in spite of a difference in reality. In most cases this difference in reality requires that the action be faster than it was in other times. But on the other hand, it demands similar actions, counting, genetic forms, and ways of thought. When I was working with the Kaxinawa Indians in Acre, I found out that they are aware of unitary fractions while working with them.

We know that the Egyptians knew these types of fractions and the 2/3, but all the others were unitaries.

In most Brazilian tribes we can find some knowledge on geometry in body painting, handcrafts, and in the building of their huts. Concepts, such as parallelism and perpendicularism, are found in body paintings and are incorporated into the language. For the "Tapirapés" from Mato Grosso, the parallelism "axapaywyri" and the perpendicularism "axapanekwap." The circle and the sphere mean the perfect shape (form) for the most Brazilian tribes as it did for the

Greeks. This is shown in the hamlet shapes, the building of some huts and in most of the handcraft.

The unit of two is another characteristic of these tribes, the *one* is always part of something. The unit is done by pairs. "Tapirapés" count by the pair, the one in their language is called "without a mate." The students have to be registered at school by the pair, every teenager ("tyre" for males), has his mate and in various other instances one can see units of two. The Mynky Indians from Mato Grosso say "only two were left," meaning very little is left. This appears in eastern civilizations as "ying-yang," as well. This phenomena can also be perceived in children's games where there are many codifications and some knowledge of ethnomathematics. The counting is almost always associated with the fingers or stones (see Borba, 1987).

In the rural zone there is a regional diversity of the measurement units, the standard measurement is not necessary yet, varying, according to the region. Therefore, we find measurements such as "alqueire," "braça," "tarefa," etc.

When the teacher applies the ethnomathematics at school to develop institutional mathematics, the field research always shows the history of mathematics which shows the stage of ethnical knowledge of the society research. Usually the genetic principle is used to develop the topics one wants to teach. That is why I argue that both the genetic principle and the ethnomathematical methods are linked and can be part of the same method of the teaching of mathematics.

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Social Needs and Reforms in Mathematics Education

Developments in mathematics education are becoming more related to perceived needs in society, and in this session the speakers present their views on this relationship from the perspective of the priorities for reform in their respective countries. Feiyu Cao, China, describes how the PRC tries to match the needs of social developments and socialist modernization in reforming mathematics education. He differentiates between strategies to suit long-term and short-term needs, common and different, social and individual needs. He reports some elated investigations and possible measures to be undertaken. Fidel Oteiza and Nadja Antonijevic, Chile, summarize a set of studies conducted to generate a framework for curriculum research and development in mathematics in Chile, in which opinions were collected of professional mathematicians, high school math teachers, social scientists devoted to the study of the relations between education and labor, personnel recruiters, and computer science instructors and specialists. Teresa Smart and Zelda Isaacson, United Kingdom, describe a Higher Introductory Technology and Engineering Conversion Course, that recruited women only and was taught almost exclusively by women, in which they developed new materials and tried out new ways of working with mature women returners. George Malaty, Finland, poses the question to ICMI what kind of reform is needed now in mathematics education and refers strongly to the Soviet example of continuous reform instead of following educational fashions so long done in Western countries.

School Mathematics Education Should Suit the Needs of Social Development

Feiyu Cao

There is great significance in understanding fully the relationship between mathematics education and society, which is both a theoretical and practical problem, and which will decide in what direction mathematics education should develop in the future.

Relation Between Mathematics Education and Society

Reviewing history briefly, we can find that mathematics education emerges and develops for the needs in the development of human society, especially for the needs of production development. In turn, a certain society requires the service of mathematics education for its continuity and development. Mathematics education at its early stage was only for a few people, and the teaching contents were comparatively simple. With the development of the society and mathematics itself, more people were required to master more mathematical knowledge. In modern times, with developed science and technology, mathematics itself develops rapidly, and its use extends day by day. Mathematics has gradually become a tool which every citizen in a modern society must master. The movement for the modernization of mathematics education launched in the 1960s was not an accidental phenomenon. It was an inevitable outcome of the demand in the development of modern science and technology.

Historical experiences show that school mathematics education is not immutable, but changes along with social development. Nevertheless, it does not change spontaneously along with social development. This is because, on the one hand, science and technology and social production develop rather quickly, and only when mathematics education does not meet their requirements, would people seriously pay attention to it. On the other hand, it is not easy for most of the people to change their traditional ideology in mathematics education and the teaching methods they usually used. But this is not to say that this backwardness is unchangeable under modern social conditions. If we bring our initiative to full play and make profound research with scientific theories in modern education, we can adapt mathematics education to the requirements of the development of the modern society.

Since the founding of the People's Republic of China, several reforms on mathematics education have been carried out so as to meet the requirements of her socialist construction. For example, in the 1950s, in order to meet the requirements of most of the graduates in the rural primary schools who would take part in productive labor, simple bookkeeping for agricultural productive cooperatives was added in the arithmetic course. In 1977, in order to meet the needs of the modernization of our socialist construction, further reforms in the teaching contents of primary and secondary school mathematics were made.

To get Information on What Social Development Requires of Mathematics Education is the First Important Task in Reforming Mathematics Education

We should seriously analyze the data collected and make clear the relationship among the varied needs. According to our practical experiences, the following relations should be handled well:

(1) *Relation between long-term and short-term needs*, especially how to handle the relation between the traditional and modern contents of mathematics well. A correct proportion should be made between the traditional and modern contents of mathematics in accordance with the level of social development. For long-term needs, much attention should be paid to the cultivation of students' thinking ability, but for short-term needs, the students' ability of applying mathematical knowledge to solving simple problems cannot be ignored.

(2) *Relation between the common and different needs.* First of all, mathematical knowledge in common needs must be selected as teaching contents of mathematics for primary and secondary schools. But even in one country, uneven economic, cultural, scientific, and technological development exists in various regions. So there must be unity as well as flexibility in school mathematics education.

(3) *Relation between social and individual needs.* Judged by the needs of the scientific and technical development in the modern society, each member should master certain basic mathematical knowledge and skills so as to make mathematics a useful tool in his life and work. Nevertheless, we must see that there is difference in individual needs for mathematics. Moreover, different students have different interests and likings for mathematics. So, in deciding the teaching contents of mathematics, the unified requirements can not be too full but must leave room for the different needs of different students.

(4) *Relation between the needs and the possibility.* Only if we get to know the needs of social development and the development of students' abilities, the conditions of teachers, equipment, etc, and take all these into consideration, can we make school mathematics education suit the needs of social development. Any designed programs that go beyond objective conditions will fail.

All-Round Reforms Must be Carried out in School Mathematics Education so as to Meet the Needs of the Society

(1) Reform of educational thoughts unfit to the needs of social development.

The change in educational thoughts is an important premise for carrying out successfully the reforms of school mathematics education. So we should make all mathematics educators know which goals demanded by the future social development should be achieved in mathematics education, what kind of person should be nurtured, and what relations exist between the teaching of mathematical knowledge and the developing of students' abilities.

(2) Reforms of curriculum and teaching materials which do not meet the needs of social development.

This is the key point in solving the problem that mathematics education does not suit the needs of social development. Besides the correct handling of the above-mentioned relations on the basis of full investigation and study of the social requirements, attention should still be paid to the following points:

(a) We should carry out reforms that can be realized, proceeding from the requirements of nurturing qualified personnel in our own country in accordance with the present development and trend in production, science, and technology. In the reforms, experiences from foreign countries should not be copied indiscriminately but used for reference.
(b) The objectives of mathematics education for different stages should be fixed and the courses and teaching contents planned properly, proceeding from the present level of the majority of students and teachers and the conditions of equipment.
(c) In order to realize the improvement of teaching efficiency and the development of students' abilities, research should be made on the excellent structures of mathematics teaching materials, the good order of mathematics teaching and the proper exercises and homework.
(d) Strengthening the relation between mathematics and the reality. Mathematics should be made a useful tool for every citizen in the modern society and used to solve simple, practical problems in production and work.

(3) Reforms of teaching methods which do not meet the needs of social development.

This is a very important guarantee for realizing the objectives of the reforms in mathematics education. Especially for the realization of mathematics for all, the enrichment of modern contents in mathematics and the cultivation of students' abilities, mathematics teachers must adopt suitable teaching methods. Stress should be laid on helping some teachers change their spoon-feeding way of teaching and learning to adopt methods which are conducive to giving full play to the students' initiative and development of their intelligence.

In Order to Meet the Needs of Socialist Modernization in China in the 1990s, the Following Major Problems Have Been Taken Into Consideration in the Reforms of School Mathematics Education

(1) To get information about the requirements of basic mathematical knowledge and skills in China's economic and social development, her present situation in primary and secondary school mathematics eduction and suggestions for improvement.

The first investigation began in 1984 and lasted two years. In these two years Curriculum and Teaching Materials Research Institute, together with the Mathematics Teaching Research Association of All China Society of Education and some provincial and municipal teaching research organizations, has conducted an investigation in 16 professions in industry, agriculture, and commerce, in different fields of study in colleges and universities and in 76 periodicals. The survey has proved that some of the traditional contents in elementary mathematics have become less needed, and the requirements of the knowledge and skills that can be used directly are increasing.

The second investigation was made among teachers and research workers. Through investigation, the present levels of mathematics teaching of most of schools and teachers were made clear, and the major problems existing in the present mathematics courses and the syllabuses revealed.

From these two investigations, reliable information and basis can be provided for the reforms in the curriculum and teaching contents for mathematics.

(2) To work out the syllabuses for mathematics (draft) in primary and junior secondary schools for the 1990s.

On the basis of the above-mentioned investigations, the syllabus for mathematics (draft) has been drawn up so as to make 9-year compulsory education universal, raise the quality of the whole nation, and lay a sound foundation for training all kinds of personnel at different levels. The following important measures have been taken:

(a) Unnecessary and difficult contents are cut down.
(b) Adding the practical contents which can be well applied in future study and personal life.
(c) Strengthening the development of students' abilities.
(d) Strengthening the cultivation of good morality and habits.
(e) Paying attention to facing all the students and teaching them in accordance with their aptitude.

(3) To study and compile mathematics textbooks and supplementary materials for primary and secondary schools in different regions and by different levels.

According to the unequal development of various regions in China, preliminary plans have been made to compile several sets of textbooks, which are for the majority of the regions with general developing levels, for coastal regions, rural schools and national minority regions.

(4) To develop small-range scientific experiments in search for effective ways and methods to raise efficiency in school mathematics education.

In present China, experiments for reforms of mathematics teaching in primary and secondary schools have been carried out extensively. Since autumn of 1984, Curriculum and Teaching Materials Research Institute has been making experiments in two primary schools. And on this basis, further experiments are being done in over 100 primary schools (50% rural and 50% urban). These experiments have achieved good results, and are supported by many local educational administrations, schools, and parents.

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On the Light of Present and Future Needs, are we Teaching the Proper Mathematics? The Case of Chile

Fidel Oteiza and Nadja Antonijevic

This paper summarizes a set of studies conducted to generate a framework for curriculum research and development in mathematics education in Chile.

We are in the middle of a scientific and technological revolution. Cultural, economical, and labor conditions are changing. Are we teaching the proper mathematics? Are mathematics educators contributing to the national development? Is our research and experimental work going to contribute towards the creation of scientific and technical capacities the country needs?

The country, as many others in the Latin American region, has to overcome heavy cultural and attitudinal limitations in order to brake its underdeveloped conditions. Among other important factors, the process of development calls for scientific and technical capacities.

The national productive system is dependent on the economies of developed countries, and its standards and technologies are those dictated by foreign technological knowledge.

Scientific and technological productivity is scarce in Chile, and notoriously insufficient to assist industry, agronomy, and other sectors of the productive system to obtain the needed levels of productivity and operational independence.

Present conditions of the educational system inhibit the possibilities that those deficiencies might be mitigated in the near future. 50% of the students leave school before grade 8, and half of the remaining students drop out before high school is ended.

On the other hand, school mathematics and scientific curricula is poor. After the reform which was due in the country—between 1958 and 1964—associated with the international movement towards "modern mathematics," national programs were revised in 1980 and changed. Chile has a centralized educational system; consequently, this modification affected the entire education apparatus.

During the last decade, the country has experienced one of the most dramatic changes in educational policies, policies that have being strongly enforced by the present government. Their applications have generated a massive reduction of the state's participation in general education and in the universities, and the matching increase of private initiatives.

These policies have had a negative impact in teachers preparation, teachers salaries, and public-school-poor-student enrollment, mainly because poor economical conditions, and because motivations are different in the private sector of those traditionally present in national public education.

It can be concluded that present conditions are not adequate to create the scientific knowledge and technological know-how the country needs to guarantee its own development.

Should a change in present conditions be possible, what could be recommended to those making curricular decisions?

In order to obtain effectiveness and integration of efforts, the first element that comes into mind is orientation.

The "What Mathematics?" Studies

In order to create the above-mentioned general framework, the following questions were posed:

What mathematics should be taught in Chilean schools, and in what manner, so as to meet scientific and technological needs, say, 5, 10, or 15 years from now?

Are we teaching the proper mathematical knowledge or is there something more to be included in school curricula? Or something to be dropped? Or different cognitive and effective objectives to be achieved?

These questions were posed to professional mathematicians, high school teachers, professionals related to education and labor, to personnel recruiters in the productive system, and to computer science specialists.

The Opinions of Professional Mathematicians

Mathematics specialists, 25 members of the Chilean Mathematical Society, active in research and among the most qualified of the country, answered those questions in an in-depth interview (Quiroz, 1986).

Mathematicians showed themselves highly motivated by the questions, and very conscious about the need to relate their research to national needs and national educational policies. They also manifested their interest in being active in working towards mathematics and scientific educational improvement.

From the point of view of the curriculum, they proposed a substantial *increase of the mathematics reasoning and the amount and quality of the contents taught by schools*. Proposed new topics do not differ too much from those presently taught in more advanced countries at the secondary school level.

Most of the propositions showed consciousness and concern about the needs to relate mathematics and science to national development; to create conditions for the existence of a national thought in mathematics; and the relations between mathematics and personal-cognitive development. The way mathematics and teacher education is to be taught was also emphasized. "Thinking is more important that contents"; "how it is taught is the real question"; "creativity, critical thinking, general problem-solving strategies, those are the important objectives."

High School Teachers' View

Thirty high school teachers from high, medium, and low socio-economic area schools, participated in the second study (Olfos, 1987).

Teachers answered the same questions previously posed to mathematicians, plus some on curriculum changes, their views and attitudes towards curriculum reforms, the perceptions they had about their participation in curriculum decisions and curriculum innovations, teaching methods; and questions about the obstacles they perceive as being the causes of poor mathematics learning in schools.

This study gave the researcher the opportunity to analyze the development of mathematics education in the country; to summarize the main barriers curriculum development and innovation might find according to teachers' perceptions; and a vivid, as well as crude, picture of the kind and magnitude of the differences within schools, in the same educational system, in the same city, but with different socio-economical background.

Teachers then felt that there is nothing to be dropped from the present curricula, and that changes should be made within the given conditions for teaching and learning as well as in the support for education, more than in contents or objectives.

There seemed to be a need for informatics and computer technologies at the high school level, but teachers reported that they were not prepared to teach these subjects. Problem-solving abilities, and the ones involved in decuctive and inductive reasoning, were stated as the most important objectives to be attained by school mathematics learning.

Teachers differed highly in their attitude towards change, in the perception of their own independence to make innovations, and in what they consider sound teaching. Interestingly, teachers in poor-area schools reported that they were more responsible for what the actual curricula was, and perceived themselves as having more freedom to make curricular changes than those working in economically gifted sectors.

School Mathematics and Labor

Social scientists and personnel recruiters were asked to relate elementary mathematics learning to labor (Diaz, 1987). This was a third glance at the same basic questions the other studies analyzed. Some other questions, mainly related with the kind of mathematics needed in the labor field, were added in this opportunity.

Some of the findings of the study were: the national educational system acts as if it were a closed system; in the mathematics component of curriculum, no attention is placed on what the actual labor performance of the students in going to be; all specialists interviewed confirmed the assumption that mathematics is an important ingredient of education and that its main effects are "cognitive development and the development of thinking abilities"; when confronted with specific mathematical objectives and/or mathematical contents, specialists had no rational or empirically based criteria for selecting, rejecting, or stating priorities.

Some of the results of the study were: a conceptual framework to continue these kinds of studies; a qualification of the general statement "educational system acts as if there were no need to relate educational actions and the productive system"; a set of guidelines for making curricular decisions on the basis of the relations under study; and a chart containing priority indexes associated with the main elementary school mathematics contents, as seen by social scientists associated with the field of education and labor and personnel recruiters.

The need to do research in order to relate mathematics education in the educational system, and the productive system needs, trends, and opportunities was emphasized.

Some of the objectives that school mathematics should enhance in all students, according to this study, were: general problem-solving abilities; the abilities to generate, use, and interpret numerical data; the abilities to find and to use scientific and technical information; the abilities related with measurement and data processing; and knowledge and abilities related to computers and new communication technologies.

Trends in the labor market indicate that people should develop capacities to generate their own sources of subsistence, and to learn from—and during—the working experience. As in the other studies, "Learning to learn" and the role of mathematical reasoning in the development of the higher levels of thinking and of cognitive development was emphasized.

Informatics and the Teaching of Mathematics

The most notorious change in the labor and social scenery is due to the existence of computer and new communication technologies. Nowadays informatics pervades all human activities, and tendencies show that this will continue in the near future.

Among other objectives, school is supposed to prepare students for a productive, socially accepted, and hopefully creative and happy life. Changes in the social environment should be reflected in schools. It can be concluded that, in the near future, the most important changes in school and university curricula should be related to informatics. What are the implications for the mathematics taught in schools and colleges? What modifications should be useful in mathematics curricula to meet needed preparation for an informatized society?

These questions were posed to computer science university instructors and to computer science specialists; and a content analysis of computer science textbooks and course programs and curriculum discussion emerged.

As a result, some guidelines for curriculum development and curriculum discussion emerged.

First, there are mathematics topics, usually considered in mathematics courses, that should be emphasized and/or treated differently: finite groups and other finite structures, Boolean algebra, relations, and standard logic are some examples.

Second, some other topics, such as: formal language grammar, recursive definition of functions, automata theory, graph theory, and clausal logic should be considered.

Third, a set of general problem-solving strategies and other recommendations for the mathematics teachers were stated, in order to prepare for algorithmic as well as heuristic reasoning.

A Product: Guidelines for Action

On the light of present and future needs, are we teaching the proper mathematics? Is there something more to be included in school mathematics curricula? Something to be dropped? What are the cognitive and effective objectives for mathematics education most suited to the kind of society children and young students of today are going to live in?

An effort was carried out to generate a general framework intended to guide curricular research, innovation, and experimentation in mathematics education in the Chilean educational system.

Opinions were collected from professional mathematicians, high school mathematics teachers, social scientists devoted to the study of the relation between education and labor, personnel recruiters, and computer science instructors and specialists.

The result is a conceptual framework that includes: a rationale for mathematics education, a set of principles that might guide mathematics teaching and curriculum development in the country, proposed areas of mathematics that are worthwhile investigating as curricular alternatives to present curricula, and a set of documented research questions.

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"It was Nice Being Able to Share Ideas": Women Learning Mathematics

Teresa Smart and Zelda Isaacson

The Polytechnic of North London launched a Hitecc course in 1987. "Hitecc" (Higher Introductory Technology and Engineering Conversion Course) describes one year conversion courses for people with A level (or equivalent) in non-science subjects who wish to enter scientific or technological careers. The PNL course was unique in that it recruited women only, and was taught almost exclusively by women. The mathematics component on this course was the authors' joint responsibility, and we used this opportunity to develop new materials and try out new ways of working with mature women returners.

We discovered that many of the women had joined the course because of practical and technical experiences which had given them confidence in their ability to work in such fields. For example, one woman had worked as a printer; another had become involved in the technical aspects of photography, a third found that she enjoyed doing 'electrics,' and had earned some money rewiring houses. However, their views of themselves as learners of mathematics were less positive. At a welcome lunch prior to the start of the course, several of the women expressed their anxieties. Typical statements were:

You aren't going to ask me a question I can't answer *in front of the whole class*, are you?
I'm not telling anybody, but I'm scared!
(To which someone responded: So am I!)
What if I can't do it?

The 'it' was clearly mathematics, which for this woman seemed like an exclusive club which you either do or do not belong to.

We did not find these statements surprising, given that they all came from people who had opted out of mathematics at an earlier stage. It was clearly going to

be important to take students' feelings about mathematics into account in our planning. We also recognized that students would bring a very wide range of background, experience, and ability to the course. Further, we had to ensure that the mathematics was appropriate to the needs of their other subjects, particularly physics and electronics. It also had to be an adequate preparation for future studies, which were likely to be in male-dominated areas such as engineering. We planned the work with all these needs in mind.

Thus, a key aim was to organize the class and approach the learning of mathematics in ways which were likely to enhance the women's confidence and promote enjoyment of the subject. To this end, we wished to provide a course which was fun, meaningful (at a personal level), and open to exploration. We were guided by our experience as secondary school teachers of mathematics and teacher trainers, as well as our reading in the field of mathematics anxiety and gender issues in mathematics education (e.g., Burton, 1986; Buxton, 1981; Mason, 1982). An aim of equal importance was to facilitate the women becoming fluent and skillful users of mathematics, particularly in scientific and technological contexts. We were aware that these two aims were potentially in conflict.

We felt it was important for the women to reflect on their feelings about themselves as learners of mathematics. At the first session, the women were asked to write their own mathematical biographies, and to focus in particular on any experiences which were especially important to them. This was followed up in group discussions which were tape recorded (see below). The women's writing and talking illustrated many important issues, such as: lack of encouragement; the effect of competitiveness; fear and panic; the image of math; and that math as they experienced it was not fun. One woman wrote:

> The maths teacher had 12 pupils, 5 of whom were girls ... I do remember him saying that girls had no place in his field. By the end of the term there were only two of us girls left and he did make life hard for us. In the end I dropped maths ...

Another commented:

> I had two English teachers who were absolutely brilliant ... it makes it feel as if liberal arts teachers have got ... an enthusiasm for their subject and that's what made it seem to me that maths didn't have the potential for sparking the imagination and being fun.

And another:

> The problems came with my A-level maths—it was awful. Firstly I was in a class of 20 and I was the only female ... I found everything was so competitive that to ask one of your classmates for help was a joke.

Several strategies seemed fundamental for creating a mathematics course which fulfilled our aims. Of these, the most important was that we would encourage group work, discussion, and sharing. To facilitate this, the classroom was rearranged each day from rows into clusters of desks—at first we did this ourselves, later the students did it. We devised materials which required discussion as an integral part of the work. Some of these tasks were computer based, and gradually, as a result, the women came to see that computers could be used as an investigative tool. We emphasized the value of and thus legitimated the 'common sense' knowledge which individuals brought to the course. All assessments were open book, and understanding concepts and applying them was deemed (by us) to be much more important than remembering rules. The potential conflict between our aims turned out to be less of a problem than we had feared. We soon discovered that structured investigations followed by practice of skills and techniques provided the appropriate balance of activity. Experience showed that, once used to this approach, students readily moved between applications and underlying mathematical concepts. For example, the study of resistors in parallel led to a spontaneous discussion of reciprocal functions. Or, an investigation of transformations of graphs led to a deeper understanding of the properties of oscillating bodies.

As the year progressed, we became aware that nearly all the students were very enthusiastic about this, for them, revolutionary way of learning. At a course committee meeting at the end of the first term, students asked whether it would be possible to be taught their other subjects in the same way as they were learning mathematics. The discussion indicated that the women particularly valued collaborative and investigative work.

One of us was also collecting data for research purposes in the form of tape-recorded group discussions, questionnaires, and the women's writing. A transcript of a discussion contains one woman's revealing 'before and after' comments:

> ... it was a nightmare—quite horrendous ... that's why the way we're being taught now, is, ... I'm really enjoying it—it's so much fun—... working in groups, just in groups—I've never ever worked in groups in maths before—it's really, it's really eye-opening ...

Another woman, who before joining the course felt "not terribly positive" about mathematics because "I wasn't very good at it at school" has "positive" feelings as a result of her experiences on the course. She wrote:

> I enjoy maths and look forward to continuing with the subject on a chemistry degree. I have also found myself 'selling' it to friends and relatives.

Towards the end of the year, immediately after a class, the women were asked to write down their views about working in groups. Some of their comments were:

> It was nice being able to share ideas.
> As we are teaching each other we all get a better understanding of the subject matter.
> Working in a group, I worked through a lot of thinking that on my own would have seemed very daunting ...
> I find that working in groups helps mathematical ideas to develop and be maintained, since discussion encourages a deeper understanding of the subject. Group work also inspires confidence ...

The course has now ended, and the women are going off confidently to further study in science and engineering. It has been a challenging and exciting year

for all of us, one in which our belief in the value of discussion and group work for learning mathematics has been reinforced.

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ICMI and the Crisis of Mathematics Education: What Kind of Reform is Needed?

George Malaty

Mathematics Cultivation and the Establishing of ICMI

The cultivation of mathematics in the European societies has been affected by different factors. One of these factors was the invention of printing. Another important factor was the progress of transportation, which facilitated the moving of mathematicians and their product from one society to another. The conclusive factor, which was also affected by the abovementioned two factors, was the education of mathematics.

The result of mathematics cultivation in the European societies, was the birth of local mathematical societies to facilitate communication between the increasing number of mathematicians in these societies.

The spread of mathematical centers crossed the Atlantic to establish the American Mathematical Society in 1888 after the Hamburg society in 1690, the Amsterdam society in 1778, the Moscow society in 1864, the London society in 1865, the society of France in 1873, the Edinburgh society in 1883, among others.

To facilitate the direct communication between mathematicians all over the world, in 1897 the First International Congress of Mathematicians was held in Zurich with 204 participants.

The Fourth International Congress of Mathematicians was held in Rome in 1908 with 535 participants. By a resolution of this congress, a central committee of ICMI was established with members from Germany, England, and Switzerland.

Why ICMI?

The 19th century saw the birth of modern educational systems in Europe including elementary education. The movement of building such educational systems increased in the last three decades of this century.

Until the beginning of the 20th century, the teaching of mathematics in the elementary schools was related to the well-known three Rs of the year 1828, where one "r" refers to arithmetic. The teaching of mathematics, especially in the elementary schools, had put emphasis on the achievement of mechanical skills.

ICMI was established to support better education of mathematics at all levels and to secure public appreciation of its importance. This means that the mathematicians, who met in Rome in 1908, were of the opinion that the development of mathematics depends on the development of mathematics education.

ICMI and Mathematics Education Until the Year 1957

In the beginning of this century, ICMI was able to organize four international congresses in 1910, 1911, 1912, 1914 and to reestablish a central committee in 1928.

Until the year 1957 the most formal activities of ICMI can be seen in the short time of the years 1910 to 1914.

The work of ICMI at that time was oriented intensively towards the emphasis of understanding of mathematics instead of the mechanical skills.

This active work of ICMI received an echo in different places all over the world. For instance, in Finland in 1913 the mathematician and distinguished pedagogue L. Neovius-Nevanlinna wrote his book *Havainto-oppi* or *Perception*, in which he mentioned the new orientation in teaching mathematics.

Despite the fact that the work of ICMI was interrupted by the First World War, the work towards the understanding of mathematics continued to form a general tendency in mathematics education in the 30s.

Different specialists in different countries have done research, which showed that the understanding of mathematics leads to better attainment. They had gotten support from the gestaltists, especially after W. Köhler's work *The mentality of apes* in 1924.

One of the factors, which decreased the importance of drills and gave a space for emphasizing the meaning, was the revision of E. L. Thorndike in 1932 to his law of exercise to make it unimportant in the learning process. In other words, finally the theory based on the behavior of cats made the place to a theory based on the behavior of more intelligent animals, the apes.

The work of the specialists in methods of teaching mathematics continued in the same direction of emphasizing the understanding and found support from other educators. In the fall of 1956, B. Bloom and

others wrote the result of their classification of educational objectives in the cognitive domain, which had been appreciated by the specialists in teaching mathematics.

ICMI and Mathematics Education Since 1957

Between the world wars, mathematicians were able to hold 5 international congresses. The last was their 10th congress, in Oslo in 1936 with 487 participants.

In Cambridge, Mass., USA in 1950, the first postwar International Congress of Mathematicians was held with 1,917 participants.

In 1952, the newly formed IMU reestablished the ICMI. Not until 1969 did ICMI start the new international ICME congresses after the "new math" movement took place in different countries. The first three congresses mainly dealt with the new curricula and the ongoing projects.

At the time of the "new math," the main problem, which specialists tried to face was the public opinion. The explanations of the specialists and the advertisements and publicity of the publishers were not able to face the anger of the parents towards the "new math" textbooks. The introduction of sets and Venn-diagrams in elementary schools caused the public press on specialists to choose the slogan "back-to-basics" in 1978.

The declaration of this slogan was the declaration of the crisis in mathematics education. Up to the present, the crisis has continued for ten years. This period is not long in the history of mathematics education, but how long will it continue? It is difficult to give an answer to this question.

ICMI—The Crisis and the Reform

Since 1980, the participants of ICME attend the congress to see if there will be a new slogan or not. The use of slogans to describe and show a new tendency, became the major expected outcome of ICME's work.

On the one hand, this reflects the need of security. On the other hand, this shows how we become users of an unscientific tool which belongs to the media and marketing activities, the slogans. The slogans' system is wrong. The use of the slogans "new math," "the revolution of mathematics," "radical changes," etc., affected not only the general opinion to support the changes in the 60s, but also affected the work of mathematical educators and teachers of mathematics all over the world.

The main challenge for ICMI is how it can lead a reform of our concepts of the work in the field of mathematics education, a reform which can enable us to work again, without noise, towards better education in mathematics and thus find a way out of the crisis.

The Interests of the 80s and the new Slogans

In the 80s, most of the specialists have been interested in one or more of the following fields: computer education, computer as a tool, problem solving, games, modeling and visual geometry.

These interests are not new; new is the attempt to have one of them as a leading tendency, which can get the support of the media and the public.

These activities can only be positive if they can bring understanding of mathematics as a starting point to fulfill the other direct and indirect objectives of teaching mathematics.

In the case of ICME 6, the most expected slogans are "Mathematics for all" and "Everyday life mathematics." These slogans can finally get the support of the public and the media, but they lead to no mathematics.

Mathematics by its nature is not that of the so-called "Everyday life." Mathematics can be for all, mathematics can be a simple and lovely subject when the students can understand it.

The Kind of Work Needed

Since 1957, with the increase of the role of the media, many slogans have been invented. Slogans cannot belong to science. One of the main properties of science is continuity, therefore we have to study the work of the specialists up until 1957, the work which had been interrupted by the "new math" and "back-to-basics" movements.

The back-to-basics movement did not take us back to that time, but to the time before the establishment of ICMI in 1908.

While the specialists in methods of teaching mathematics tried to place emphasis on understanding until 1957, the dissertation of L. Saad in 1957 showed that the teaching of mathematics was far from fulfilling this objective. The number of mathematics educators now is much higher than in the year 1957. Their work can help in fulfilling this objective.

ICMI and the Other School of Education

The work in mathematics education in the USSR presents a special case, from which we have to learn.

Mathematics education in this society has been a continuous work since Leonard Euler came to St. Petersburg in 1730 to work at the Emperor's Academy and teach mathematics in the Academy's school (gimnazii). One of the specific characteristics of mathematics education there lies in the structure of the curriculum. This curriculum is organized to introduce gradually to all students the abstract, deductive nature of mathematics from the 1st grade on and the

axiomatic structure from the 6th grade on. Gifted students receive special education from the 7th grade on, mathematical clubs and competitions take place from the 4th grade on.

As a result, mathematics is the most popular subject. To show how successful they are in teaching mathematics there, let us take a look at the following example:

The 4th grade student of ten years of age is asked in one of the textbook exercises to prove that the vertically opposite angles are equal. To reach such a level, a gradual preparation is begun in the 1st grade. The specialized methodologists of the first three grades of the Academy of Pedagogical Sciences of the USSR, M. I. Moro, M. A. Bantova, G. V. Bel'tyukova, A. S. Ptshyolko, and A. M. Pyshkalo wrote the textbooks of these grades about 20 years ago, but they are continuously developing these books on the basis of continuous experimental work. On the other hand, their textbooks are further developed versions of the previous textbooks.

In 1967, a reform committee under the leadership of Kolmogorov, refused the kind of reform taking place in the Western countries. For instance, the reform which the committee accepted included the use of the concepts of Sets from the 4th grade, but not the writing of a special topic in Sets. They accepted the use of more of the transformations approach in geometry, but within a more rigorous axiomatic structure of the Euclidean geometry from the 6th grade. The committee did not accept the unification of geometry and algebra in one textbook, to keep to geometry its lessons and importance.

The continuous improvement of mathematics education under the supervision of the most well-known mathematicians all over the country is supported by public appreciation of mathematics and its structure. The appreciation of the structure of mathematics and its beauty is one of the main factors which lead to the resolution of 1985 to separate the computing education in the 9th and 10th grade from the teaching of mathematics to keep to mathematics its importance.

The appreciation of mathematics and its structure is a result of mathematics cultivation, which has been affected by the reform of education of the common people at the beginning of this century. The teaching of mathematics to these common people was based on the work of the distinguished mathematician and pedagogue A. P. Kiselev (1852–1940). Kiselev's phenomenon is a result of the long cultivation of mathematics in this society, his textbooks of the end of the 19th century have been used until the 60s of the 20th century. Kiselev had continuously revised his textbooks to some of about 40 editions. After his death, his books were revised again. Until the present, these books form the basic reference material for building the curricula and writing the textbooks. On the other hand, these textbooks have relations with the previous textbooks.

In conclusion, the education in mathematics is a continuous work, which has not been influenced by any slogan or mode.

The question now to ICMI and also to ICME is: Can we learn something from the other main school of education, which represents the USSR school?

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Educational Institutions and the Individual Learner

Individual and Social Learning Motivations

Mathematics is learned in a social setting and the significance of social motivations needs to be better understood by mathematics educators. Anne-Nelly Perret-Clermont and Maria-Luisa Schubauer-Leoni, Switzerland, consider the learning of mathematics as originating from an intersubjective construction between the teacher and students and illustrate how the "didactic contract" controls all learning and teaching. They also address the social constructions of meanings of mathematical activities and conceptual functions that the contract permits. Timothy E. Erickson, USA, discusses certain attributes of cooperative exercises and cooperative materials, developed by the EQUALS Project, and how they work together to help produce successful experiences for both students and teachers. Albrecht Abele, Federal Republic of Germany, reported on an analysis of videotaped situations to show the interdependence of social learning and mathematics learning.

The Social Construction of Meaning in Math Class Interactions

Anne-Nelly Perret-Clermont and
Maria-Luisa Schubauer-Leoni*

In previous publications we have presented data concerning the role of social interactions in the development of cognitive skills and operatory structures (Perret-Clermont, 1980; Perret-Clermont & Schubauer-Leoni, 1981; Perret-Clermont, 1982). In a current research project we are examining the content of communication between teachers and pupils dealing with mathematics within the school context. We find ourselves at the crossroads between the sociology of educational institutions and curricula, the social psychology of individual strategies and the psychology of learning. But at the same time, we want to take into account the specific nature of what is supposed to be learnt in this communicative process: mathematics, a corpus of knowledge and skills loaded with many more symbolic values than we are commonly aware of.

Individual Learners and Individual Teachers Within a Structured Social Context

A Structured Social Context

Schools are social institutions with legal foundations, roles, and norms defining the nature of the professional roles of their agents, the time and space of the activities, and the curricula of lessons and examinations.

Educational institutions are part of a wider social and cultural project. Traditions and political choices define the nature and forms of the adult-child relationship, the rights and obligations of different socializing agents (schools, professional training, parents, media, etc.), and the knowledge (its nature, form, and uses) which is thought worth transmitting from generation to generation. Mathematics is part of this cultural transmission.

These institutional, social, and cultural factors produce (more or less) explicit definitions of school math (its nature and symbolic values) as well as definitions of the formal roles of teachers and pupils.

Many Elements of the Social Context Remain Implicit

However, norms, values, and expectations of the social context are not always explicit nor conscious, especially when they belong to larger social role configurations (i.e., sex, class, or professional roles).

Concerning mathematics, we find that the mathematical tradition as well as the pedagogical tradition have cultivated a series of habits and styles that seem to constitute a type of "ethos" which is never made explicit (e.g., what do mathematicians mean when they refer to "elegance"?).

These norms, rules, values, habits, and the "ethos" define the teaching and learning process. They constitute the web of implicit "didactic contracts." That is, within the school context, pupils and teachers interact according to (more or less) tacit social contracts which define their respective social roles, the form of their mathematical "encounters," and the content of their communication (Schubauer-Leoni, 1986).

However, the social context is not the only determinant of the terms of these "encounters" between pupils and teachers. These persons as individuals (and/or as members of families or larger social groups) elaborate strategies aimed at gaining benefit from their school experience or, at times, just surviving it! The balance of "costs" and "benefits" in these strategies can vary from case to case. Individuals entering into negotiation processes within the above-mentioned "social contracts" do so according to their aims and strategies.

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Thus, we have observed (in teacher and pupil discourse, in classroom observations, and in experiments) that mathematics is very seldomly considered "in abstracto" in school. Adults and children always think of mathematical concepts and tasks within the specific classroom context.

Mathematics are not Considered "in Abstracto"

On the Pupils' Side

In a series of investigations (Schubauer-Leoni & Perret-Clermont, 1980; 1985) we have repeatedly found that 7 to 8-year-old children, who can easily deal with elementary arithmetic additive and subtractive problems within the classroom, seldom use the formal arithmetic notations when asked by a psychologist to write down what happened in a problem dealing with candy, flowers, or dice. The formal mathematical "code" taught in schools was only used for those tasks when the setting was such as to explicitly suggest to the children that, in accordance with the habitual "didactic" social contract, they were faced with a "mathematical problem."

In another series of research (Schubauer-Leoni, 1986b), pupils who were asked to prepare mathematical problems for their cohorts seemed to concentrate more on inventing problems that would assure their supremacy in the "teacher's" role (controlling the mastery of the solution, the difficulty of the task, etc.) than on formulating interesting problems (an idea they never seemed to have thought of!).

On the Teacher's Side

Interviews with primary and secondary school teachers suggest that these professionals only consider mathematical concepts within the context of their teaching: i.e., in relation to specific pupils and curricula. For instance, teachers talk more easily of the chances of success and failure of solving given algebraic problems by specific groups of pupils than of their aims in presenting those problems to their pupils. When asked their criteria for choosing a given exercise, they refer to the history of recent classroom interactions with their students. For these teachers, mathematics is essentially considered as a set of information and tasks which require presentation followed by learning assessments. They tend to insist that this has to be done in a very systematic way, in accordance with the (more or less) implicit "didactic contracts" that regulate their interactions with their students. They know that pupils consider the "didactic contract" transgressed if they are tested on a matter which has not been taught to them by their current teacher or if the test uses unfamiliar notation or wording. This results in the development of a kind of idiosyncratic "jargon" between the teacher and his pupils that serves as a metalanguage in dealing with "school math." When pupils change math teachers, the "jargon" then has to be verified, updated, or redefined!

Teachers also adapt their perceptions of what mathematics is in accordance with their representations of the learning difficulties of perceived types of students in their classroom. They seldom speak of the difficulty of a math problem without referring to some idiosyncratic characteristic of a specific pupil who they are thinking of at the moment.

Being a Student in Math Class: A Specific Socio-Cognitive Skill

It seems that skills learned in math class are specific to the local context of the teacher-class and teacher-pupil interaction. The learning of appropriate algorithms of responses to specific sets of school tasks seems to be the result of, on the one hand, strategies for school success within very detailed and refined sets of norms, expectations, and "jargon" that regulate the modalities of social and cognitive performance, and on the other, the individual's capacities to manage the emotions elicited by the encounter of two distinctive sets of strategies: that of the teacher and of the pupil within the wider classroom, school and socio-cultural context.

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Cooperative Learning in Mathematics: A Way to Engage All Students

Timothy E. Erickson

In the United States, young women and certain minority groups are underrepresented in higher-level mathematics courses and in math-based careers. There is little credible evidence that these groups are naturally poor at mathematics; instead, the problem seems to be the result of a complex web of environmental and societal forces. What can we do in schools to help previously underserved groups be more successful in mathematics?

The small but growing movement towards cooperative learning techniques holds great promise for helping make mathematics education more equitable. In this paper, rather than describing research results (which are generally positive)[1] or the specific techniques teachers use in cooperative classrooms (which are numerous)[2], we will explore the social issues inherent in cooperative learning in mathematics, the implications for educational equity, and special problems cooperative learning faces in the school setting.

We will make a distinction in this paper between *group learning*, in which students simply work together to learn "content," in this case, mathematics, and *cooperative learning*, in which the teacher and students value social skills as well. Exactly what these skills are varies from scheme to scheme, but generally, they are communication and group-process skills like being polite, active listening, encouraging others, consensus-building, interpreting what others say, and summarizing. On the surface, these skills have little to do with mathematics. We believe they are essential parts of mathematics education.

Implications for Equity

This paper's original interest in cooperative learning is as a tool to promote educational equity. How does that work?

First and most important, effective cooperative learning engages all students. This can go wrong, of course; it is possible that one student might take over a group, or that many students might lose interest in a cooperative lesson. But that is one of the main problems cooperative curriculum development addresses, as we shall see. We certainly hope that many more students will be engaged in this than would be in a traditional lecture.

Once underserved students are engaged by a cooperative lesson, they will experience success in mathematics, many of them for the first time. Interest and success feed each other; if we can get more students interested in learning math, we can expect higher achievement in mathematics.

We have said that cooperative learning can help underserved students. How do we expect certain techniques magically to help the students who need it most? The answer comes partly from our beliefs as to why certain groups have done less well in mathematics until now.

First, there is well-documented teacher bias against certain groups in heterogeneous mathematics classrooms. For example, teachers are much more likely to call on boys than on girls.[3] Though it may seem callous, cooperative learning removes the teacher for part of the learning experience, ameliorating some of the bias the teacher unconsciously introduces.

Second, different students learn in different ways. Teaching mathematics in a lecture format and isolating students for problem solving will teach math best to the students who thrive in that environment. We do not suggest that teachers give up entirely on "teaching by telling," but a mixture of teaching modes will serve a mixture of learning modes more equitably. In this way, too, students who are strong in social skills may have a chance to shine.

A related comment: A student who learns well by lecture and in isolation will not learn poorly in a group. On the other hand, this type of student may benefit most from learning the social skills part of a cooperative curriculum.

Special Problems Implementing Cooperative Learning in Mathematics

The change to cooperative learning is not easy. There are many problems we need to acknowledge.

The first is that cooperative learning goes against the grain of many educational establishments. A cooperative classroom is noisy; it appears chaotic to the traditional observer; and the control of activities frequently rests with the students, not with the teacher. It is hard for some teachers to abandon their traditional power, and hard for some school administrators and parents to allow it.

Another problem is assessment. Cooperative learning educators distinguish between *grading*, a judgment of student achievement, and *assessment*, a way for teachers and students to evaluate student learning. Grades are usually summative, assessment in this context is usually formative.

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[1] See, for example, Jeannie Oakes (1988) in *Getting untracked*, National Education Association, or Robert Slavin (1986), *Cooperative learning in student teams: What research says to the teacher*, National Education Association.

[2] There are many systems; in the USA, the most well-known are by Elliot Aronson, by Robert Slavin, and by David and Roger Johnson. There are many practitioners elsewhere, for example Sharan and Sharan in Israel.

[3] You will find a wide-ranging review of the literature and suggestions for intervention in *Gender expectations and student achievement*, by Delores A. Grayson (1985), Los Angeles County Office of Education.

Assessment can be complicated in the cooperative classroom. Teachers and students both need to know what each student understands to be able to plan intervention, remediation, study, and future lessons. But how can you tell what individuals know when the students are in a group?

There are many schemes for administering traditional tests to assess groups and individuals and to increase motivation. But many of the most interesting and effective assessment methods are qualitative by nature. Informal observation is very powerful and efficient, though not comprehensive. Teachers can also perform more formal mini-ethnographic studies to evaluate individual contributions to group work. They may cycle through the groups, being willing not to observe every student in every session. Individual writing, where appropriate, can reveal much about student understanding. Student-centered assessment is possible as well, gradually letting students take more responsibility, not only for their own learning, but also for its assessment.

Another problem with implementing cooperative learning is sorting through the various cooperative learning schemes and the objections they have to one another. Experts differ on the precise social skills that should be in the curriculum and in what order they should be taught. They differ in the degree to which they feel teachers ought to intervene. They differ in their beliefs about assessment, ranging from the "carrot and stick" approach of group testing to the distributed assessment schemes that require student responsibility. Most important, they differ in the language they use to describe their programs.

Finally, there is a problem finding materials that work in mathematics classrooms. Most of the existing literature focuses on social skill-building. There are a few classic activities (such as the cut-up squares activity[4]) that are ideal for the cooperative math classroom, that incorporate both social skills and mathematics content. There are less well-known activities, including some simulations, that fit in as well. It is also possible to use existing mathematics problems and ask students to work on them in groups. But there is not a large body of mathematics curriculum that is designed with a cooperative classroom in mind. Many teachers have had to create much of their curriculum themselves.

Some Criteria for "First" Cooperative Mathematics Activities

As teachers and students become more accustomed to cooperative learning, classes can tackle more sophisticated group projects, can engage in jigsaw lessons, and can take traditional work and adapt it to the cooperative classroom. But what about the class that is starting out? The above discussion leads us to some recommendations for curriculum materials.

One of the critical considerations is that everyone participate. One way is to divide up roles in the group. Most cooperative learning schemes distribute roles such as Recorder, Facilitator, and Encourager. But in mathematics, we can also create problems in which we distribute the necessary *information* throughout the group. In that way, everyone can (and must) contribute to the solution of the group's problem.

In distributing the information needed to solve a problem, though, we must remember that the problem must be worthy of group solution. If an individual could solve it easily alone, there is no point in making it hard for a group. That is not to say that some students will not feel hampered by their groups; but that is a problem in the social domain, not in mathematics.

These are only two guidelines of many we might propose. Curriculum developers are now beginning to produce more materials for mathematics teachers who want to incorporate cooperative problem solving into their classrooms. The author has just finished a book (Erickson, 1989. *Get it together*. Berkeley, CA: EQUALS) of problems that are appropriate for a wide range of abilities in many different areas of mathematics and adhere to these principles of distributed information and groupworthiness.

[4]Bavelis, A. (1950). Communication patterns in task-oriented groups. *Journal of the Acoustical Society of America, 22*, 225–230. As far as we know, this is the original reference to this remarkable and ubiquitous activity.

Socialization and Learning Mathematics

Albrecht Abele

The pedagogical demand to increase the use of small-group instructional methods with regards to socialization in mathematics instruction strikes many mathematics teachers as contradictory. They certainly do not dispute the possibility of applying small-group instruction methods in appropriate areas of mathematics curriculum or at special stages of skill. They emphasize, however, that this can only be accomplished by ignoring or reducing learning in mathematics. They reject small-group instruction methods in mathematics classes because it can occur only at the cost of mathematics learning, or to put it bluntly, because mathematics as subject matter is too important to waste as aid for teaching socialization.

On the other hand, there are many teachers, especially from the social sciences and the humanities, who consider it impossible that small-group instruction methods could be conducive to mathematics learning. Their conception of mathematics as a com-

plete, artistically constructed scientific edifice allows, in their view, only teacher-centered instruction in which interaction between one student and another has no place.

On contrast to these two types of prejudice, we offer the following thesis:

Thesis

Socialization and mathematics learning do not proceed independently of one another, but they depend on each other. That is:

(1) Learning mathematics is promoted or restricted by the social events in the group of learners (class or small-group).
Derived teaching behavior: creation of a healthy social milieu in the group of learners, in order to promote and stimulate the process of learning mathematics.

(2) Social cooperation is determined—promoted or restricted—by the methodical presentation of the subject matter.
Derived teaching behavior: preparation of the mathematics lesson informed by different methods, so that in dealing with mathematics there is the unavoidable necessity for student-student communication. Thereby social learning processes are introduced.

Within the scope of this contribution, an example (videotaped lesson) will be used to show how communication processes in a school group arise from favorable methodical arrangement; and, on the basis of socially intact group events, how students arrive at the solution of the mathematical problem. The example presents the following characteristics with the specified consequences:

(1) The mathematical problem is posed in the appropriate methodological form, therefore it is especially useful as the natural basis of conservations in the group. This will become clear when the pupils can, for example
(a) formulate hypothetical solutions,
(b) attempt to verify these,
(c) provide reasons for their procedure, and
(d) perform computational checks on the results;
all this independently of the teacher.

(2) Student involvement in the class is practically inevitable when the student is granted the right to share in the decision making in a period of instruction, as was the case in the example just cited. The participation of the students in learning intensifies because they have made the instruction their own.

(3) Guidelines from the teacher will be accepted willingly and will have a positive effect on learning when they are well-grounded and not done only for the sake of appearance in relation to the students' remarks.

Example

A group working situation:
Problem of ordering in the number system of base 4.
A group of four children (grade 3) work with four matrix plans 4 x 4.

First task: The pupils represent the number 0 to 31 by the multi-base blocks putting the blocks onto the first two plans.

Second task: The pupils have to put the pieces of the game "Quadrimath," which can be used for an isomorphous representation, onto the second two plans in such a way that "the solution fits as well as possible with the plans already laid out." The Quadrimath-plates differ in color (two ways), in form (four ways), and punching (four ways). Thus, the principle of variation has been accounted for. The intended learning objectives require that the pupils recognize the isomorphics of the solution arrived at in the first and in the second task and can apply them in subsequent individual tasks.

Waiting for the children's work, we have the following questions:
- What is the behavior of the children with such a problem-oriented situation when left without a teacher (teacherless task)?
- Is the group capable of working when difficulties arise?
- How do the pupils overcome difficulties, which come from different frames of thinking?
- Do the children consciously or unconsciously develop certain strategies for problem solving?

As we can see, the problem is solved by the children in four phases:
(1) solving the multi-base blocks problems;
(2) free experimenting with Quadrimath pieces;
(3) partially systematic essays, but sometimes not coordinated;
(4) adoption of a general plan for a total solution.

The main difficulty came up after the third phase: Despite some good individual suggestions for solving partial problems, the children do not have an idea of how to come to an overall total solution. They are not yet able to place the plates on the game boards, because they are caught in a contradiction of two opposite conceptions.

The discussion of the pupils follows this line: They argue within a first frame created by the concept of cardinal number when they say:
"An individual (of multi-base blocks) corresponds to 1 hole (of the quadrimath pieces), 2 individuals correspond to 2 holes and 3 individuals to 3 holes,"
or:
"We should have a plate with no holes (which doesn't exist!) to place at the beginning of the rows."
The second frame is created by the concept of place value when the pupils say:
"A rod corresponds to 4 holes,"

or:
"We place the plates with 4 holes at the beginning of the rows."

This conflict situation is settled when the children agree about a common new conception of frame which is built up by taking their bearings from the *problem* situation, i.e., the solution of the second problem has "to fit as well as possible" with the solution of the first task. The children change their behavior: No longer searching for special possibilities for putting the plates onto specific fields of the plan, they now ask for rules of translation like:
"What does blue actually mean in this?"
"We have to agree on what the 'form' means."
"We'll take this form as individual blocks."
"Four holes mean: a new row begins."
"We'll take the next form for the second row."

Conclusions

(1) From a methodological point of view the problem was formulated along the lines of Heckhausen according to the principle of "fit" ("Prinzip der Passung"): Problem 1, easy, building on what is known; Problem 2 with clear challenges, one which at first can not be grasped as a whole, but which offers the students immediate and easy access. In this case it was the simil: rity with a known situation (Problem 1, with solution). The challenge of the task was taken on by the students; this led to a lively exchange of opinions among the students and finally also to the solution of the problem.
(2) Since at first the path to the solution of the problem is not obvious to the children, they make use of several possible paths in the course of testing their usefulness (methodological principle of "Assoziativität" along the lines of Fricke). In most cases this methodological process leads to spontaneous student-student interaction.
(3) In the process of finding a solution, the moment of questioning between the third and fourth phases leads, by means of autonomous exchange of opinion, to a new orientation: The previous frame of mind "cardinal number" or "place value" will be replaced by the more productive idea "transformation from one form of representation into another."

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Cultural Influences on Learning

Children come to school as culture-bearers and the importance of this for mathematics learning is discussed in this topic. Analucia Dias Schliemann and Nadja Maria Acioly, Brazil, provide data on how everyday experience with numbers affects the way people solve problems and analyze how the numbers and the arithmetical operations involved in the problem affects the efficiency and the strategy used to find the solutions. Frederick K. S. Leung, Hong Kong, discusses four characteristics of the Chinese culture that might influence mathematics learning: the Chinese stress on filial piety and respect for superiors, the Chinese stress on memorization and practice, the characteristic of the Chinese language, and the high parental expectation of the Chinese. Martin R. Hoffman and Arthur B. Powell, USA, discuss the use of writing activities in mathematical curricula as a vehicle for student reflection and empowerment.

Numbers and Operations in Everyday Problem Solving*

Analucia Dias Schliemann
and Nadja Maria Acioly

Problem solving in schools has often been dealt with as a stepwise activity in which the child is supposed to (a) read the problem, (b) decide which arithmetic operations he has to perform, (c) write the corresponding mathematical sentences, (d) perform the operations and, finally, (e) produce the answer. Analyses of uses of mathematical knowledge in everyday activities, however, show that different strategies may be used, and the answer may appear even before a mathematical operation is chosen or thought of (see Carraher, Carraher, & Schliemann, 1985; and Lave, 1987).

Another belief frequently held by math teachers is that, to solve a problem, the child must first learn how to perform the operations required. This may, in fact, be true when the problems involve large numbers which can only be operated through the use of written algorithms or calculators. When, however, problems are solved outside of school and oral procedures are

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called into play, a different picture may appear, and the numbers involved, more than the operations to be performed, may determine how easy a problem is and what strategy will be used.

The number system and the relationships between the numbers are not experienced in the same way by every subject. Children and adults in the United States know, for instance, that four quarters make a dollar without having to add or multiply 25 by 4. This is a known relation that may not be so evident to children in other countries. Different uses of numbers privilege different groupings, creating particular patterns. Whenever instances of these groupings appear in a problem, the relationship between them is so evident that the answer to the problem emerges in a sudden fashion, with no need to perform any arithmetic operation. For those who understand the decimal system, 5, 50, and 500 is one such grouping, and the answer to a problem like "How much does one item cost if 10 cost 50?" may not result from the division of 50 by 10 but from knowledge about the properties of the decimal system.

In this study we provide data on how everyday experience with numbers affects the way people solve problems. The issue of how the numbers and the arithmetic operations involved in the problem affect the efficiency and the strategy used will also be analyzed.

Subjects were 20 adults who worked as bookies in a special kind of lottery game in Recife, Brazil (see Schliemann & Acioly, in press, for a thorough description of the game). At work, among other types of bets, bookies have to determine the price of groups of bets made on the permutations of the digits in a number, taken three by three and four by four. The players usually state the value they want to bet on a certain number and all its permutations. For instance, one may choose 2583 and ask to bet on all 3 and all 4-digit permutations of the digits 2, 5, 8, 3. The bookie determines the overall value to be paid by finding, through inspection of tables or by memory, the number of permutations among the digits in that number and multiplying it by the value the player is betting. In this case the player will be betting on 24 3-digit numbers and 24 4-digit numbers. The structure of the game and usual bets are such that 12 and 24, with their multiples, become privileged groupings.

The way problems appear to bookies at work, most often, calls for multiplication. Only rarely do they meet the inverse form that requires division. In this case, players state the amount they want to pay for the whole set of bets, and the bookie has to find the value of each bet. Most of the subjects in this study reported not knowing how to perform division, a fact that by itself would suggest that any problem calling for division should be more difficult than problems calling for multiplication. However, in problems which involve privileged groupings, the operation may be of lesser importance. To test this hypothesis, the bookies were asked to solve two series of problems. The *first series* was composed of six multiplication problems, *similar to those most often occurring at work*, with the price for one bet stated by the player and the bookie determining the price for the whole set. However, in this case the *numbers involved were not part of the privileged groupings* that were likely to develop as a result of the subjects' work experience. The *second series* was composed of six division problems, which *rarely occur at work*, where the whole price to be paid was stated and the bookie had to determine the value of the bet for each number that made up the set. In this series, the problems involved *numbers that were part of the privileged groupings* that were more likely to be developed as a result of the bookies' everyday work.

Analysis of data presented in Table 1 shows that division problems with privileged groupings were easier to solve than multiplication problems with non-privileged groupings. Although most subjects did not know how to perform division, the percentage of correct answers to division problems was higher than that for multiplication problems.

Table 1
Percentage of Responses for the Total Number of Problems Presented to the Subjects

Type of problems	Correct answers	Immed. answers	Written proced.	Mental comput.	Hypoth. testing
Multiplication (non-privileged groupings)	68.3	0.8	64.2	4.2	0.0
Division (privileged groupings)	76.6	45.0	24.2	55.8	22.5

The most popular strategy to solve multiplication problems (with non-privileged groupings) was the use of a series of operations solved through written school procedures that usually followed the same stepwise sequence for each problem. In contrast, for division problems mental computation was preferred and, in nearly half of the division problems, the answers were given immediately and were followed by explanations or computations that were meant to show their appropriateness. These immediate answer to problems that rarely occur at work indicate that the numbers involved were part of the subject's privileged grouping. One example of this approach is the following answer given by an illiterate subject, asked to compute the price of each bet if one were to bet 9,600 cruzeiros (the Brazilian currency at the time) in all 3 and 4-digit permutations of the digits in 587643:

Here you have 360 thousands and 120 hundreds [that is, 360 four-digit and 120 three-digit permutations of the six digits in 587643, adding up to a total of 480 bets]. It's 20 cruzeiros on each, because 480 plus 480 makes 960. If it were 10 on each that would be 4,800. At 20 it makes 9,600.

When the numbers involved were not part of a privileged grouping, the most common procedure was a hypothesis testing strategy, used at least once by 17 subjects, where a value for a single bet was hypothe-

sized and then tested by means of multiplying this value by the total number of single bets; if the result matched the total value stated in the problem, the answer was given; if not, a new value was tried, until the answer was found. One example of this strategy is shown in the answer of a schooled subject solving the same problem:

It is 120 hundreds and 360 thousands. [Writing the computations bellow, the subject talks aloud:] It makes 480. If it were at 5 each, it would make 2,400. Let's increase it a little bit. If it were 10 cruzeiros in each, it would make 4,800. If it were 15 ... I'm going to add. It would make 7,200. There is still some more, you have to add more. Ready, it is 20. I added by fives.

120	480	2400
360	0	7200
480	4800	9600
5	2400	
2400	7200	

Here, after finding the total number of possible 3 and 4-digit permutations of the digits in the 6-digit number, the bookie tried 5 and then 10 as possible solutions, testing it by multiplying 480 by 5, and then by 10. As neither answer was adequate, she tried out 15 as a solution, adding together the result of the multiplication by 5 to that of the multiplication by 10 and, finally, tried out 20 by adding the result obtained for 15 to that obtained for 5, finding an adequate answer.

These data show that, firstly, even though unable to perform division as taught in schools, subjects who deal with computations at work may develop effective strategies to solve division problems through exclusive use of addition and multiplication. Secondly, the sequence of steps to solve a problem varies greatly, depending on how the relations between the numbers involved in the problem appear to the subject. Thirdly, everyday experience allows the development of knowledge about the relations between the numbers and the constitution of privileged groupings that greatly helps finding the solution to arithmetical problems, without need to go through a fixed sequence of steps. These three trends conflict with the school-prescribed ways, where problems have to be solved through a pre-determined sequence and are meant to demonstrate knowledge of a particular set of number facts, which differs from those learned outside schools. This may contribute to the high rates of school failure among children who start very early to deal with the properties of the number system while dealing with money at work. Recognition of the knowledge they bring to school and of the appropriateness of their own strategies to solve certain problems, should contribute to more interesting problem-solving classes and to more effective teaching of other more formal strategies when these are required.

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The Chinese Culture and Mathematics Learning

Frederick K. S. Leung

The Social Orientation of the Chinese

Compared with Western cultures, which stress independence and individualism, the Chinese emphasize integration and harmony. Related with this "social orientation" (as opposed to individual orientation) are characteristics like obedience, compliance, respect for superiors and filial piety. Psychologists claim that there are distinctive cognitive styles associated with these characteristics, and cognitive styles, of course, affect the way people learn mathematics.

More directly related to the learning of mathematics was a recent project called 'Mathematical Understanding of Taiwan Students' (MUT) which was essentially a replication of the CSMS study in the UK. Lin (1988), in a paper discussing some of the findings of the project, pointed out that Taiwan students were much less susceptible to the so-called "child-method" which was so prevalent among British subjects of the CSMS study. One piece of evidence that Lin used was the narrower facility spread in the Taiwan data. In a replication study (Algebra) done by the author in Hong Kong, the standard deviations of the facilities for the limited items ('2a + 5a =' and the three items discussed below) with results in all the three places available were computed. The s.d. for England (23.3, 22.7, 19.8 for age groups 13, 14, 15 respectively) were found to be much larger than those in Taiwan (24.6, 11.2, 7.7) and Hong Kong (18.7, 10.4, 8.7). Now Hong Kong is a society which is rather different from Taiwan in many aspects, but the people in the two places are of the same race (i.e., Chinese). This suggests that the similarity cited above may be due to cultural factors rather than other societal factors (like politics or economy).

Lin attributed the different performance of the English and Taiwanese students to the different emphases the educators of the two places had on mathematics learning and quoted the Cockcroft Report to support his argument. But he did not discuss why there were different emphases after all. The cultural differences of the two places may provide us with some hints. The UK is a typical Western country where the rights of the individual are viewed very highly. In such a culture, it is not surprising that students are encouraged

to use their own (child) method. Taiwan and Hong Kong, on the other hand, are Chinese communities where conformity is the norm. So it is just natural that students do not use methods that are not taught to them.

The Chinese Stress on Memorization and Practice

A couple of years ago, one of the expatriate colleagues in our department gave a lecture to some 130 PGCE students on the topic of "meaningful learning." At the start of the lecture, he showed the students a 6-digit number, and towards the end of the one-hour lecture, he asked those who could still remember the 6-digit number to raise their hands. The purpose of this exercise was to bring home the message of the importance of meaningful learning, and memorizing a 6-digit number, being a "meaningless" task, should prove to be difficult. But to his surprise (and embarrassment!), nearly all of the 130 Chinese students raised their hands and were able to recite the 6-digit number!

In fact, researchers have reported in the literature on the seemingly superior performances of Chinese subjects on memorization tasks. Hoosain (1979), for example, found that the mean digit-span for Hong Kong undergraduates was about two digits higher than that reported for comparable samples in the West, and other findings pointed to the same conclusion (e.g., Huang & Liu, 1978). Now digit-span is a measure of our short-term memory and short-term memory plays an important role in performing mathematical tasks (Bell, Costello, & Kuchemann, 1983). So a difference of two digits might have significant implications on the learning of mathematics.

Related to this stress on memorization is the emphasis the Chinese put on practice. It is a rather well-known fact that students in Chinese schools are required to do much more homework than their counterparts in the West. Stevenson (quoted in Cunningham, 1984), for example, reported that Taiwanese grade 1 students spent more than five times longer than American grade 1 students on homework. He also found that Taiwan students performed much better in mathematics achievement tests than their American counterparts, and he explained the latter result using the former. Other international comparative studies in mathematics achievement (e.g., IEA SIM study, see Brimer & Griffin, 1985) have shown that the Hong Kong students out-performed many of their Western counterparts. Could the Chinese stress on memorization and practice be a possible explanation of the phenomenon?

The Characteristics of the Chinese Language

The first relevant characteristic is the short pronunciation duration of numbers in Chinese. Hoosain (1984) found that the shortest average pronunciation duration of a Chinese number is 265 ms, significantly shorter than the corresponding average of 321 ms in pronouncing a number in English. As pointed out above, this has special relevance to the learning of mathematics.

The second characteristic concerns the Chinese orthography. In a comparative study by Lesser, Fifer, and Clark (1965), Chinese subjects were found to have better visual-form discrimination or space conceptualization abilities, and Lesser, in a later study (1976), reported that these cultural patterns had remained quite stable. Some researchers accounted for this better performance of the Chinese by their childhood experience with learning the Chinese orthography.

The third relevant characteristic of the Chinese language is its use of classifiers between every cardinal number and the objects being quantified. This "unscramble(s) the confusion that otherwise surrounds conservation of numbers." Also, the Chinese language "has the advantage of explicitness and pragmatic retention of the essential semantic elements in the vocabulary it uses for mathematics" (Brimer & Griffin, 1985, p. 23).

The fourth characteristic concerns the relationship between language and symbol. Lin (1988) in his discussion about the MUT and CSMS studies mentioned above, found the surprising result that in the algebra test, "the most difficult item (item 3) for the English student is proved to be the easiest item ... for Taiwan students." The items concerned and their facilities are:

Country	England			Taiwan			Hong Kong		
Age group	13	14	15	13	14	15	13	14	15
1. $2a + 5b + a =$	.40	.60	.66	.09	.51	.64	.64	.80	.86
2. $3a - b + a =$	.27	.47	.56	.13	.52	.64	.73	.82	.89
3. $(a - b) + b =$	.15	.23	.32	.42	.54	.65	.42	.64	.72

Lin explained the results by quoting Kuchemann's (1981) claim that English students found item 3 difficult because they regarded letters as a shorthand for objects (e.g., 3a as 3 apples), while Lin conjectured that Taiwan students, being not so familiar with the English letters, regarded the letters as objects in their own right. It would be interesting to replicate the test to Chinese students in Hong Kong, since, although they are Chinese ethnically and they live in a predominantly Chinese community, they start to learn English (together with Chinese) when they are very young. In a replication study by the author, it was found that item 3 was found most difficult for Hong Kong students (same as the English case) but item 2 proved to be the easiest! Looking at the answers given by the students, the most common error for item 1 is 8ab (36% of the error), and about half of the students who make this error have item 2 done correctly. It is not easy to account for this strange pattern of performance of the Hong Kong students, but may be the absence of a coefficient (i.e., coefficient = –1) for b in item 2 forces the students to combine 3a with a to get the correct answer, whereas the 5b in item 1 prompts

to students to add the coefficients together. But it seems that Hong Kong students, like their counterparts in Taiwan, do not regard letters as a shorthand for objects (if a is apple and b is banana, 8ab would make not sense!) either.

Lin also found some other interesting differences between the performance of the English and Taiwan students (for example, the difference in performance in the placing of 3/5 on the number line, with the typical English error of putting it between 3 and 4, while the typical Taiwanese error is to place it between 5 and 6), and replication of these parts of the study in Hong Kong is still going on.

The High Parental Expectations of the Chinese

Oriental parents and teachers are known to expect more of their children in academic achievement than Westerners. Their expectations are manifested in terms of their attributions of the success and failure of their children, their feedback or reactions towards their children's academic performance, and the demand of the standards to be achieved by the students.

Stevenson, Lee, and Stigler (1986), for example, found that

> U.S. mothers are significantly more likely than Japanese mothers to believe that innate ability (as opposed to effort) underlies children's success in mathematics. (Note that the Japanese are very much influenced by the Chinese culture.)

Hess, Chang, and McDevitt (1987), in a study comparing the attitudes of mothers and children from Chinese, Chinese-American and Caucasian-American families, obtained similar results. They found that the American mothers were more likely to attribute their children's failure to the lack of ability, while Chinese mothers usually attributed the failure to lack of effort and the success to the teaching in the schools. In the same study, Chinese mothers were also found to use punishment and threatened punishment more often than the American mothers, who provide "rewards without setting higher standards" more often.

In the terminology of the attribution theorists, Chinese parents were more likely to attribute their children's success and failure to controllable factors. These attitudes were communicated to their children through feedback/reactions, and that in turn caused their children to put more effort in their studies.

The Chinese classrooms were also reported to be characterized by more demanding teaching contents, and Chinese textbooks were found to contain more difficult problems when compared to the West (see, for example, Wares & Becker, 1983).

Conclusion

I must admit that many of the preliminary conclusions about cultural differences reached in the discussions above are based on rather weak evidences when judged by the rigorous standards of educational research. But the nature of the problem itself makes rigorous evidences tremendously difficult, if not impossible. It is difficult to establish causal relationship between culture and other variables, for culture is so all-embracing that it is practically not possible to "control for" culture. Nevertheless, I hope I have argued that there are sufficient findings of different mathematical learning behaviors among students from different cultures that suggest culture does make a difference.

As educators come to realize more and more that learning cannot be "culture-free," I hope this short paper has provided insight into how one particular culture affects mathematics learning. For those who have heard "rumors" about the superior performance of Chinese students in mathematics, this paper may throw some light on the trustworthiness or otherwise of these "rumors." But most important of all, studies of this sort should remind us to be more sensitive to the influence of culture on learning. In helping students learn mathematics, we must take into account the culture that our students come from and avoid the simple transplantation of educational theories. These studies should also forbid us to make superficial judgment on the performance of students from different cultures simply based on internationally administered tests. And hopefully, future international comparative tests will take more into account the different contexts of the participating countries in designing the tests and in interpreting the results.

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Mathematical and Commentary Writing: Vehicles for Student Reflection and Empowerment

Martin R. Hoffman and Arthur B. Powell

In this paper, we discuss the use of writing activities in mathematical curricula as a vehicle for student reflection and empowerment. The activities described have been used with underprepared mathematics students in university settings. Due to their previous scholastic experiences, these students have become estranged from mathematics. Consequences of this estrangement include poor performance on standardized examinations and the belief that mathematics is not a thoughtful activity. This belief and the companion view of mathematics as a static collection of facts which must be memorized contribute to their poor performance and feeling of alienation from mathematics. The writing activities discussed below are intended to challenge this belief; they are designed to promote and increase students' awareness of and facility in mathematics as a way of thinking.

In mathematics classes, writing is usually restricted to copying the problems and solutions of others and to providing short numerical or written responses to questions posed by presumed authorities—textbooks and instructors. The purpose of these writing activities is to provide a basis for external evaluation of students' learning. We have found broader purposes for writing which include providing opportunities for students to reflect on their understanding and supporting their attempts to negotiate meaning in mathematics. Ultimately, these activities are designed to empower students as learners capable of understanding and generating mathematics. We conceive of writing activities which accomplish these ends to fall into two categories: mathematical writing and commentary writing. Space allows only brief descriptions of these activities.

By mathematical writing, we mean the writing of meaningful mathematical statements which goes beyond the answering of closed-ended questions which require only a single answer. The emphasis is on self-expression in mathematical notation. The stimulus for these expressions are often open-ended situations and tasks, some of which are created by students themselves. For example, in one writing activity, known as "Question the Answer" (Gattegno & Hoffman, 1976) or "Reverse Problem Solving" (Boisset, 1985), the instructor supplies the 'answer' and the students write problems for which the given answer is appropriate. Thus, instead of

$\frac{19}{23} + \frac{17}{23} =$ or

$2x \cdot 4x^2 =$,

one might ask for problems whose answers are

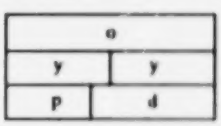
$\frac{36}{23}$ or $6x^3$.

Similarly, students can be asked to create a word problem which corresponds to a given equation, for example:

$0.10(6000 - x) + 0.0825x = 0.09(6000)$.

The variety of answers will not only reveal the students' computational facility, but will also indicate their level of understanding of the operations and relationships involved. These types of writing activities often lead to explorations not anticipated by the instructor. Traditional curricula present few situations where this type of activity cannot be used.

A second type of mathematical writing can be generated in conjunction with manipulative materials, such as Cuisenaire rods (Gattegno, 1971) or geoboards. For example, the rod pattern



can yield:

$5 + 5 = 10$, $5 + 5 = 6 + 4$, $10 - 4 = 6$,

$5 = \frac{1}{2} \times 10$ $2 \times 5 = 10$, $10 : 5 = 2$

$\frac{1}{2} \times 10 < 10 - 4$, and so on.

The mathematical richness of such activities when compared to traditional textbook exercises is apparent.

Restricted domain activities are a third type of mathematical writing that we use. The restrictions determine the focus of the activity, yet such activities are characterized by virtually unlimited possibilities of expression. An example of this type of activity is the following: Create expressions for the integers from 1 to 25 using only the signs 2, 3, 5, x,—In this case the focus is on multiplication, subtraction, exponentiation, and the order of operations conventions. We have found the following to be an especially well-received and challenging exercise for our students: Find expressions for as many integers as possible from 1 to 100 using the integers 1, 2, 3, and 4 each exactly once in each expression. Endless variations of

these and other open-ended mathematical writing activities await an instructor willing to yield some control of the learning environment.

Many underprepared mathematics students conceive of mathematical notation as consisting of meaningless signs that are manipulated according to prescribed rules. In the activities described above, students have opportunities to endow the signs that they write with meaning and, consequently, their awareness of these signs as the conveyers of meaning is promoted. This awareness is a necessary requisite for students to operate fluently and meaningfully with mathematical notation. As underprepared students begin to develop such facilities, they become empowered with an ability that is rarely the focus of traditional mathematics instruction.

Another empowering pedagogical technique, for both students and instructors, is commentary writing. We use the term commentary writing to denote activities in which students write in prose about their learning of mathematics. These activities encourage and promote students' reflection on their doing and thinking about mathematics and on their feelings in relation to mathematics. To engage students in reflecting on their mathematical experiences, we have used a number of different writing activities; among them are free writing, weekly journals or learning logs and multiple-entry logs.

Based on our experience of the last several years, we have noticed that the stylistic range of students' writings can be divided into two main categories, non-personal writing and personal writing. Within each, the content varies from commentary about mathematics, to feelings about it, to issues of learning and studying, to miscellaneous seemingly unrelated topics.

By non-personal writing, we mean writing which is a report, a narrative description of events which appear to be external to the writer. Writing of this type tends to describe classroom activity as if the writer were preparing a lesson plan of which the following is an example.

Today in math class we learned about factorials. The majority of the students understand the factorials.

Another type of writing in this category describes perceived feelings of others toward classroom activities.

The small groups that you formed in our class are something that really helps us do our work better. The homework also helps us do our work better and helps the student to develop their habits of studying.

These examples are typical of non-personal writing. They indicate a lack of engagement and commitment to the activities of the course, or at least to the writing activity. They appear to be marginally beneficial as a learning vehicle.

In the second category, personal writing, the writer is clearly presenting his or her personal views. Some examples, taken from journals, are the following:

(1) $x^2 + 11x + 35$ is not factorable because the number 11 is an impediment for this polynomial to be factorable. In this case, there is no number whose addition is 11 and whose multiplication is 35. If instead of 11, it was 12, this polynomial would be factorable.

(2) Today I managed to get an emergency loan for my books. Then I ran into a guy that I met at registration and finally I paid my bill.

(3) It was like a counting chart, but it includes negative numbers. There are plenty of patterns. Patterns going horizontally, vertically and diagonally.

(4) Major problems! I have no idea what to do. I need to know the rules ... It's just how can you subtract 3 + (–5). It's addition. Why did we subtract? Why does (–5) – (–3) = (–2) and (–3) – (–5) = 2? Why the difference in answers—same problem ... Also problems 8a and 8p. I want them answered before I go crazy.

Excerpts 2 and 3 above are descriptive, non-reflective in nature; the former focuses on issues external to the course, while the latter, although unelaborated, has a mathematical focus that could be developed into a more useful learning exercise. In contrast, excerpts 1 and 4 are more reflective and thus more useful as learning tools. In excerpt 4, one senses emotional tension as the student raises focused questions and indicates a desire for a reasoned response. Whereas in excerpt 1, the student provides analytical responses to the implied question. These excerpts only begin to illustrate the variety of responses that this activity can generate. Our objective is to promote personal, reflective writing, in which the content is mathematics and students' affective responses to it, as it best supports mathematical thinking.

For promoting reflection in writing, another useful pedagogical tool is, what we call, multiple-entry logs. These are created by dividing a sheet of loose-leaf paper width-wise into three equal sections. Into the left-hand column, students copy portions of "text" which particularly interest them. In the middle column, they write a comment, an evaluation, a summary or any other type of reaction to their "text" selection. The word "text" is used to mean words, written or spoken, mathematical symbols and mathematical problems. At regular intervals, students are requested to review and reflect on previous logs and, in the right-hand column, to update their reflections on the original "text" selections. The three excerpts below are from the multiple-entry log of one student.

Text	Comments	Reflections
I'm having difficulty with the concept of $x^2 = x \cdot x$, and x^2 and x being unlike terms.	This doesn't seem logical to me. It seems to be one of those rules in math that just "is." If x is a variable and represents ... never mind ... I think I understand. The term $3^2 = 3 \times 3$ and the term $3 = 3 \times 1$. Right? [Instructor's query: Would you explain further how your explanation has answered the question for you.]	x is a term and x^2 is a separate term. x by itself equals x. x^2 equals $x \cdot x$ or x times itself.

The above excerpt illustrates several interesting aspects of the types of writing students produce using multiple-entry logs. First, the entries in the left-hand and middle columns are examples of personal writing which is reflective and analytical in character. After stating the source of the conceptual difficulty, the student then discusses how the two statements juxtaposed appear to be illogical.[1] The student further implies that if they are considered logically consistent, then this simply confirms her view that mathematics "just is"; that it is a given body of immutable, external truths with little connection to one's sense of the world. The tone of this first sentence indicates an affective response to this piece of mathematics. The student then interrupts to announce a conceptual breakthrough and describes it by specializing the statements, using the number 3. The act of writing reflectively afforded the student an opportunity to consider these mathematical statements in some depth.

Other aspects of students' writing are embodied in the third excerpt from this multiple-entry log. It was prompted by the instructor's query. Unlike the second excerpt, the third does not contain affective statements, and the writer is removed from these statements. The function of these statements is to inform the reader how to understand and interpret the statements "$x^2 = x \cdot x$, and x^2 and x are unlike terms." In contrast to the specialized example and the expressive function of the second excerpt, the third is transactional[2] and contains general statements about mathematical concepts. These two features of the third excerpt indicate a feeling of confidence in the veracity of the insights held by the student. In general, movement along the expressive-transactional continuum, like that displayed in this excerpt, is indicative of students' increased confidence in their understanding of a particular concept or in their improved ability to perform mechanical manipulations.

Writing in the mathematics classroom, as we have described it in this paper and elsewhere (Powell, 1986; Powell & López, 1989), is used primarily as a means for students to learn mathematics and about themselves as mathematics learners, and not just as a means to measure information acquisition. Students write to articulate their beliefs about the nature of mathematical knowledge and their emotional responses to the mathematics they are doing; to construct and negotiate meaning; and to reflect on and monitor their learning and affectivity. Writing is an externalization of thought, less transient than memory, and less ephemeral than speech. Because of these features of writing, it is a visible vehicle for students to examine their conceptualizations and to revise them in light of further or contradictory evidence.

Using writing as a support for mathematics learning has benefits for both students and instructors. It allows for instructors to "hear" from each student and for students to know that their concerns and ideas will be heard. The personal communication established between instructors and students also serves to alleviate some anxieties that students have toward mathematics. Furthermore, students' writings reveal what they understand and do not yet understand. In writing, students reveal their misconceptions, variant conceptualizations, beliefs, and patterns of thought. This record provides evidence to both students and instructors of areas which need further development and interests which can be explored more fully.

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[1] From a conversation that one author had with this student, it is known that her use of the word "this" refers to the two statements considered together.

[2] The terms 'expressive' and 'transactional' are used here in the sense that Britton et al. (1975, pp. 88–90) have provided: Transactional writing uses language "to get things done: to inform people (telling them what they need or want to know or what we think they ought to know), to advise or persuade or instruct people." It is used whenever an "accurate and specific reference to what is known about reality" is needed. Expressive writing is "'thinking aloud on paper.' It has the function of revealing the speaker, verbalizing his consciousness ... submits itself to the free flow of ideas and feelings ..."

Are Girls Underprivileged Around the World?

The issue of girls' underachievement in mathematics has been discussed for some years. In this topic, special consideration was given to the social context surrounding the relation of girls to mathematics. Gila Hanna, Canada, reported on the results of the mathematics achievement of girls and boys in 20 countries, using the data collected by SIMS, which show that in the majority of the countries girls are not underprivileged in relation to their achievement in mathematics in grade 8. This changes in grade 12, however, and she compared attitudes in the countries in which significant sex differences were observed to attitudes in countries in which such differences were non-existent to generate interesting hypotheses about the factors responsible for girls' inferior or superior achievement in mathematics. Erika Kuendiger, Canada, presents data on the attitude scale "Gender Stereotyping" of the SIMS study and discusses that in light of differences in the general learning environment. Frank J. Swetz, USA, reviews findings from non-Western societies which indicate that mathematical abilities and attitudes are culturally related and—as in the reported case of two Malaysian studies—that sex-related differences may contrast sharply with opinions popularly held in the West.

Girls and Boys About Equal in Mathematics Achievement in Eighth Grade: Results From Twenty Countries

Gila Hanna

The purpose of this paper is to investigate gender differences in mathematics achievement among 8th graders, making use of the data of the Second International Mathematics Study (SIMS) conducted by the International Association for the Evaluation of Educational Achievement (IEA). In IEA terms, the 8th grade in the United States and Canada is called Population A, the grade in which the modal student age lies between 13.0 and 13.11 years by the middle of the school year.

The study used a very large random sample stratified by region and school type in each of the 20 participating countries. Due to the very large scale of this study, precise and generalizable conclusions can be drawn about all the aspects of the teaching and the learning of mathematics that were measured, and certainly about the scope of gender differences.

The IEA survey sought detailed information from each of the participating countries on three interrelated aspects of mathematics teaching: the intended curriculum, the implemented curriculum, and the attained curriculum. Extensive achievement tests were designed to measure students' knowledge and skills in five areas of mathematics deemed important and appropriate: arithmetic, algebra, geometry, measurement, and statistics.

The Data

The 20 countries participating in the study formed two groups. The first group consisted of the 7 countries that took part in the longitudinal version (a pretest at the beginning of grade 8 and a posttest at the end of that year), while the second consisted of the 13 countries that took part in the cross-sectional version (tested once only).

The present analyses are based on both the cross-sectional and the longitudinal data. In the cross-sectional part of the study, there were 175 items, 40 items in the Core test and 33 or 34 items in each of four Rotated forms, while in the longitudinal part, there were 180 items, 40 in the Core test and 35 in each of the four Rotated forms.

Procedure

Every response to an item was coded into one of three categories: correct, wrong, or item omitted. Within each country the achievement results were obtained as follows: For each item the percentage of correct responses (p-values) was calculated separately for boys and for girls, with the student as the unit of analysis. Mean p-values were then obtained for each sex separately and for each country by averaging the p-values for the individual items in that country. Finally, the mean p-values for subtest, country, and sex were averaged to yield an overall (for all the countries) average mean p-value for each sex for each of the subtests, with the item as the unit of analysis.

The analyses focused on:

(1) Overall differences between boys and girls by subtest for all countries taken together.
(2) Overall differences by subtest and item difficulty.
(3) Sex differences within and between countries (multivariate analyses of variance and paired t-tests were used).

Results

Table 1 shows the number of girls and boys in each participating country as well as the mean p-values and standard deviations for the algebra subtest.

Table 1
Descriptive Data on the Algebra Subtest

	Girls		Boys	
	N	Mean (sd)	N	Mean (sd)
Belgium Flemish	718	61 (14)	652	56 (13)
Belgium French	874	57 (16)	1001	53 (16)
British Columbia	1080	52 (18)	1066	51 (18)
Ontario	2401	44 (16)	2421	44 (16)
England	1338	43 (13)	1247	42 (13)
Finland	2091	46 (17)	2303	41 (15)
France	4629	51 (15)	3586	53 (15)
Hong Kong	2726	44 (15)	2822	44 (14)
Hungary	908	51 (19)	844	47 (16)
Israel	1651	48 (16)	1711	50 (16)
Japan	3924	61 (17)	4167	60 (15)
Luxembourg	1016	36 (18)	989	38 (15)
the Netherlands	2667	49 (17)	2769	50 (17)
New Zealand	2574	39 (13)	2629	38 (12)
Nigeria	389	30 (17)	1040	34 (17)
Scotland	627	45 (16)	729	44 (14)
Swaziland	485	26 (12)	414	26 (12)
Sweden	1660	33 (16)	1830	32 (14)
Thailand	1833	39 (16)	1988	37 (14)
USA	3452	43 (14)	3202	42 (13)
Total	37043		37410	
Percent	49.75		50.25	

Mean p-values and standard deviations (in brackets) by gender for the 20 countries taken together are as follows:

	Girls	Boys
(1) Arithmetic	50 (15)	50 (13)
(2) Algebra	45 (13)	45 (12)
(3) Geometry	42 (15)	44 (15)
(4) Statistics	55 (17)	55 (16)
(5) Measurement	49 (20)	51 (18)

The major results are the following:

Arithmetic: The median of mean differences between girls and boys (defined as girls' percent correct responses minus boys' percent correct responses) in the arithmetic subtest (62 items) was zero. The distribution of the differences is shown in the boxplot: the median difference is zero, the range is -9 to +8, and the distribution is more or less uniform across that range. Thus, half the questions favored boys while another half favored girls.

Algebra: The boxplot displaying the differences for the 42 items reveals that for 41 items the differences ranged from -3 to +4 with a median of +1, while one item was an outlier with a difference of -7. Girls had a higher success rate than boys on 50% of the items of this subtest.

Geometry: In geometry the picture is slightly different: the boys' success rates were higher than the girls' for 75% of the items. As the boxplot shows, there was a difference of 21 percentage points on one of the items in that subtest. The girls' performance was higher on one quarter of the items.

Statistics: On the 18-item statistics subtest girls did slightly better than boys on 50% of the items. The boxplot shows that the differences between the sexes ranged from -4 to +3 percentage points.

Measurement: Boys did better than girls on the measurement subtest. The differences ranged from -9% to +5%. The boys' performance was higher on about 70% of the items.

Overall Differences by Subtest and Item Difficulty

A number of points may be of interest:

(a) In arithmetic, differences between boys and girls tended to be in the boys' favor for the more difficult items (index of difficulty up to 35%), and in the girls' favor for the less difficult ones (index above 65%).

(b) In algebra, the only item that showed a difference exceeding 5% is in the 45% level of difficulty. The differences in the girls' favor are spread over the entire range of item difficulty, as are those in the boys' favor.

(c) In geometry, boys did better than girls on the majority of the items over the entire range of difficulty; when results were better for girls, they were also spread out over the entire range. The item on which boys were considerably more successful than girls (the difference being 21%) was not the most difficult; its average rate of success was 59%.

(d) Girls tended to do better than boys on the easier items in the statistics subtest. The range of difficulty for the items on which the boys were more successful was from 22 to 60, while girls were more successful on items in the difficulty range from 45 to 85.

(e) In measurement there is a clear pattern of greater success rate for boys on the more difficult items. Boys did better than girls on all the items in difficulty range from 18 to 45. For the items in the difficulty rate of 46 or more, there is no clear pattern in the distribution of p-values.

Differences Within and Between Countries

For each country the mean percent age of correct responses for girls was compared to that for bc ys, using the paired t-test with the item as the unit of analysis. Statistical significance in that context means that if these items were a random sample from a large set of items, then the average difference between the sexes for that set of items is not zero. (Because there are 20 separate t-test analyses, the level of significance was set at .005.)

The results are presented in Table 2. A positive difference in mean percent correct responses represents

a higher mean percent for girls, and a negative difference a higher mean for boys; a dash (–) indicates that the difference was not statistically significant.

The countries in the two groups (7 in the longitudinal group and 13 in the cross-sectional) are in order, from countries for which there were more subtests with differences in the girls' favor (top) to countries where there were more differences in the boys' favor (bottom). Likewise, the five subtests are in order from left (most favorable to girls) to right (least favorable to girls). Thus, the first subtest is algebra (since this is a subtest in which the girls did better in 5 countries), followed by arithmetic (in which girls did better in 3 countries), statistics, measurement, and, finally, geometry (in which boys outperformed girls in 10 countries).

As Table 2 shows, most of the differences between girls and boys did not reach statistical significance at the .005 level. Moreover, the differences that did reach statistical significance were not large; they ranged from +5 to –7. Looking at each subtest separately it appears that for two of the five topics, measurement and geometry, the significant differences occurred consistently in the boys' favor: In 7 of the 20 countries boys had higher p-values and in 10 countries boys had higher p-values in measurement and geometry respectively.

The pattern of results by country indicates that relative to boys from the same country, girls were more successful in Belgium Flemish, Thailand, Belgium French, Finland, and Hungary, and least so in France, Nigeria, Israel, and the Netherlands.

Table 2
Mean Percent Differences of Correct Responses Reaching Statistical Significance at the .005 Level, by Country and Subtest

	Alg (32)	Arith (62)	Stats (18)	Meas (26)	Geom (42)
Belgium Flemish	+5	+4	–	–	–
Thailand	+2	+2	–	–	–
British Columbia	–	–	–	–	–2
USA	–	–	–	–2	–2
Ontario	–	–	–	–3	–2
New Zealand	–	–2	–	–3	–3
France	–2	–4	–3	–5	–6
	Alg (40)	**Arith (46)**	**Stats (18)**	**Meas (24)**	**Geom (47)**
Belgium French	+4	+2	–	–	–
Finland	+5	–	+4	–	–
Hungary	+4	–	–	–	–
England	–	–	–	–	–
Japan	–	–	–	–	–
Scotland	–	–	–	–	–
Swaziland	–	–	–	–	–
Sweden	–	–	–	–	–
Hong Kong	–	–	–	–	–2
Luxembourg	–	–4	–	–	–6
Nigeria	–4	–3	–	–4	–3
Israel	–2	–3	–5	–6	–5
the Netherlands	–1	–4	–3	–7	–5

Discussion

As mentioned earlier, the breadth of this survey makes it possible to draw precise and generalizable conclusions concerning sex differences and differences among countries in mathematics achievement (with an experimental study, it would not have been possible to draw such conclusions, due to the limited sizes of samples).

Following is a summary of the findings:

(1) Differences among countries were greater than sex differences.
(2) Overall, no sex differences emerged in arithmetic, in algebra, and in statistics.
(3) Overall differences were observed in geometry and in measurement.
(4) In five countries, sex-related differences favored girls in one or two subtests.
(5) In five countries, no sex-related differences were observed.
(6) In five countries, boys outperformed girls in one or two subtests.
(7) In five countries, boys were superior to girls in one to three subtests.

The purpose of this study was to determine if, among 13-year-olds, there are important sex differences in mathematics achievement. The results suggest that at that age such differences vary from country to country and that they are at most very small. Since it is very unlikely that biological differences between the sexes vary from one country to another, the SIMS data tend to contradict those theories that attempt to explain boys' superiority in mathematics on the basis of biological differences.

In fact, a male advantage is non-existent in many countries. When it is observed, particularly in geometry and in measurement, it is in the mathematics subjects which were least taught in class (as the data on the attained curriculum show). It appears that girls and boys assimilate the subject matter taught in class equally well: arithmetic, algebra, and statistics. Thus, it seems reasonable to conclude that sex differences at that age, when they exist, are due to out-of-class experiences and to psychosocial processes rather than to biological differences.

In order to determine which psychosocial factors contribute to girls' lower mathematics achievement (in certain countries), one would have to analyze data from all the countries on the parents' and teachers' attitudes towards mathematics learning by girls and boys and on the students' own attitudes towards this subject. Analyses along these lines could further our understanding of the impact of the socio-cultural models transmitted by the environment on the achievement of students of both sexes.

Such attitudinal data were collected by the research teams of the Second International Mathematics Study, in fact, and are available for further research. Hence, it is quite feasible to compare attitudes in the countries in which significant sex differences were observed to attitudes in countries in which sex differences were non-existent. Such comparisons could generate inter-

esting hypotheses about the factors responsible for girls' inferior (or superior) achievement in mathematics. Furthermore, these data could be useful to educators interested in improving girls' achievement in mathematics and in related subjects.

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Mathematics—A Male Subject?!

Erika Kuendiger

The perception that mathematics is a subject, in which mostly men are interested, and in which they generally perform better than women, plays a major role when gender-related differences in achievement and career choices are investigated.

Depending on the theoretical framework used for explaining gender differences, the conceptional meaning of the above perception changes.

Researchers who look for genetically related factors to explain achievement differences consider the above perception as mirroring the assumed higher potentials of men. Thus, with regard to their research paradigm, the perception that mathematics is a "male subject" is considered a logical consequence. Its existence is regarded as an additional piece of evidence.

In contrast to the above stands the view that mathematics-related differences favoring men are due to differentiated learning processes. Researchers using this conceptual framework, explicitly or implicitly, assume that perceptions like the above influence women to withdraw from mathematical activities, and consequently women do not use their potentials in the field of mathematics. Thus, the perception "mathematics—a male subject" is rather considered as causing gender differences than as mirroring a biological fact. To date different models are available that describe the complex relationships assumed to relate mathematical achievement, career choice, stereotyped perceptions and other variables (see, e.g., Eccles, 1986; Fennema, 1985; Schildkamp-Kuendiger, 1982).

In the Second International Mathematics Study (SIMS)—besides others—information about the cognitive and attitudinal learning outcomes of students was obtained. In the attitudinal questionnaire a scale called "Gender Stereotyping" was included. This scale is made up of four statements. Students were asked to indicate their opinion on 5-point scales, reaching from strongly disagree to strongly agree. The four statements are:

- Boys have more natural ability in mathematics than girls.
- Boys need to know more mathematics than girls.
- Men make better scientists and engineers than women.
- A woman needs a career just as much as a man does.

The answers were coded so that a higher score indicates a more favorable attitude, meaning that girls are as able as boys, and that they need mathematics and careers as much as boys, etc.

Although the scale "Gender Stereotyping" provides only limited information as to the degree students perceive mathematics as a male domain, the obtained data have the advantage that random samples stratified by region and school type were used in the participating countries.

Results of the following countries/provinces are presented: the two Canadian provinces of British Columbia (CAN BC) and Ontario (CAN ONT), Finland (FIN), Hong Kong (HK), Hungary (HUNG), Israel (ISR), Japan (JAP), New Zealand (NZ), Scotland (SCO), Sweden (SWE), Thailand (THAI), and the United States of America (USA).

For each of these countries information is available from two groups of students: Population A and Population B.

Population A (Pop A) can be described by the grade level in which the modal age of the students is 13; this is grade 8 for Canada and the USA.

Population B (Pop B) includes students of the last year of secondary school who specialized in mathematics. Only in Hungary no specialization at the end of high school has taken place, so that Pop B is drawn from all students attending this grade (for further information on the SIMS see the forthcoming IEA publication by Pergamon Books, London).

The graph shows arithmetic means for Pop A and Pop B of each country. The x-coordinate represents the mean of the girls ($\bar{x}_{female}$). To highlight differences between female and male students, the y-coordinate represents the difference between girls' and boys' means ($\bar{x}_{female} - \bar{x}_{male}$). If there were no difference be-

tween the average attitudes of girls and boys, a country would be represented by a point located an the x-axis. Moreover, the more favorable the girls' attitude, the bigger the x-coordinate (maximum value being 5, minimum value being 1).

The graph reveals that for all countries and both groups of students (Pop A and Pop B) the mean results for girls are bigger than 3, the middle position on the scale, indicating a favorable attitude. The most favorable attitude is found in Sweden at Pop B level, and the least favorable one in Japan at Pop A level.

In all countries the attitude of boys differs from that of girls. These differences always indicate that boys have a less favorable attitude. The smallest difference is found in Hungary at Pop B level, and the biggest differences are found in Ontario and the USA at Pop A level.

For all countries the results of this attitude scale change from Pop A to Pop B. Yet, as the graph indicates, there does not seem to be any regularity as to the direction of the change or its magnitude.

In summary the SIMS data show that students of different countries vary considerably in their response to the "Gender Stereotyping" scale. Girls in general seem to perceive mathematics less as a male domain than boys.

Further inspection of the data indicates that this attitude is not related to the amount of single sex schools a country has, and that there is no direct correspondence between gender-related achievement differences (Hanna & Kuendiger, 1986; Hanna, 1988) and these attitudinal differences.

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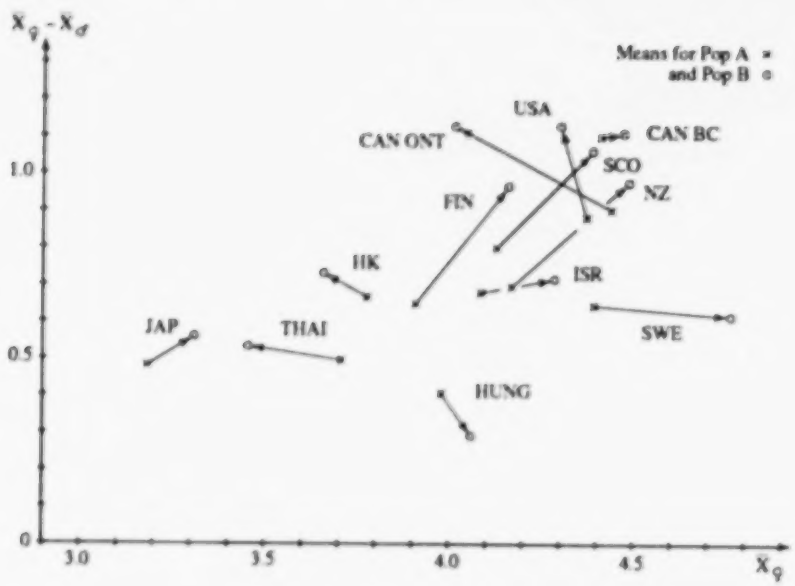
Gender Stereotyping



Cross-Cultural Insights Into the Question of Male Superiority in Mathematics: Some Malaysian Findings

Frank J. Swetz

Gender Differences in Mathematical Attitudes and Achievement: Some Malaysian Findings

In 1980, a survey of attitudes of Malaysian students towards mathematics, school, and school learning was undertaken (Swetz, Langgulung, & Johar). From selected urban and rural schools, the attitudes of 1,006 13-year-olds, 534 males and 472 females were solicited. The survey instrument was devised by randomly combining items from two distinct scales of the International Study on Mathematics Achievement: Attitude Towards the Place of Mathematics in Society, to be called "Mathsoc"; and Attitudes Towards School and School Learning, "Schola" (Husen, 1967). A Likert-type response was required to each item on the scale and attitude indexes were computed for each subject's responses. An attitude index close to 1 indicated a very positive attitude, and a value close to 5 a negative attitude.

Although Western-based research would indicate that females at this age should exhibit less positive attitudes towards mathematics than males, no significant differences for Mathsoc means between males and females was found ($Z = .965$, $p < .05$).

This finding naturally leads to the question of possible differences in mathematical performance between Malaysian males and females. A concurrent but independent investigation examined the problem-solving abilities, attitudes, and background of students preparing to enter the National University, Univerisiti Kebangsaan Malaysia, UKM. As part of this investigation a test of problem-solving ability was administered to 117 men in four classes served by four separate teachers and 82 women served by a team of two teachers. Problems were chosen on the basis of (1) they were most likely of a form not previously done by students and (2) the students would have sufficient mathematical knowledge to solve them. Each of the six problems to be solved was designed to probe certain aspects of the student's problem-solving ability, specifically:

Problem:

(1) Intended to test logical thinking, the use of diagrams and applications of modern math concepts, i.e., sets and Venn diagrams.
(2) Numerical pattern recognition and inductive thinking.
(3) Analytic thinking requiring a use of diagrams and applications of algebraic techniques.
(4) The interpretation of a physical process as the area under a curve which would be obtained through the use of integration. Students had been exposed to such models in their previous studies in mechanics. This question tested whether they could transfer their knowledge to a new situation.
(5) Required the use of geometric insight and application.
(6) Intended to test synthetic and analytic thinking in a geometric setting. Diagrams had to be employed.

A one-hour time was allowed for the test and each problem was worth 10 points in the scoring scheme.

UKM matriculation students are 17 to 18 years old. According to the findings of Western-based surveys a and research, this is the age where large male-female, mathematical performance differences could be expected especially in problem-solving exercise. Yet the Malaysian population sampled, is amazingly homogeneous in attitudes and performance ($Z = .806$, $p < .05$). The profiles of difficulty for the problem-solving survey questions differed for males and females by one entry. In problems involving geometric insights and spatial relations, i.e., 3, 5, 6, there is either no significant difference in performance—problems 3 and 6, or females perform better than males—problem 5 ($Z = 2.194$, $p < .05$).

Discussion and Conclusions

It should be noted that in both Malaysian studies, as originally conceived and carried out, gender differences were not a factor of investigation but when extrapolated from the data, they produced interesting findings. Of particular note is the homogeneity of the male-female results for the UKM study. Three factors: attitudes, ability, and educational background, two of which were actually tested, i.e., attitudes and ability, would seem to account for this result. Attitudes and ability for the male-female populations are similar, what about educational background? Matriculation students are selected from all over Malaysia, and while not a completely random sample, they are certainly a mixed population coming from many different secondary schools. Schools in Malaysia follow rather inflexible curricula that dismiss personal choice and expose males and females, as long as they stay in school, to the same mathematical background. Matriculation classes, while taught by different teachers at different locations, also follow a strict syllabus set by UKM. Thus, the mathematical background of both sexes tested, for all practical purposes, was also the same. Fennema and Sherman (1977) have noted that one important factor that contributes to the deficiencies of female response and performance in mathematics in the United States is mathematical background. Malaysian findings would seem to support this claim as does some recent research in the U.S. (Stones, Beckmann, & Stephens, 1982). The institutional permissiveness by which females are allowed or prevented from pursuing mathematical studies is a societal decision and intrinsically reflects cultural expectations.

Maccoby in her work on sex-related differences (1966) notes that:

> Members of each sex are encouraged to, and become interested in and proficient at, the kinds of tasks that are most relevant to the

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roles they will find currently or are expected to fill in the future. According to this view, boys in high school forge ahead in math because they and their parents and teachers know they may become engineers or scientists; while girls know they are unlikely to need math in occupations they will take up when they leave school.

Her comments reflect Western experience. Malaysia is a traditional society moving towards modernization. Modern science and technology are just beginning to establish a presence in the country. Mathematics has not yet been culturally stereotyped as a male profession; thus, apparently there are no sex-related biases associated with it. Also in the traditional societies of Asia, Africa, and Latin America, sex-role modeling may be more conducive to familiarizing girls with mathematics and its uses than in the West. In the Third World, women are frequently small business entrepreneurs. Selling crafts and produce in the markets, they become accustomed to using weights and measures and undertaking calculations. In some of these societies, women manage the family accounts and actually own the family land. The weaving of geometric patterns, whether in rattan floor mats, baskets, or cloth, develops an appreciation for geometry in its forms. In non-Western societies, weaving is womens' work and part of a young girl's training. The Malaysian girls sampled appeared comfortable in problem-solving situations involving geometric manipulations and spatial visualization. Thus, it would seem that in Malaysia, and possibly other non-Western societies, cultural and societal constraints do not discourage women from participating in mathematics and may even perhaps encourage their participation.

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Societal Determinants of Learning

This topic concerns the influence which the children's society experts on their mathematics learning. Joan Bliss, Ruffina Guttierez, Vasilios Koulaidis, Jon Ogborn, and Haralambos N. Sakonidis, United Kingdom, have investigated a cross-cultural study in the United Kingdom, Greece, and Spain of children's ideas about what is really true in four curriculum subjects—mathematics, science, history, and religion—and report on their findings. Gustav Adolf Lörcher, Federal Republic of Germany, describes the specific problems in mathematics education which result in the fact that—both in developing as well as in industrialized countries—many children learn mathematics in a foreign language, and tries to offer some ways to deal with them. Ali Rejali, Islamic Republic of Iran, presents a study of the causes for which students are not interested in studying mathematics in Iran and some suggestions to solve the problem. Bernd Zimmermann, Federal Republic of Germany, demonstrates that "mathematics for all" and "mathematics for the gifted" are not only not exclusive alternatives, but the main issue "mathematics for—or better with—all" can be proliferated by experiences and research in the special area of mathematically gifted students.

A Cross-Cultural Study of Children's Ideas About *What is Really True* in Four Curricula Subjects: Science, Religion, History, and Mathematics

Joan Bliss, Ruffina Guttierez, Vasilios Koulaidis, Jon Ogborn, and Haralambos N. Sakonidis

Introduction

The present study, which is being carried out in Greece, Spain, and England stemmed from an earlier study carried out only in England (Bliss & Sakonidis, 1987). Most pupils have opinions about the nature of the different subjects they learn at school, and we might well ask about the origins of such opinions. One of the underlying assumptions of the study is that teachers' views of the subject and their instructional practice could significantly influence pupils' attitudes towards the subject. However, our first task was to look at the pupils' perceptions about *what is really true* in the four curricula areas mentioned.

Method

A questionnaire was first given out to children between the ages of 11 and 16 in secondary schools in England, and then to pupils of the same age in Spain and Greece. The pupils were all in secondary school with the exception of Greece, where children between 11 and 12 are in the last year of primary school. The questionnaire read as follows:

WHAT DO YOU THINK?

We would all like to know what is really true (well, most of us). What do you think about whether these subjects tell you things that are really true?

- what you learn about science,
- what you learn about religion,
- what you learn about history,
- what you learn about mathematics?

Say what you think here (choose one for each of the following subjects).

SCIENCE TRUE because ...
NOT REALLY TRUE because ...
CAN'T DECIDE because ...

Similarly for mathematics, religion, and history.

Results

From the results in Table 1, mathematics and science show similar trends in England, Spain, and Greece with the majority of children considering them both to be "true." History shows a different trend in the three countries with the majority of children in Spain and Greece judging the subject to be true, but this is not the case for England, where less than half the children judge it to be "true," with quite a substantial number of children finding it difficult to decide. With religion, the English pupils are extremely skeptical with only 15% considering it to be "true." A fairly substantial proportion of the Spanish pupils consider religion to be true (67%), although for the Spanish child religion is less "true" than history. The picture is reversed in Greece where a large proportion of pupils believe that what they are taught in religion is "true" (79%) and this is more so than in history for the Greek pupil.

For mathematics the "not true" judgments are roughly the same in all three countries and similarly for the "can't decide" judgments, whereas for science there are more English than Spanish or Greek pupils who "can't decide" whether or not the subject is true, although approximately the same very small percentage of pupils in all three countries believe it "not to be really true."

Table 1
Judgments About *What is Really True*

		Mathematics %			Science %			History %			Religion %		
	Σ	NRT	CD	T	NRT	CD	T	NRT	CD	T	NRT	CD	T
England years 1–4 11–15	151	3	14	83	4	28	78	20	37	43	46	44	15
Spain years 1–5 11–16	118	3	11	86	0	7	93	3	14	83	3	30	67
Greece years 1–3 11–14	418	5	15	80	2	7	91	10	17	73	5	16	79

Comparisons Between Subjects

In the original study in England, in order to understand better the relationships between subjects, six comparisons were made between: mathematics and science, mathematics and history, mathematics and religion, science and history, science and religion, history and religion. The counting was as follows: X truer than Y (or equal) gives three possible categories:
X truer than Y ... category of marks of 1 for X,
X less true than Y ... category of marks of 1 for Y.
If both subjects are equally true, then a score of one is given to the equality category.
"Can't decide" wins over "not really true."

Since in Spain and Greece all four curricula subjects are judged by a fairly substantial number of pupils to be "true," comparisons between subjects are less revealing. However, the differences between the three countries for mathematics in relation to other subjects are of interest, and are summarized in Table 2.

From Table 2 it can be seen clearly that in the comparison between mathematics and history, approximately 2/3 of both the Spanish and the Greek children give these two subjects equal status, whereas for the English pupils mathematics is 'more true' than history 50% of the time. When the comparison between mathematics and religion is made, the Greek pupils differ from pupils in the other two countries in

as much as they accord equal status to mathematics and religion about 2/3 of the time. For the Spanish children this drops to only 56% of the comparisons, and with the English pupils mathematics is 'more true' than religion in 84% of the comparisons. The comparison between mathematics and science is the area in which the pupils' judgments from the three countries are the closest, with the majority of children from all countries believing these two subjects to be 'equally true.'

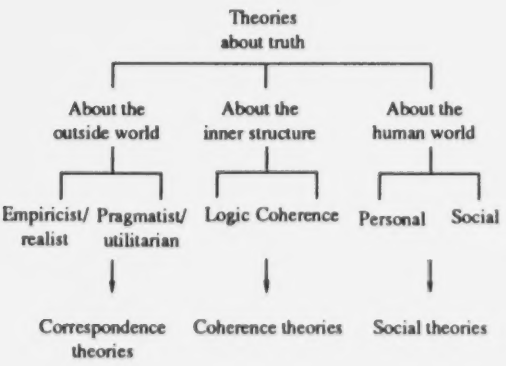
Table 2
Comparisons Between Mathematics and Science, Religion, and History

		Math/Science %			Math/Religion %			Math/History %		
	Σ	M>S	=	S>M	M>R	=	R>M	M>H	=	H>M
England	151	17	69	14	84	13	3	50	41	9
Spain	118	4	86	10	33	56	11	18	68	14
Greece	418	5	78	17	17	68	16	18	66	16

Explanations for Judgments

In order to develop categories for the pupils' explanations, we are considering three general types of theories about 'truth,' correspondence, coherence, and social theories. These are set out in the network below.

The network is still being developed and will consider issues such as authority, conformity, etc. within social theories, and finer distinctions will also be developed in the set of categories to do with coherence. For the moment the explanations given for mathematics and science have been looked at in some detail, and the picture follows.



For mathematics the predominant explanation for its 'truth' has to do with its coherence in all three countries, although with fairly substantial differences between Greece, England, and Spain. However, in Greece a fairly substantial number of children view the 'truth' of the subject in utilitarian terms, that is, in terms of their careers and the sorts of jobs they will get. This type of reason is less predominant in England and Spain. In Spain and England—a little more than in Greece—explanations in terms of social reasons play an important role, and in most cases this has to do with the authority of the people creating or teaching the subject.

For science, the profile is very different to that of mathematics, with the predominant explanation for its 'truth' having to do with empiricism, that is, that evidence can be found through experiments and this for all three countries. The English also see that science is logical as well as empirical. As with mathematics, for the Greek pupils utilitarian reasons play an important role. For the Spanish children social or utilitarian reasons account for just over a third of the responses.

Conclusions

There are some interesting differences between the three countries in terms of the judgments about the 'truth' of different curricula subjects, with the English tending to be extremely skeptical about religion and fairly doubtful about history. While the Spanish and Greek pupils tend to consider all four curricula subjects as reasonably 'true,' the Spanish are a little more skeptical about religion whereas the Greek pupils have their doubts about history. The explanations reveal fairly substantial differences between the three countries, with the Greek pupil seeing subjects very much in utilitarian terms, and the Spanish children being more concerned with social or authority reasons. There is a common thread of coherence reasons for mathematics and empirical reasons for science in all three countries. These results are, of course, only provisional, since the study is ongoing. We hope it will be completed within the next year.

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Learning Mathematics in a Foreign Language

Gustav Adolf Lörcher

In my contribution, I will try to throw some light on the question "How can failure in math education be reduced?" In order to find out what can be done, we need more insight into the questions "Which children fail?", "Where do they fail?", "Why do they fail?".

My contribution is based on work in teacher education during the last ten years including:

- about 1,000 hours of remedial teaching for individual children from 6 to 20 (both German and immigrant) with special problems in mathematics education;
- studies of methods, curricula, syllabuses, textbooks in mathematics education within West Germany and the main home countries of immigrant children (Turkey, Yugoslavia, Italy, Greece, Spain);
- empirical research: investigation of the written work in the main content areas of mathematics with about 1,000 10-year-old and about 3,000 15-year-old children (1/5 immigrants), analyzing also the methods children used and the errors they made.

Mathematics: A Foreign Language

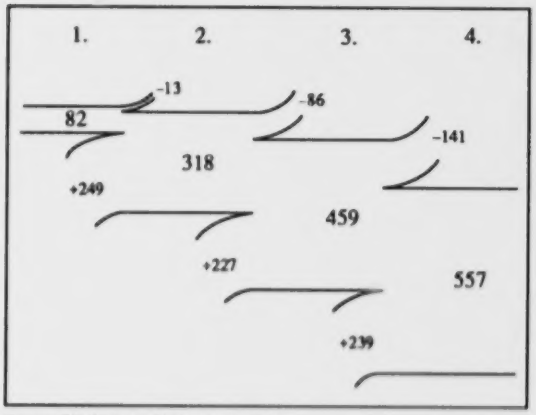
Looking at mathematics as a language makes it clearer that there is not such a strong hierarchy as often claimed; mathematical concepts form a kind of network, offering many different approaches and ways, that can be made more or less accessible or inaccessible by the experiences the teacher is referring to, by the rules he formulates, and by the vocabulary he is using.

In its *semantics*, *syntax*, and *vocabulary* mathematics has many characteristics of a foreign language.

What makes sense *(semantics)* and what is mathematically important in a verbal problem in order to solve it is often very different from what is important in real life; so for many children math is not based on or in accordance with their experience, but is like a dead language of a foreign culture.

The rules *(syntax)* are often hidden for the children, they seem arbitrary and more or less depending on the teacher's mood. The *vocabulary* is often enormous and not limited by didactical considerations. An investigation of the special vocabulary in a German mathematical textbook for the first four school years gave the following picture: The amount of the mathematical terminology is about as big as in a foreign language. About 1/3 of the words were out of use in the following year and half of the words were used only on one page in the textbook.

How Many Mathematical Terms in a Textbook Series: Grade 1–4



Source: Die Welt der Zahl - Neu [1975].

Success and Failure in Mathematics

Failure in math often leads not only to failure in school, but also to a life-long feeling of intellectual inferiority. In order to avoid this failure, children need success in math. Success and failure in math in German schools depend mostly on children's ability to solve test items.

Investigating the *solution process* in working out a problem can help to see the main steps and the main obstacles for a pupil trying to solve a problem more clearly.

In a research group we investigated the written work of a random sample of 1,000 15-year-old pupils each year during the last three years in the final mathematics examination of the "Hauptschule" (bottom half in the ability range). We found five main steps (and obstacles) in the solution process:

Step 1: starting the problem (13% failed tackling the problem, only 87% could continue to the next step);

Step 2: finding the right approach (20% of the pupils who had stated the problem used a wrong approach; so only 70% of all pupils reached the next step);

Step 3: completing the approach (10% of the remaining children missed, e.g., an operation);

Step 4: right calculation (14% of the remaining children made calculation errors);

Step 5: finding the right result (3% of the remaining children made minor errors like copying errors; so only about 50% succeeded in coming to a successful solution).

The main *obstacles* for all children seem to be:
- starting the work (semantical reasons);
- finding the right approach (semantical, syntactical, or vocabulary reasons);
- doing the right calculation (calculators were not allowed).

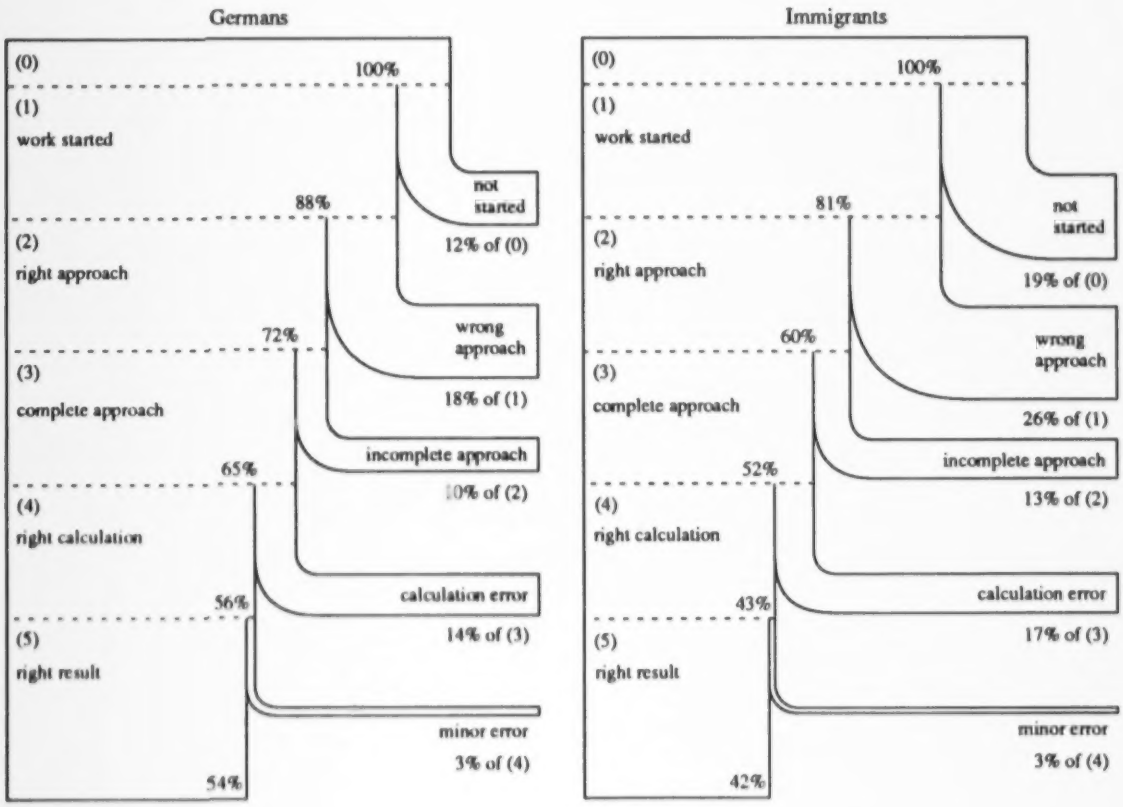
Reducing failure in math means concentrating on these three steps.

Additional Obstacles for Immigrants

In most industrialized countries immigrant children form a considerable part of the school population (in the FRG between 1/10 and 1/5). They are less successful in school. But not much is known if and how math contributes to this failure in school. In our research we compared success and failure of immigrant and German children during the solution process of the examination items:

The overall picture is: Immigrant children are a disadvantaged group. Their final success rate is about 20% lower than that of German pupils. In all three years the picture was nearly the same, though the sample was different.

Comparison of the Solution Process



A more detailed analysis of the errors pupils made during the solution process revealed: Immigrants failed more often than German children at the first three steps:
- starting the problem,
- finding the right approach, and
- completing it.

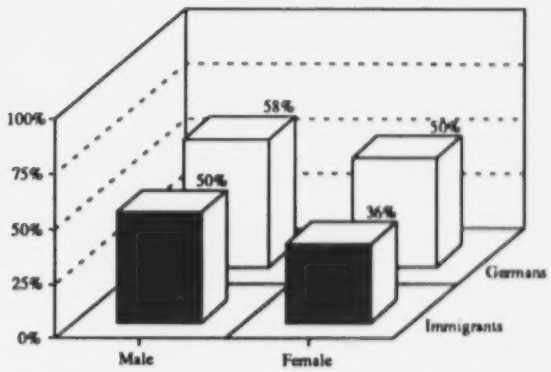
They had no increased calculation problems.

Additional Obstacles for Girls

A second disadvantaging factor in mathematics education is being a girl. The final success rate of German girls was 15% lower than that of German boys. The difference between immigrant sex groups was even bigger: Here girls' final success rates were about 25% lower than that of boys.

It turns out that *immigrant girls* are the group which is most disadvantaged (and perhaps most neglected) in mathematics education. They have additional problems not only in the first three steps of the solution process but also in calculation.

Success Rates



Discussion of the Results

Immigrant children have increased difficulties compared with German children. Their success rate is reduced by about 1/5. Their difficulties arise at the very beginning of the solution process: More often than German children they feel unable to even attempt to find a solution, and more often they cannot find the right approach (i.e., the right formula or the right transfer from given numbers to the formula or the right operation).

A similar pattern arises with *girls*. Their success rate is nearly 1/5 lower than that of boys. They too fail more often when trying to tackle the problem and to find the right approach, but they also have some additional difficulties in calculation (dealing with large numbers or decimals).

When both factors (different mother tongue and female gender) come together, the failure rate rises more than expected. Both disadvantaging factors seem even to reinforce each other. *Immigrant girls* seem to be an entirely disadvantaged and neglected group in mathematics education.

What are the *reasons* for these disadvantaging effects? One reason, which may explain some of immigrant children's additional difficulties, could be a *language* factor. Another reason, which may explain some of girls' additional difficulties, could be called *experience* factor.

In applications and geometry items it is often a male reality the examples are taken from, and boys who have got more experiences in these concepts and numbers may have considerable advantages. Lacking experience in the 'reality' of many test items may also explain why many immigrant children failed at special items.

In pure arithmetic and algebra items which neither depended on these experiences nor on language, differences in achievement were insignificant between (German) girls and boys and between (male) immigrants and Germans.

A third factor which could explain why minority groups around the world have greater difficulties in math may be called a *discourage* factor. The danger of inconsiderate arrogance (the knowing teacher behaves as if he is superior and gives the learning child a feeling of inferiority) is especially high in math and seems to have disastrous effects on all weaker groups.

Proposals

The results of our research cannot explain success or failure of single pupils. But they can give insight into the question of where it is possible to reduce failure—a failure that is most threatening for minority groups.

Many of these difficulties also arise with German children. It is not the kind but the amount of problems that is specific for immigrant children. Many difficulties usually hidden or overlooked turn up clearly in the context of immigrants' learning problems. Mathematics is a foreign language for all children, but a foreign language inside a foreign language for immigrants. That is why dealing with immigrants' math problems could make us more aware of all children's learning difficulties in mathematics education. To overcome some of these difficulties, long-term and short-term ideas are necessary.

In the long run, it is necessary to stop the total neglect of children's *mother tongue* in teaching mathematics. Immigrant children should have the right to learn in their own language. In addition, also their present and future life conditions should be taken into consideration in the classroom much more.

In the short run, teachers should try to *reduce language difficulties* by:
- using more non-verbal approaches,
- simplifying their vocabulary,
- trying to clarify what is really important, and
- giving a clear structure.

They should try to diagnose immigrant children's learning problems and give them special support. This support should include learning

- how to make use of tools (most of all *calculators*, but also measurement and drawing tools),
- how to understand the basic terminology and the language used in verbal problems.

Of great importance are special preparations for the examinations, which often are decisive for immigrant children's future lives. One of the most important ideas may be to find more ways for *encouraging* children in mathematics and giving them more confidence into their own capabilities.

Lack of Interest of Students for Studying Mathematics

Ali Rejali

There exists a world-wide lack of motivation for studying mathematics. In Iran, as the statistics show, each year, the number of high school students majoring in mathematics decreases. For example, the percentage of mathematics students in 1975 was 29. This figure fell to 12 in 1977 and came down to 7 in 1983. This decrease occurs more in the deprived provinces (Department of Mathematics, Research Organization of the Ministry of Education, Islamic Republic of Iran, 1988).

In 1983, an investigation committee was formed to seek the causes of lack of interest in mathematics. This committee, called CILIM (Committee for Investigation of the Causes of Lack of Interest in Mathematics) was organized by the Research Organization of Ministry of Education. The author, a member of CILIM, wrote a paper explaining the causes for the problem and making some suggestions for its solution (Rejali, 1983).

In the present article, the findings of CILIM are presented along with a summary of the conclusions obtained in questionnaires filled out by some sample students and their teachers from intermediate and high schools all over the country (Department of Mathematics, Research Organization of the Ministry of Education, Islamic Republic of Iran, 1988).

Causes

The most important cause for this lack of interest was found to be of social and economical background. Uncertainty about the future and the advantage of having a job in the medical profession are the main determining factors in not taking mathematics seriously. 25% of mathematics students from the above-mentioned sample wanted to continue their studies in medicine, whereas the overall sample of students believed that a job in the medical profession was the most important.

Here we will discus the educational causes: Fear of mathematics as being difficult, abstract, and easily forgetable, being unaware of its applications, and having better opportunities in other easier subjects left the students with no motivation at all for mathematics. Even students who have already overcome the above-mentioned problems are not satisfied with the available sources. According to a report (*Second international mathematical summary report for the United States*, 1985), lack of self-confidence in students for studying mathematics is a world-wide problem. However, attitudes towards mathematics differ in different societies. For example, it is reported that "most of the American students believe that mathematics helps them to think logically and is a good subject for creative people." The author has not seen this strong belief about mathematics in Iranian students, which is mostly due to the above-mentioned problems and also to the following.

Another important cause is lack of motivation in teachers. Unfortunately, most of the teachers are not interested in their jobs. (The author saw the same problem in France and the United States as well.) This may be due to several social and economical problems of teachers.

Also, lack of experienced and well-trained teachers is a problem. Most of the teachers questioned by CILIM said that there are few experienced, well-trained, and interested mathematics teachers in our society, and a good and effective inservice teacher training program does not exist.

In addition, the curricula have been changed without the necessary preparations. This and the lack of well-trained teachers, who are not able to keep their knowledge up-to-date, and who neither have the time nor the opportunity for further studies, have had terrible effects on the education of mathematics, which is a teacher-oriented subject.

Lack of logical and systematic relations between mathematics taught at different levels, lack of well-educated specialists in the field of mathematical education, the wrongly planned program for mathematics due to the great amount of material presented which is over and above the students' capacity of handling it, and also not presenting subjects such as geometry, number theory, and combinatorics are some of the other causes.

Another important cause is the unfamiliarity of the families with mathematics, its applications, and its role in the logical thinking. This is important because, for instance, 80% of the students asked by CILIM chose their field of study upon recommendation by their family.

Finally, 35% of the students did not choose mathematics because they were not interested in the subject, 25% because of their fear of mathematics, and 25% because there was no mathematics field in their area.

Suggestions

(1) Social and economical problems of mathematics and engineering graduates and teachers should be solved.
(2) The teachers' identity and dignity should be reestablished.
(3) Teachers should be accommodated with welfare, something they really deserve.
(4) More teacher centers should be established.
(5) More books and journals for teachers and students should be published.
(6) The teachers should be trained for their jobs.
(7) Inservice teachers' training programs should be improved.
(8) Mathematics should be popularized.
(9) Students should be helped to think mathematically. This can be done by putting more emphasis on subjects such as geometry, number theory, and combinatorics, and by dropping subjects that can be taught later.

The author welcomes suggestions on the part of the audience and the readers on this article.

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Mathematics for All *and* Teaching the Gifted

Bernd Zimmermann

The following remarks refer mainly to the situation in the Federal Republic of Germany. Our focus is on the teaching of mathematics in the 9-year elementary school ("Hauptschule").

There are several *constraints inherent in the society*, which have an impact on the mathematics learning of students:

(1) In West Germany there are *different school types*, which serve *different needs of the society*. In reality, students in the 9-year elementary school belong mainly to the "lower" class and can expect corresponding jobs after school (if they do not become unemployed).
(2) There is a high level of conformity pressure on the students (and on the teachers) as a result of the whole *examination system*. Especially the specific entrance examinations of vocational and trade organizations call for specific knowledge learned and stored in a specific way.
(3) The whole *testing and grading* system has a tremendous impact on the "real" educational objectives (but—unfortunately—not vice versa).

Some *consequences* of these constraints for the mathematics learning of students are as follows:

(a) Students as well as teachers have or are presented with, a rather restricted *conception of mathematics*, in which it is mainly conceptualized as a bag of tricks, rules, and formulae. Educational objectives for the "basics," sometimes condensed in so-called "foundation lists," often reinforce this very restricted picture of mathematics:

> A curriculum based on the "foundation list" is likely to fail to confront less successful pupils with experiences which demand more general, formal and powerful mathematical concepts and techniques. Not only will this inhibit their further mathematical development; it will deny them a broad understanding of the nature and context of mathematical activity. (Ruthven, 1987, p. 250)

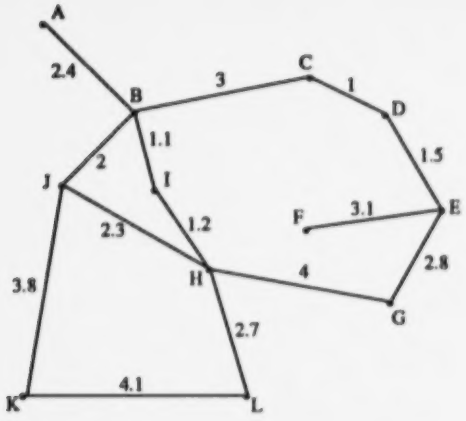
(b) This narrow and rigid picture of mathematics, as well as the examination and grading system, have serious consequences for the conception of *"mathematical giftedness."* At the very least, these two factors support the assumption that it is possible to differentiate "giftedness" for a well-defined subject from "very good" to "poor" in a linear manner. In this way, the possibility of the existence of different types of giftedness (cf., e.g., Krutetskii, 1976) is ignored, and there is a pandering to ability stereotyping (cf. (a)).
(c) Because of the aforementioned factors, there are fewer possibilities in the "traditional" mathematics lesson to meet the different needs of the students, who are mathematically gifted in different ways. Experience of success and resultant high motivation can be gained only by those students who fit or adjust best to these "boundary conditions."

The author enjoyed some very encouraging experiences with teachers and students of a 9-year elementary school, using a number of ideas and problems originally selected or created for a project to foster gifted students. These experiences demonstrate that this unsatisfactory situation can be changed!

Topics like "Tower of Hanoi," different versions of the game "Nim," searching for "minimal spanning trees," tessalations of the plane (cf., e.g., Kißwetter & Rosenkranz, 1982; Wagner & Zimmermann, 1986) were taught not only by the author, but also by teachers who never had a formal education in mathematics (except their own high school lessons).

To give a concrete impression, we refer in the following description of our experiences repeatedly to the following problem ("minimal spanning tree"):

> The following figure represents a road map (towns A–L, distances in miles). The roads are covered with snow. The road net is to be cleared in such a way, that all towns are connected and the total length of all roads to be cleared is a minimum.



We could make the following observations:

(1) There were so-called "poor" and unmotivated students who showed extraordinary *creativity*. For example, some discovered the recursive pattern for the minimal number of moves for the "Tower of Hanoi," several found the two main algorithms for the aforementioned street-net problem:
   (a) the "building up" algorithm: one can start in any town and must always look for the shortest road to a new town;
   (b) the "reduction algorithm": one looks for "loops" (closed paths) and one omits the longest road in this circle. One continues in this way until there are no more "loops."

   It took our "gifted" students in a Saturday-morning session more than half an hour to find these ideas, while three of our "poor" students presented these solutions after less than five minutes!

   These examples demonstrate that there is also a broad *potential of creativity* in so-called "poor" students, which should (and *can!*) be met better by suitable, non-routine problems, by more pithy mathematics, which belong to a *broader conception of mathematics!*

(2) These examples additionally demonstrate that "richer mathematics," which requires hardly any prerequisite knowledge, gives more possibilities for *differentiation*.

(3) We could register *success experiences* and *motivation* in students who were hitherto more frustrated by their mathematics lessons!

(4) The experiences with the street-net problem demonstrate that students can be motivated for and stimulated by the process of *mathematical modeling*. In this way, the students could apply or enrich their *heuristic competence* like "examine special cases"/"concept formation" (when analyzing "loops," cf. (b), or "progressing inductively," cf. (a)). Furthermore, more "nonlinear" *network thinking* is supported, which focuses on different solution processes and different relations between mathematical model and reality.

(5) In addition, it could be proved that there are teachers (non-experts in mathematics!), who are willing and able to enrich their picture of mathematics. Moreover, they started to change their grading system by relating it to this broader conception of mathematics (e.g., by shifting their focus more from "the right answer" to "good ideas and conjectures").

We had similar experiences in some classes in comprehensive schools in Hamburg when mathematics syllabus was revised according to the conception of mathematics presented above. Analogous studies are being carried out in some comprehensive schools in Helsinki, Finland (cf. contribution of Pehkonen on ICME).

Therefore, the main hypothesis, derived from these pilot studies, is as follows: "Mathematics for all" and "mathematics for the gifted" are not only not exclusive alternatives, but the main issue "mathematics for all" (better: "mathematics with all") can be proliferated by experiences and research in the special area of mathematically gifted students.

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The Social Arena of the Mathematics Classroom

The special context for learning which is the "mathematics classroom" has particularly important social features which were explored in this topic. Josette Adda, France, shows how the taboos, the forbidden acts, and compulsory rituals in the mathematics classroom create an artificial world, as far from the mathematical "world of ideas" as from the external "world of everyday life" and discusses why this sociological context is in favor of pupils from a high socio-cultural level. Tom Cooper, Australia, analyzes primary mathematics teaching in terms of its role in the social and political function of schooling in reproducing the present society. He demonstrates that mathematics teaching is to be seen to have a formidable role in reproducing crucial ideologies, a role made more powerful because of the notion of objectivity that surrounds mathematics itself and masks hegemonic functions. Andrea L. Petitto, USA, describes the structure of classroom discourse and dis-

cusses the significance of this kind of discourse for the difficult and often gradual process of disambiguating arithmetic procedures and linking them to children's own intuitive understanding of numbers. Terry Wood, USA, analyzes the evolving regularities or patterns identified in whole class interactions as the teacher and children discuss solutions to arithmetic activities; focusing on the implicit, taken-for-granted obligations that the teacher and students feel under in particular situations, and on their expectations from each other.

The Mathematics Classroom as a Microsociety

Josette Adda

Let us consider the title of our theme and the metaphor in it, "the social arena of the mathematics classroom": in that arena, are there wild beasts, is there a martyr, is there a tamer?

We will try to answer this by describing, in the manner of the ethnographer, the interactions between the personages and the discourses exchanged in the school institution and especially in the mathematics classroom. Ethnographers and ethnologists study a social group with its communication rules, its implicits, its presuppositions: ethnographers describe them, ethnologists try to understand and interpret them.[1]

We will speak of *normal*, standard classes in France (but the observations that I have made in other countries do not contradict what is said here), because we can note that the situation is not the same for experiments (especially when there are researchers looking at the classroom) or for individual lessons (the distinction was already explained by S. F. Lacroix in 1805 in "Essais sur l'enseignement en général et sur celui des mathématiques en particulier") and even with group lessons in nature classrooms (e.g., research by Y. Paquelier with classes at seaside or ski resorts) where the relations between pupils and teacher are very different and school activities become games. But we could say that the study of those privileged situations is not without interest when looking for a positive transformation of the present classroom.

The teacher enters into the arena with two missions: to *teach* mathematics and to *evaluate* pupils in order to orientate them. He knows that he will succeed in none of those missions if he is consumed by the "lions"; so he has to tame them, to dominate them, and, for that, he has two means at hand: *seduction* and *terror* which are the two usual arms of power in all societies, the two means of authority of chiefs ("carrot" and "stick"). The proportion of one in regard to the other is variable, for instance teachers without confidence in their pedagogical ability and/or in their mathematical proficiency are inclined to compensate their fear of being devoured by an excess of authoritarianism.

So, the *persuasion* of the pupil by his teacher has little to do with the conviction of the mathematician (see S. Lang, 1985 asking a young student if his agreement to a demonstration was "by conviction or by intimidation.")

The teacher not only has the authority of a chief, he also has that of a *sorcerer:* He is the one who knows, people trust him, people believe in what he asserts, and that is also a behavior opposite to the mathematical critical behavior: So, in the "new math" times, many pupils *repeated*, by intimidation, some activities with Venn diagrams without being discomfortable in front of many contradictions. I could quote aberrations pronounced by teachers and repeated without reaction by their class. By a behavior of parrots, students are able to accept and repeat any proposition asserted by the teacher. We have to note that even in a situation where the exchange is not only a dialog, this type of relation hierarchized by the knowledge still exists because a student may also repeat, without criticism, what is asserted by a schoolmate whom he considers to be superior to him (see, for instance, the cribbing of tasks). Everyone also knows how the "leader phenomenon" in group activities works.

On the contrary, the notion of *mathematical truth* is so perverted that, even when they produced a correct demonstration or a good calculation by themselves, the pupils felt it necessary to have the teacher's agreement to know what is true.

Actually, when pupils reason in the mathematics classroom, it is at a second level: They do not reason about mathematical objects and their properties but they argue, often by analogy, about what the teacher is looking for. This is most obvious when they have to answer to a question, to solve an exercise or a problem.

Everybody knows many examples of the famous perverse effect called "the captain's age" coming from the fact that children are raised to "understand" (sic!) that such and such a word, such and such a type of story will induce that operation (compared to the behavior of the well-known horse Hans raised to calculations ...). Walter Szetela asking the question "A construction truck can carry 18 m^3 of soil. How many truck loads are needed to carry 3,750 m^3 of soil?" remarked that 37% of grade 7 pupils made a division but "do not understand the requirement of a whole number answer."

We all know the classical exercise "Peter and James have the same amount of money. Peter gives 23 F to James. How much more does James have than Peter

[1] For this methodology of didactical research and especially the observer's paradox and necessity and limits of participant observation, see my communication in group T5: *Ethnology of math classrooms.*

now?"; there is no occurrence in this text of the "2" necessary for the multiplication needed and this exercise is always a trap in school although the question would seem obvious in a real situation. The true question in the pragmatics of school questioning is the metaquestion "What is the answer expected?" and the answer given is only a sham.

Actually, the students have to feel (and so do the "able" students and do not the students in failure) that the "trains," the "bath tubs," the "markets," and the "mammas" of those problems are also sham ones (the "kitsch" of school mathematics) and that it would be very dangerous to answer to school questions by referring to the price of the sweets bought this morning before coming to the school, for example.

Moreover, as in a politicized society where nobody dares to say that the king is naked, nobody would say in a math classroom that the captain's age has nothing to do with the shipload. What is said by the teacher is not to be doubted and though young children very often ask "why?" at home, the schooling has the effect to provoke the inhibition of that question. It is particularly grave to see this taboo in the mathematics classroom, where, on the contrary, the ideal should be to conduct students to that curiosity (even, for example, "why did I make that mistake?"; the custom of the schooling is such that each year I need a long time before students at the University dare ask that type of questions).

The *role* distribution is clear: in the classroom it is the teacher who asks questions (definitely not the students!) and this is linked to his *second function:* that of an evaluator, of a judge. This fact explains why the pupil would be hesitant to reveal what he is ignoring.

The second function has some perverse effects upon the function of formation: instead of teaching mathematics, one teaches the art of passing examinations (this was already written in the letter by E. Galois *Sur l'enseignement des sciences*, Gazette des Ecoles, 2 January 1831). So pupils receive an initiation to *pass-rituals* ("rites de passages"), and that explains the parrot behavior evoked here: the mathematical practice in school is reduced to a practice of a ritual type (so the solving methods degenerate into recipes, tricks, and rites).

In the past, the selection of an elite was done through studies of rhetoric. Now this role is attributed to the sciences, but it is not at all different, because the *rhetoric* aspect is very important in the mathematics classroom also: People develop a certain style there, a social behavior (docility mixed with authority in the sham), each one playing his/her role[2] according to the *stereotypes* (see works by Bishop, 1985; Leder, 1980). Here we find the "hidden curriculum": the social influence of the class is more important than what is learnt there (J. H. Newman noted that for Oxford, and it is true in France, not only for the "Grandes Ecoles," but also for the entire school system).

So the microsociety constituted by the mathematics classroom is a perfect reduction of the *surrounding macrosociety*, and this explains why it functions as an instrument for social *reproduction* (in the meaning of Bourdieu, 1970; see also Bourdieu et al., 1987).

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Negative Power, Hegemony, and the Primary Mathematics Classroom: A Summary

Tom Cooper

Abstract

There is evidence that teachers see mathematics as a crucial subject for reproducing existing social values, and that they modify mathematics curriculum material accordingly. Stake and Easley (1978) describe how American teachers believe that mathematics functions as a "behavioural badge of eligibility" for the various privileges of society. Romberg and Carpenter (1986) report on a study by Stephens (1982) which documents how American teachers deemphasize meaning in curriculum material. Anyon (1981) reports how American teachers modify curriculum material to suit the social class of their students. Cooper and Meyenn (1984) describe how, even when alternatives exist, Australian teachers resist changing mathematically undesirable practices if they believe change may put at risk instruction in social values.

This paper analyses primary mathematics teaching in terms of its role in the social and political func

[2]This point is developed in my communication *Pygmalion effect and socio-cultural 'handicap'* in group T4.

tions of schooling in reproducing present society. The analysis is based on the notions of *hegemony* (Gramsci, 1971) and *negative power* from non-decision making (Bachrach & Baratz, 1970). It is argued that teacher decision making is confined and controlled because the majority of teachers ensure that their actions are in accord with the dominant ideology of their schooling situation and their society. The correspondence theories of Bowles and Gintis (1976) are extended to include the notions of cultural capital (Bourdieu and Passeron, 1977; Bernstein, 1977) and resistance, penetration, and limitation (Willis, 1977). Finally, mathematics' role in the social outcomes of schooling is analyzed using the theories of Apple (1980) and Giroux (1980). Mathematics teaching is seen to have a formidable role in reproducing crucial ideologies, a role made more powerful because of the notion of objectivity that surrounds mathematics itself and masks its hegemonic functions.

Introduction

The thesis of this paper is that mathematics can be a crucial subject within the primary school in reproducing the existing social values of a society: Particularly, in a Western capitalist society such as Australia, the social values relate to the means of production. In the way it is taught, mathematics can emphasize and reinforce the values and relationships that underlie what is produced, how it is produced, and for whose benefit: particularly, in a Western capitalist society, the values that are the basis of authoritarianism, individualism, competition, and division between labor and management. This crucial role of mathematics has been exacerbated by the growth of technology.

It is the further thesis of this paper that many teachers recognize and accept this role for mathematics, even if only tacitly (Polanyi, 1962), and wish to know how to modify curricula and textual materials to this purpose.

The basis for these theses are the neo-marxist and critical theory (Kemmis & Carr, 1986) perspectives described below and the experiences from a research project in a primary school which attempted to change teaching in mathematics classrooms to a more problem-solving and child-centered orientation (see Cooper & Meyenn, 1984; Cooper, Redden, & Seidenstein, 1985, for some descriptions of this project).

It should also be noted that these theses do not apply in all cases. The interactions between teachers, their practice, and the forces on them from society are complex and involve resistance and contradiction that a summary paper can not cover. However, these theses do provide an alternative explanation as to why teachers persist with mathematically inadequate teaching practices and resist change to more effective procedures.

Perspectives for the Theses

Insight into the functioning of the mathematics classroom is offered by the theories of Apple (1980) and Giroux (1980), who suggest that a major function of schools is to sort and rank the student population. They argue that this sorting process is not arbitrary or random, but corresponds to the hierarchical organization of society.

Initial work on the social role of schooling was done by Bowles and Gintis (1976) in identifying a "hidden" curriculum that taught student skills and attitudes necessary for a compliant workforce in a capitalist society. This work made a significant contribution to our understanding of the stratification process in schools. However, the more recent critiques of Apple and Giroux demonstrate that Bowles and Gintis' notion of "correspondence" between student experiences and adult roles is inadequate in that it assumes student passivity in the stratification process. Willis (1977) found that students from working class backgrounds actively resisted the imposition of conflicting cultural messages. Willis called this resistance "partial penetration," because, while students accurately recognized the oppressive nature of schooling, their active resistance prevented them from acquiring the knowledge and skills (the cultural capital) necessary for social mobility. Willis' working class students understood that, for them, school was not going to provide equality of opportunity and individual choice, but, in doing so, they rejected mental labor and, therefore, accepted the separation of labor and management.

The social role of schooling is to produce cultural capital in a form that reflects the dominant ideology of society (Bourdieu & Passeron, 1977; Bernstein, 1977). At the basis of this is the notion of hegemony (Gramsci, 1971): the ideas, beliefs, values, and attitudes which define the existing social order as right and legitimate. Capitalist production continues with little coercion, because even those whom it impoverishes accept the hegemony which supports it. The function of schooling is, therefore, to allocate cultural capital and "to legitimize this hierarchically organized stratification process"—to pass on the hegemony (Christensen, Gerber, & Everhart, in press).

One method of control within this reproductive process is non-decision making (Bachrach & Baratz, 1970). Very little coercion is necessary, because most participants in schooling, in fear of being isolated as not legitimate, ensure their actions are in accord with those of the dominant ideology. Within all decision making, there are limits on the agenda of alternatives, limits which are masked by the dominant hegemony.

Applications to Primary Mathematics

Weaknesses in the preparation of primary students in mathematics have tended to direct attention to the quality of the teaching (e.g., Shuard, 1986). Conventional wisdom is that primary teachers tend to rely on worksheets and textbooks, to use an imitative style of teaching (Education Department of Western Australia, 1978) and to teach instrumentally (Skemp, 1978), focusing on rules without meaning. There appears to be an overdue emphasis on computation and correct answers, with little focus on problem solving and applications.

With some exceptions (e.g., Bourke, 1985), there is a lack of research documenting what happens in primary mathematics classrooms to support or deny these assertions. What support is offered tends to come from reports (e.g., Cockcroft, 1982; Shuard, 1986) and from studies of mathematical achievement (e.g., Lindquist et al., 1983). The emphasis in mathematics education research has been on the psychology of the learner, appropriate instructional methods and the epistemology of the discipline itself. Within instruction, there is particular interest in the cognitive aspects of achievement (e.g., Silver, 1985) and techniques of instruction (e.g., Silvey & Smart, 1982) and some interest in affective traits, particularly for girls (e.g., Leder, 1982). There is a paucity of research into the social context of primary mathematics teaching (Bishop & Nickson, 1983). Most research seems to assume that mathematics is value free in both cultural and social status terms and is, therefore, objectively assessed.

There are indications that social analysis may offer insight into the functioning of mathematics classrooms. Stake and Easley (1978) found that traditional teachers saw science and mathematics as "heavily laden with social values," recognized that scientific knowledge "may function more and more as a behavioural badge of eligibility for employment" and wanted help in adapting newer instructional materials to support the inculcation of work ethic values they saw as important in present society's means of production (Easley, 1978). How teachers amend instruction is documented by Stephens (1982). Stephens found that activities designed for children to construct meaning had been abandoned or modified by separating the tasks from their mathematical contexts. Joffe (1986) reported that many mathematics teachers in English schools modify material from Nuffield primary mathematics texts by removing the discovery activities.

Anyon (1981) argued that this adaptation is in response to the social class of students. She studied mathematics teaching in five elementary schools, all of which used the same mathematics text. The two working class schools in her study focused on procedures without explanation or connection to thought processes. The middle class school seemed to offer more flexibility and made efforts towards understanding. The "professional" class school placed emphasis on discovery and experience as a basis for children constructing their own knowledge, while the executive class school extended this further through explicit instruction on problem solving and encouragement to justify the reasonableness of answers.

Mellin-Olsen (1987) described the role of ideology, hegemony, and resistance in the mathematics classroom, providing directions for activity that may enable a critical awareness of this situation to be developed.

Implications

The tantalizing conclusion to applying the sociological perspective above is that mathematics has a special place within the reproductive functions of schooling. Experiences in primary mathematics classrooms (e.g., Cooper & Meyenn, 1984) seem to indicate that both teachers and students view mathematical knowledge as related to the future economic role of the learner—mathematics seems to be cultural capital highly indicative of future economic success. Those who aspire to middle class professional positions are seen to require a large amount of it. Those who aspire to executive positions are seen to require a certain type—the part of mathematics that enhances problem solving, logical thinking, and expression. Those who aspire to clerical positions are seen to require rote procedures. For the remainder, manual laborers, unemployed and mothers, only basic survival skills are seen as necessary.

There is evidence that teachers modify their mathematics programs to suit their beliefs about the future requirements of learners (e.g., Anyon, 1981). This is supported by research on girls and mathematics (e.g., Spender, 1982). It is evident that mathematics in schools is presented in an abstract and verbal logical (analytic) form, which may suit the children of the dominant class, when a more visual spatial (geometric) presentation seems to be as effective mathematically (e.g., Krutetskii, 1976) and may be more appropriate for working class children.

Mathematical ability is very public. The nature of instruction and the form of assessment ensures that achievement is openly discussed in the classroom. In many classrooms, mathematical ability is related to correct answers. These are considered to be independent of the marking whims of the teacher. Achievement is given the appearance of objectivity. Yet, there is no discussion of the questions, and the closed form of their presentation, which sets up this right-wrong dichotomy, a situation that, in purely mathematical terms, is invalid. This is particularly so for rote instructional techniques. In this way, the form in which mathematics teaching is presented provides a powerful legitimization for its stratification function. Furthermore, the fear of being incorrect prevents meaningful challenge. Any penetration of the irrelevant nature of mathematics instruction is most likely, therefore, to result in rejection of the mental activity that is the

basis of mathematics. This conclusion leaves many students unwilling and unable to undertake executive functions in society.

Hegemony and Negative Power

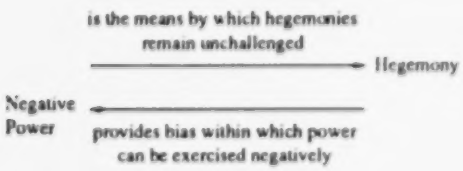
As mentioned earlier, two important notions in the interrelationships that enable the social reproduction above to continue are hegemony and negative power. Hegemonies are sets of ideas, values, and beliefs that define existing social relationships and orders as right and legitimate, and they exercise a pervasive influence. The primary mathematics classroom has to operate within the dominant hegemonies of the society in general, the school in particular, and mathematics as the specific discipline. It is tantalizing to argue that

(1) the general hegemony of the present technological society provides the special place for mathematics in leadership and high status vocations;
(2) the particular hegemony of schooling provides the competitive and authoritarian nature of much mathematics instruction; and
(3) the specific hegemony of mathematics stresses individualism and division.

Negative power arises from the non-decision making of Bachrach and Baratz (1970). It is the indirect face of power where actions that are not in the interest of those in charge are suppressed, thwarted, and prevented from being aired without the elite having to initiate or support any actions or exercise any powers of veto. It is the process by which the agenda of alternatives in decision making are limited by the non-elite members' unwillingness to initiate or support any action that they see as not being in the elite's interest. It is again tantalizing to argue that negative power is the reason why:

(1) many teachers seem unwilling to follow any practice that they see as not being approved by more experienced teachers, principals, or inspectors; and
(2) many children seem to spend inordinate amounts of time attempting to determine and do "what the teacher wants."

As is described more fully by Cooper (in press), negative power and hegemony reinforce each other:


is the means by which hegemonies remain unchallenged
Hegemony
Negative Power
provides bias within which power can be exercised negatively

Conclusions

Much of this paper is conjecture. The social critique of primary mathematics attempted in this paper, necessarily preliminary though it is, should therefore be seen only as providing fruitful ground for future research and analysis.

However, for the present, there appears some indication that mathematics has a formidable role in reproducing crucial ideologies, such as the separation of labor and management, that support capitalist production. This role is made more powerful because an ideology of objectivity surrounds mathematics and masks its hegemonic function.

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The Structure of Mathematical Discourse Among Teachers and Children in Elementary School

Andrea L. Petitto

The purpose of this paper is to describe the structure of classroom discourse and discuss the significance of this kind of discourse for the difficult and often gradual process of acquiring an understanding of elementary arithmetic. Elementary math embodies many sophisticated concepts. The typical algorithms taught present a host of ambiguities for the child's mathematically naive mind. The process of disambiguating these procedures is a difficult task.

The Teacher as a Flexible Expert Responds to the Many Variations in Children's Understanding

Teachers create lesson plans and then alter them in response to individual children's varying and sometimes unexpected interpretations of arithmetic presentations. Teacher/student discourse is analyzed within a cognitive framework which provides a way of directly linking discourse elements to the conceptual content of the lesson, and thereby trace the interplay of teacher and student input with respect to the lesson content. The structure of teacher/student discourse and its implications for learning are compared across children varying in mathematical skill. A theory of expert guidance characterizes teaching as "improvisation on a plan," and indicates the kinds of competence necessary for the teacher's ability to improvise, as well as showing how, within this theoretical framework, the best laid plans can go awry.

What do Children Learn From Each Other?

The major distinction here is the teacher's control over the whole task versus the more distributed control among a group of peers where no one is an expert. Teachers guide their students in a flexible way, but always maintaining a high level of procedural coherence (except when things go awry). Among peers, however, the overall structural coherence must evolve through interpersonal coordination. The nature of this coordination is potentially more complex and problematic than the guidance of a flexible expert.

Conceptual Conflict and its Resolution

Interpersonal interaction brings differing points of view into conflict, and individual internal processes of equilibration may act to resolve these conflicts by creating higher-level concepts—à la Piaget. But is this process effective only in universal cognitive achievements—those that we are somehow "built" to acquire? To acquire the more complex cultural achievements—e.g., math—it may be that interpersonal interactions must also carry the resolution of those conflicts. What

is the form of this resolution? What is the role of each individual in the process of resolution? And how are these individual roles reflected in individual cognitive change—that is, learning?

Pooling of Resources and Coordination of Effort

To address the above question, it has been necessary to identify specific aspects of peer interaction which are major components in any collaborative effort. Two major components are discussed: pooling of individual skills and knowledge (cognitive resources) and the structure of socially organized coordination.

Individuals often take on specific roles which they adhere to consistently throughout an extended group effort. These are thought of as intellectual niches which particular individuals fill. An analysis of group interaction sheds some light on the nature of these niches and the people who fill them.

A detailed analysis of small-group dynamics showed that children's roles were determined by differences in both personality and technical knowledge, and that resolution of conflicting points of view involved a combination of technical and political group dynamics. Effective communication and coordinaton depended upon the existence of a common basis of understanding the technical demands of the task at hand, and on effective leadership qualities of at least one group member. Although there are many gaps in extent to which all group members understand each other's concept of the task, an effective group leader is able to coordinate the fragmentary efforts of each member into a coherent and effective solution.

Whole-Class Interaction as the Negotiation of Social Contexts Within Which to Construct Mathematical Knowledge

Terry Wood

My colleagues, Paul Cobb and Erna Yackel, and I have recently completed a teaching experiment in one 2nd grade classroom based on a constructivist philosophy. In the traditional teaching experiment (Steffe, 1983) a mathematics researcher teaches one child by creating learning opportunities based on the child's current level of mathematical understanding. We attempted to expand this concept to the reality of a traditional classroom with 20 or more children and one teacher.

The basic instructional approach was small-group problem solving followed by whole-classdiscussions of the children's solution methods. The instructional activities were developed from conceptual models of young children's construction of arithmetical knowledge (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & von Glasersfeld, 1988). These activities were developed for the entire 2nd grade year and were designed to be problematic for children at various conceptual levels. The richness and diversity of these activities provided the children opportunities to offer a variety of solutions to problems as they discussed them in whole-class and small-group settings.

The extent to which the children did in fact talk about their understanding of mathematics can largely be attributed to the social context in which mathematics was taught and learned. The nature of the teacher and student interactions that occurred within the whole-class discussions were crucial to establishing the social norms necessary for developing a setting in which the children would feel psychologically safe to express *their* mathematical thinking. Mutual obligations and expectations were negotiated and renegiotiated by the teacher and students as they established an interaction pattern that would form the basis for creating what has been referred to as a "problem-solving atmosphere" (Silver, 1985).

The teacher's intention during class discussions was for the children to express their current thoughts about mathematics, which would provide opportunities for both the teacher and children to learn. On the one hand, the children would need to reconstruct and explain their own solutions, listen and consider alternative viewpoints, and resolve conflicts. On the other hand, the teacher would have an opportunity to view things from the child's perspective and would come to understand individual children's methods.

However, her intention and expectations for children's actions was quite different to the children's previous experiences in school and in this classroom in other subjects (Wood, Cobb, & Yackel, in press). In those situations the children were expected to learn what the teacher wanted them to know rather than express their own thoughts (Voigt, 1985).

The manner in which the mutual construction of this different set of norms for the class discussion evolved is reflected in the early classroom interaction patterns in which two levels of conversation occurred. In one conversation, they talked about mathematics, whereas in the other, they talked about talking about mathematics. The mutual construction of a different set of norms for math lessons was initiated by the project teacher as she exerted her authority in order to help the children reconceptualize their role during mathematics instruction. She wanted the children to think about mathematics, to figure things out for themselves and talk about their ideas in the public arena of whole-class discussions. Consequently, her expectation that the children would express their thoughts placed the children under the obligation of having to reconstruct their solutions and explain them to others.

Conversely, if she had certain expectations of the children, then she had to accept certain obligations for her actions as well, because from the children's point of view a certain amount of risk was involved in fulfilling the teacher's expectations. It is one thing for a child to think about how to solve a problem privately, but quite another to express those thoughts to your

peers. In this situation the child's thinking can be publicly scrutinized and evaluated, which leaves open the possibility of embarrassment and feeling incompetent. From the children's perspective, if they were to accept the obligation of expressing their thoughts, then they expected the teacher to accept and respect their thinking as such. She was obligated not to evaluate their solutions or to try and impose her ways of doing mathematics on them. Thus, she had to reconceptualize her own role during mathematics instruction. This evolving, interlocking network of obligations and expectations was beyond the conscious awareness of both the teacher and children, but nevertheless was mutually constituted. The following example which occurred during the first week of school, illustrates the manner in which the teacher and students mutually constructed a social context in which children would express their mathematical thinking. The discussion centers on the word problem "How many runners altogether? There are six runners on each team. There are two teams in the race."

T: Peter. What answer-solution did you come up with?

Peter: 14.

T: 14. How did you get that answer?

Peter: Because 6 plus 6 is 12. 2 runners on 2 teams. (Peter stops talking, puts his hands to the side of his face and looks down at the floor. Then he looks at the teacher and then at his partner, Ann. He turns and faces the front of the room with his back to the teacher, and mumbles incoherently).

T: Would you say that again. I didn't quite get the whole thing. You had ... Say it again please.

Peter: (Turns to look at the teacher). I made a mistake. It's wrong. It should be 12. (He turns around and faces the front of the room.)

The teacher had intended that the emphasis of the discussion would be on the children's solution methods. However, Peter realized that his answer was wrong and interpreted the situation as one that warranted acute embarrassment, as indicated by his response of turning away from the class to avoid looking at the other children. His action would be appropriate with respect to the social norms of traditional instruction where students were expected to produce correct answers by using prescribed methods. However, this construal confounded the teacher's intention and she realized that in making Peter obligated to explain his solution, she put him in the position of having to admit that his answer was wrong, publicly.

She recognized that if she was to expect the children to express their own solutions to problems, then it was her obligation to ensure that they feel no shame or embarrassment when they presented erroneous solutions in front of others. Consequently, the teacher had to modify her intentions immediately and renegotiate the social norms with the children. To this end, she initiated a second conversation in which she and the children talked about talking about mathematics.

T: (Softly.) Oh, okay. Is it okay to make a mistake?

Andrew: Yes.

T: (Softly.) Oh, okay. Is it okay to make a mistake, Peter?

Peter: (Still facing the front of the room.) Yes.

T: You bet it is. As long as you're in my class it is okay to make a mistake. Because I make them all the time, and we learn from our mistakes, a lot. Peter already figured out, "Oops, I didn't have the right answer the first time" (Peter turns and looks at the teacher and smiles) but he kept working at it and he got it.

As she emphasized that Peter's attempts to solve the problem were appropriate in every way, she simultaneously expressed to the other children her expectation that their thinking about mathematics was what was important in this class, not right answers. Peter, as he publicly admits his mistake, recognizes that the teacher's reaction is one of support and encouragement rather than judgmental. Thus, he is able to face the class with a smile.

The teacher and children initially negotiated obligations and expectations at the beginning of the school year, which then made possible the patterns of interaction that allowed the children to express their thinking about mathematics for the rest of the school year. While these norms were initially established in the context of the whole-class discussions, they were also crucial to the development of the norms for the small-group interactions in which cooperation and collaboration were critical to the success of the group and to the individual's learning of mathematics.

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Learning Under Difficult Conditions

For some students, learning mathematics is especially difficult. Typically their problems are considered as psychological, but in this topic there was a focus on the social nature of their problems. Marilyn Frankenstein and Arthur B. Powell, USA, explore the needs of "non-traditional" learners, "outsiders" of the educational system like women, ethnic and national minorities, and other working class adults who are severely underprepared to successfully meet the challenges of the traditional curricula of higher education. They present alternative approaches of "non-elitist" mathematics education which meet their intellectual needs in mathematics. Françoise Cerquetti-Aberkane, France, describes the mathematics education program in special classes for children with difficulties and criticizes certain parts of that "special pedagogy." Nick Taylor, Republic of South Africa, reports on some findings from case studies conducted among 13-year-old students in Soweto into the methods employed in the solution of problems on equivalent fractions. His findings place a question mark over the utility of those psychological perspectives on learning which seek to separate the cognitive and affective domains, and those multicultural and ethnomathematical considerations which see cultural aspects as providing a handle for lifting "underprivileged" learners into the mainstream.

Mathematics Education and Society: Empowering Non-Traditional Students

Marilyn Frankenstein and Arthur B. Powell

Non-Traditional Students: Societal, Political, and Educational Contexts

In the United States, as in other nations, large numbers of individuals are prevented from realizing their potential in the context of mathematics and mathematics-related fields. This mathematical disempowerment of individuals and the accompanying alienation that many experience are due, in large part, to effects of the interactions among the social, political, and economic structures of the U.S. In regards to scholastic achievement, for example, attitudes toward race, gender, and class, which are both given birth and reinforced by the political economy of the U.S., restrict the access that ethnic and racial minorities, women, and working class students have to mathematics and mathematics-related fields.

In the late 1960s, arising from the social and political pressures of the struggle against U.S. involvement in the Vietnam War and the Civil Rights Movement, higher educational institutions opened their doors to new kinds of students, ones traditionally considered as "outsiders." A significant proportion of these outsiders or, as we shall refer to them, "non-traditional" students, were not adequately prepared to meet the conventional expectations of post-secondary institutions. Indeed, in their secondary institutions, many non-traditional students were tracked into non-college bound curricula and, in mathematics, have had their academic self-esteem crushed.

The mathematical disempowerment of non-traditional students is exaggerated by their misconceptions and beliefs about mathematics, ones which were promoted by their primary and secondary school experiences. For instance, they tend to believe that mathematics is an arcane body of knowledge, containing immutable, eternal truths which are external to humans and, therefore, can be discovered only by "gifted," "sagacious" minds. In accord with this belief about the nature of mathematical knowledge, non-traditional students also hold the misconceptions that mathematics is learned by completing repetitious exercises and arduously memorizing facts. Most insidious, however, is their belief that none of the mathematics that they are expected to master cannot be developed, least by them.

To respond to the large influx of non-traditional students into tertiary educational institutions, remedial mathematics courses have been established. But most of the dominant instructional methodologies have not been successful. Giroux's (1982) categorization of the ideologies underlying various approaches to native-language literacy is useful in understanding the problematic nature of traditional mathematics pedagogies and the promise of some alternative ones.

Giroux's category of *instrumental ideology* focuses on principles of prediction, efficiency, and technical control. Knowledge is seen as objective and external to the knower. Facts are stripped of the subjectivity of class formations, race, and gender and celebrated for their supposed neutrality. The focus of instruction is on atomized content, the mastery of which constitutes the central problematic. This ideological perspective underlies traditional mathematics pedagogies and has its crudest expression in remedial instructional methods which concentrate on lower-order cognition—mechanical proficiency and rote memorization—omitting any insight into the nature of mathematics as a way of thinking and seeing and the uses of mathematics in understanding the world.

Giroux's category of *interaction ideology* centers on the human dimensions of knowledge, viewing knowledge as a social construction. It regards meaning, as opposed to mastery of content, as the central problematic. This ideology underlies "humanistic" mathematics pedagogies which focus on alleviating "math anxiety," individualizing instruction at the students' own paces, and problem solving with a stress on process over product. In such pedagogies, there are important moments when students can grasp the nature of mathematics and their ability to do mathematics. Though interaction ideology is concerned with cognitive dissonance and moral development, it omits notions of political conflict and power differences among

socio-economic classes and racial, ethnic, and gender groups.

Giroux's third category, *critical ideology* does take these factors into account. This ideological perspective, which owes much to Freire's (1970) pedagogy of the oppressed, extends the complex role schools play as institutions that mediate and sustain the logic of the state and the imperatives of capital, to include the important concept of human agency where, in a dialectical process, people both participate in their own oppression and struggle to resist. Furthermore, critical mathematics pedagogies take the view that the cognitive processes owned by speakers of any language are akin to those involved in thinking mathematically; that students require situations to mathematize and only a minimum of givens; and that from these they can generate mathematics both inductively and deductively (Gattegno, 1988). This critical ideology and its pedagogical manifestations are the foundations of the three instructional techniques—reading, articulation, and ethnomathematics—elaborated elsewhere (Frankenstein, 1987; Hoffman & Powell, 1987; Powell, 1986) and discussed briefly below.

Reading and Articulation: Constructing Meaning in Mathematics and Using Mathematics to Construct Meaning in Society

By reading, we restrict our reference here to non-textbook readings related to mathematics and mathematics learning. Students read excerpts from the mathematics education literature and critically reflect on the reported findings and their mathematics learning in light of these findings. Their reflections are made public in small groups, class discussions, and through writing. Students also read newspaper articles on topical social, political, and economic issues, and the mathematics of these issues are examined. Here the focus is on understanding and interpreting, through discussions and in writing, the statistical information and arguments presented.[1] Furthermore, we focus on how statistical data can clarify issues and reveal aspects of the underlying structures of the society.

Critical interpretation and reflection are processes which also can be stimulated and promoted by articulation activities. In such activities, students are requested explicitly to reflect on cognitive and affective aspects of their mathematical experiences and to articulate these reflections in speech or in writing. Talking about mathematics is facilitated by small-group, collaborative work. The results of these small-group deliberations are publicly heard and debated in class discussions. Collectively and not without dissenting positions, views or meanings are thus constructed.

Articulation activities in which writing is a focus require students to engage in exploratory, speculative writing as a primary mechanism to externalize their reflections. This type of writing can be a valuable end in itself or can be used as a starting point in which feedback and revisions are grounded. These revisions then are part of a process through which students write more formally about mathematics and their learning. Several types of writing activities are used, including multiple-entry logs (Hoffman & Powell, 1988), free writing, and learning logs or journals (Powell & López, 1989).

Ethnomathematics, Cultural Affirmation, and Non-Traditional Students

Ethnomathematics, a cultural anthropology of mathematics, is defined by D'Ambrosio as "the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" (D'Ambrosio, 1985). Ethnomathematical research focuses on methodological differences in various cultures' mathematics, such as the cultural uses of the period and comma in mathematical notation or how learning is affected when notational conventions of numeration developed for one cultural-linguistic group is uncritically adopted to the cultural-linguistic system of another (Powell, 1986); it can involve analyzing conceptual differences in various cultures' mathematics, such as the classification structures of other languages, where, for example, in the African language Setswana, things are classified by what they *do*, rather than by what they *are*, as in Indo-European languages (Berry, 1985). Ethnomathematical research has shown that the basic notions of mathematics are similarly developed in all children, regardless of social class, race, and culture (Ginsberg, 1982); that what counts as mathematical knowledge needs reconsideration, since, for example, the geometric knowledge of basket weavers (Gerdes, 1986) and the group structure of kinship relations (Ascher & Ascher, 1986) have been found to embody complex mathematical structures; and that the contributions of various cultures to mathematics has been distorted and hidden, such as the false portraits of historically famous mathematicians like Euclid, depicted "as fair Greeks, not even sunburned by the Egyptian sun" when "there is not a shred of evidence to suggest that he was anything other than [an African person of Egypt]" (Lumpkin, 1983; Zaslavsky, 1983).

We have found four important reasons for integrating an ethnomathematical perspective into the curriculum. First, it gives a more accurate account of the history of mathematics and the contributions of non-Western peoples to it. Second, an ethnomathematical teaching perspective encourages instructors to have students examine their methods and ways of conceptualizing mathematics. Such examinations can be done through class discussions, writing, and student-

[1] Frankenstein (1987). For a text that teaches basic mathematics through real statistical data about the world, see Frankenstein (1989).

instructor interviews. Instructors can then build from the mathematical structures understood by their students. Third, while students reflect on and reconceptualize their mathematical knowledge, they come to realize that they already know more mathematics than traditional evaluations reveal and develop confidence that they can learn even more mathematics. Fourth, since our students are culturally and racially diverse, they are culturally affirmed by the results of ethnomathematical research. They gain respect for the contributions of their and those of other communities to the history of mathematics as well as respect for their own intellectual work, breaking through the view that "the intellectual activity of those without power is always characterized as non-intellectual" (Freire & Macedo, 1987, p. 122).

Conclusion

Both as a sector within the U.S. society and as individuals, non-traditional students are not without power, especially intellectual power. The atmosphere created in mathematics classes must encourage this realization and can best be done when the instructor is not the focus of attention. To accomplish this aim, one which can lead to the empowerment of non-traditional students, as with any other group of students, instructors must engage them in dialog. In this dialogue, students must be considered and treated as equals. This dialogue could include discussions of teaching styles, positions on the nature of mathematics and learning, and the constraints and shortcomings of the system in which the course exists. Further, instructors and students should interact as co-researchers discussing and designing appropriate instructional and learning techniques[2] and ways of effecting institutional change. Together, instructors and students can develop ways of overcoming the effects of racism, sexism, and classism on instructional methodologies and student achievement in mathematics. These efforts, by necessity, will be directed towards structurally transforming the negative educational and societal conditions in which the dialogue exists.

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[2]For an example of this point, see Powell and López (1989).

Teaching Mathematics in Special Classes for Children With Serious Difficulties in France

Françoise Cerquetti-Aberkane

Types of Classes

In the French teaching system there are different types of classes for children with serious difficulties. Some are flexible structures really allowing the children to place themselves back into the "normal" system. On the contrary, some others do not or hardly allow the reinsertion.

At primary level, in those flexible structures we find the "first-year adjustment class" which allows children coming out of pre-school to take one year to prepare themselves for the first year of primary school. Unfortunately, it is not infrequent for children to stay in that class a second year, from which they go on to an "improvement channel," which rarely allows a rejoining of the normal cycle.

There is also a "supporting second year," which makes it possible for children to do the second year in two years.

The path followed by these children is usually this: first-year adjustment class, supporting second year, second-year primary.

The number of pupils in those special classes is smaller and, in theory, the teacher is specialized. But beginners with no special formation are often in charge of such classes.

There are also classes with different levels especially for foreign children who speak no or little French and also for children of nomads and barge-people.

Finally, there are so-called "improvement classes" with different levels which take children who have not succeeded in the "normal" cycle. Created at the beginning of this century, they should normally be for mentally retarded children. The children who come to those classes have stayed in the same class several times, and the older ones have no chance whatsoever of getting back into the traditional school system.

In the secondary education system there are preprofessional classes at CPPN level and pre-apprenticeship classes (CPA), which take children between 14 and 16 coming from the "improvement classes" of the first and second years of secondary school.

The "sections d'éducation spectalisée" SES (specialized teaching sections) take the children who have very great difficulties when they leave the primary system.

Finally, there are the IMP "instituts médico-pédagogiques" (medical-pedagogical institutions) and the IMPRO "instituts médico-pédagogiques professionnels" (medical-pedagogical professional institutions) which normally take in mentally sub-normal children, but also school-maladjusted or extremely overactive or difficult children.

Socio-Cultural Origin of the Children

All these children have approximately the same characteristics. Their socio-cultural origin is mostly the non-qualified working class, either working, or on the dole, or out of work because of invalidity or long illness, with very large families. To this are added huge family problems (alcoholism, delinquency), or parents either separated of divorced or deceased. Some children are put in the cane of foster families which are themselves in difficult situations. Some children are beaten, ill fed. Add to this various housing problems, insalubrity, improper hygiene. In those classes there are also children who are physically handicapped either from birth on or because of accidents (cf. Adda, 1980).

The socio-cultural problems of the children with which I worked were very serious. They often had to face multiple difficulties, either personal or in the family.

Difficulties Linked With the Socio-Cultural Background Encountered in the Teaching of Mathematics

Some of the main difficulties linked with the socio-cultural background that we came across especially in the teaching of mathematics are:

- The children are often absent one after the other, so that it is very difficult to have any coherent, sustained activity with them.
- Another handicap is the difficulty they have in keeping their attention on their work for a given time. This is partly due to the consecutive failures driving them to passivity and lack of interest for anything having to do with school. There are also other causes such as lack of proper food (some children come in the morning having eaten nothing since the previous night), wearing insufficient clothes in winter, the violence of family relationships and brutal treatment.

They also have trouble adjusting to quickly changing instructions within the same activity. This is mainly due to the lack of stimulation within the family with the use of educative toys. Moreover, the accessory knowledge needed for mathematics has not been acquired. Many children do not know the names of colors or the rules of simple games such as card games, dice, the game of goose, as they do not play them within the family.

Some children do not know how to count objects one by one with their fingers as they did not learn this at home. Others have pronunciation problems, since they are not corrected at home. Frederic preferred to say any number except 5 or 6, because he was ashamed to not pronounce those words correctly. Some children do not understand a picture drawn in perspective. Also, if there are no books in the house, those children have no interest for written material.

Circular Notes, Syllabus, and Specific Activities

The adjustment, support, and migrant children classes have no specific syllabus. They just adapt the usual contents of the primary syllabus. There are no special books, and the teachers use ordinary primary books. For the improvement classes the motto is to offer their children "concrete" problems relating to "reality." Most often the texts offered are pseudo-concrete (cf. Adda, 1976; 1984).

For the CPPN and CPA classes, the circular notes clearly show that the children should be given concrete problems in direct relationship to the world of work.

It is interesting to note that in the recent official syllabus or mode of instructions, as in the previous ones, there is no mention of the classes for children in great difficulty. They are only mentioned in statistics to note the number of children accommodated.

Most of the time those children are not offered mathematics. They are taught basic arithmetic (the four operations) and how to use a train schedule, fill out a check, and so on. Of course, these things are necessary, but it is a pity to cut down mathematics to only those tasks.

The teaching value of the project is often used in those classes as a therapy against failure. The idea is to take on a long-term project and carry it through, for instance to organize a trip, or to repair a horse-drawn carriage. Through this project the children will use French, mathematics, history etc. Each child is then in a success situation, but the main problem is that if the project lasts for the whole year, the offered activities are very limited. Also, each child will most often tend to take by what he knows best already and will not progress in subjects where he feels ill at ease. Moreover, the problem of reinvestment of the capacities acquired in school into a real-life situation remains complete as the children have not been prepared for it.

People think that those children are only capable of being interested in concrete situations. Actually, the socio-cultural handicap added to the ambiguities of the wording of math problems which call upon a "false reality" (cf. Adda, 1976), the lack of progressiveness in the introduction of new concepts and notions are largely responsible for creating the difficulties encountered by those children in the learning of mathematics. However, my experience and the results of my research have shown me that given the right circumstances, those children loved and knew how to solve abstract math problems (cf. Cerquetti-Aberkane, 1981; 1985).

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"Let Them Eat Cake" Desire, Cognition, and Culture in Mathematics Learning

Nick Taylor

Cognitivism: A Sufficient Explanation?

Consider the following two questions:

(1) A loaf of bread must be shared equally between five children. What fraction of the loaf does each child get?

(2) A cake must be shared equally between six children. What fraction of the cake does each child get?

It could be argued that the cognitive tasks involved in answering the two questions are identical, or at least so close as to elicit respective responses from any subject which may differ slightly in quantitative performance but not in qualitative method. Case studies conducted with four 13-year-old students in Soweto early in 1987 refute this expectation (Taylor, in preparation). The differences in approach to the two problems adopted by Paul Nkosi, for example, are most illuminating.

Paul: In question 1, I said a loaf of bread ... er, by the way, you cut into the slices. You, you got a ten ... ten slices, and ten slices makes a loaf of bread, and then I said five ... five people can share a loaf of bread by getting a two-tenth of it.

Paul: Question 2 ... we have a ... a ... cake, and we must share this cake to six children, and each children it's mu... each childrens must get a ... a one-sixth of it.

NT: Okay, why do you say one-sixth?

Paul: When .. because when ... when you divide a ... cake into six parts, you got one-sixth, and each children gets one-sixth o... of ... of the cake.

In Paul's second answer, the word "cake" could be replaced with "whole" or "unit" without altering the sense of his response. In complete contrast to this formal solution, his first answer derives directly from his experience of the specific properties of a loaf of

bread: His description conjures up a picture of a loaf being cut into ten slices, and these being distributed equally amongst five children.

What these differences indicate to me is that Paul's mathematical activity is not merely a rational process, but is somehow tied in to profound emotional and/or cultural forces.

Culture: Purely Deterministic?

Bread plays a central role in the diet and indeed the family life of Paul. He lives with his grandparents and brother. He feels very strongly that the health education he received at primary school was quite correct in its teachings concerning brown bread. He can feel it nourishing him when he sits down to a breakfast of two slices—thick like this, indicating with thumb and forefinger—on returning from his run past Baragwanath Hospital every morning. His grandmother is waiting when he arrives home from school for lunch: it is she who cuts the bread and serves the accompanying stew.

Paul does not often eat cake: usually only at birthday celebrations. But he cannot eat very much at a sitting: too sweet, he insists, pulling wry faces and holding his stomach. He makes occasional purchases for the kitchen at the corner shop. Bread, milk, maize meal, vegetables, sugar, tea, and coffee are common items or his shopping list. Does he ever buy things for himself? No. If he had money, what would he buy? Fruit: apples, pears, oranges, bananas. Sweets? Never. Only fruit.

The Unconscious: Drive or Displacement?

The respective roles played by bread and cake in Paul's life obviously affect his solutions to the above two problems. Yet, if this was merely a matter of cultural determinism, one would expect children of similar cultural situations to be subject to similar influences. Again, my Soweto studies undermine this hypothesis. Although the four students interviewed were drawn from the same school classroom, residential neighborhood, and socio-economic bracket, the two respective solutions of Camel were virtually identical to those of Paul, whereas Valeria and Brenda produced formal responses to both questions.

While not wishing to overemphasize the generalizability of these results, they do highlight the need, firstly, to consider factors other than cognition and culture when attempting to make sense of mathematics learning, and, secondly, to theorize how these elements are mediated.

I stumbled across some indirect evidence pertaining to this problem when I discovered that both boys aspire to be policemen when they leave school. This struck me as a most puzzling choice, in view of the political situation existing at the time. High school students have been centrally caught up in the South African struggle since the historic events in the Soweto schools in 1976 (Muller, 1987). Amongst the more traumatic events of the wave of civil war which swept the country in 1985/86 and which would have affected my four students directly, were two incidents in which students at their school were shot dead by security forces on or very near the school premises, followed by the occupation of the school, and the large scale detention, interrogation and alleged beating of students.

When I spoke to Paul about his occupational dreams, it was clear that the horror of his personal everyday existence was filtered out, and replaced with an altruistic idealism.

NT: What would you like to do when you leave school, Paul?

Paul: Highway Patrol.

NT: What do you like about Highway Patrol?

Paul: Well, meneer [master, or sir], at Highway Patrol you can go to school and study accounting ... and ... and I also want to catch people stealing cars. Sometimes these boys steal cars in town, and then they drive very fast and ... and ... rough. Not like the people who are buying the cars. And then sometimes there are children playing in the street and they can get hurt.

In the midst of chaos, poverty, and oppression Paul latches onto the bourgeois dream of the work ethic, ownership and the defence of property.

Conclusion

Cognitive and cultural processes undoubtedly play a prominent role in the production of mathematical motivation and meaning. Cognitive theories of learning tend to view the subject as making completely rational choices from alternatives provided by her environment; affective aspects, if taken into account at all, are considered as having a purely additive influence.[1] Multi-cultural approaches to learning tend to view cultural forces as uniformly deterministic, as providing a framework for the compensatory education of culturally deprived students,[2] or for making education relevant to the life experiences of the student.[3]

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[1] For a typical description of the cognitive science approach see Gardner (1985).

[2] This is particularly so in South Africa, where those in power typically view the black majority as "milieu deprived" and requiring "compensatory education." See, for example, Carstens (1986).

[3] This last aspect is true also of recent cognitive accounts. See Resnick, Cauzinille-Marmeche, and Mathieu (1987).

My study indicates that, in ignoring the role of unconscious desire, these approaches cannot fully account for learning. Furthermore, far from being a victim of biologically and/or politically produced desires, the individual subject adopts a specific position within her context or discourse. Desire is a complex mix of the most obvious elements in the personal life of the subject, and the displacement[4] of unpleasant experiences by the unattainable dreams projected by society. While Paul accepts the role of custodian to these dreams, his classmates devise ever more inventive initiatives for the maintenance of the momentum in the struggle for a new South Africa.

Whereas both cognitive and cultural accounts of mathematics learning might want to say that the curriculum should be restructured so as to make school mathematics more accessible to the child, my work suggests that such solutions would result in no better than a hit and miss success rate if they fail to problematize the issue at its most fundamental level. The kind of psychoanalytic account I am attempting to signpost here (Hendriques, Hollway, Urwin, Venn, & Walderdine, 1984) would indicate that school mathematics is inaccessible to students because it is not premised on an adequate theory of the subject.

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[4] I use the term in its Freudian sense (Freud, 1953).

Mathematics Education in Multi-Cultural Contexts

Within the mathematics classroom there can be serious cultural conflicts which will challenge both learners and teachers to negotiate satisfactory practice. Raymond A. Zepp, Macau, asserts that a new way of thinking about the relationship between language and mathematics is necessary, and describes the new directions that research has begun to move towards. Ina Kurth, Federal Republic of Germany, reflects on some special problems of children learning mathematics in a foreign language and illustrates her presentation with examples from the material particularly developed for such children. Helen Watson, Australia, discusses her work with teachers who in their everyday practice are confronting the problems of teaching and learning mathematics in two worlds: Yoruba teachers in Nigeria in West Africa, and Aboriginal-Australian teachers in the Tanami and Laynhapuy region of Nothern Australia; together they have developed understanding of the relation between conceptual systems and considered the perplexing problems of mathematical education in a bicultural situation. Norma C. Presmeg, Republic of South Africa, explores ways in which cultural continuity may be fostered in the context of mathematics education. She is well-experienced in working with multi-cultural classroom situations and with adult students, who might be in a situation similar to learners in developing countries around the world.

New Direction in Research on Language in Mathematics

Raymond A. Zepp

One of the most important and controversial cultural variables which affect the learning of mathematics is that of language. The early 70s saw an awakening of interest in the effects of language on mathematics education. The UNESCO symposium on "Interactions between Linguistics and Mathematics Education," held in Nairobi in 1974 did much to highlight the need for investigations into the relationships between language and mathematics.

One of the pioneers in the field was John Gay (1967), who published a study of mathematical reasoning by speakers of the Kpelle language in Liberia. His results, which showed marked differences in logic between American speakers of English and Liberian speakers of Kpelle, served as an example for other researchers in exploring the question of Whorfian differences in thought.

The adjective "Whorfian" is derived from the name of Benjamin Lee Whorf, who hypothesized that a person's thinking and logical processes are dependent on his first language. To attempt logical reasoning in a second language is difficult, if not impossible, because the logic in the mother tongue is different and pre-emptive. The case is stated succinctly by Sampson (1980, p. 89):

> One might take the Whorf hypothesis to mean that even logic is relative to language, so that, say, if Aristotle had been a Hopi then modern logic as well as modern physics would have developed quite differently.

The extent to which the Whorfian hypothesis is true is of course tremendously important to mathematics education. If it is true in its strongest version, then mathematical logic in one language might be quite different from that of another language. Students whose first language was, say, Swahili, could not be expected to study mathematics in English, since their brains were programmed with Swahili logical patterns.

Gay's original work focused on the Whorfian issue, and his studies with the Kpelle appeared to confirm the Whorfian hypothesis. The method used by Gay was based on a natural assumption: The ease or difficulty with which a logical concept or principle can be expressed in a language contributes to the ease or difficulty in acquiring and using that principle. For example, Gay found that the Kpelle were more adept than Americans at forming disjunctive ('or') concepts, but weaker at forming conjunctive ('and') concepts, and he attributed these differences to the relative simplicity of the Kpelle expressions for inclusive and exclusive disjunction, and their relative complexity in expressing conjunctive concepts.

This method was to become the model for other research: If students from one language group were experiencing difficulty with a mathematical concept in English, the structure of their mother tongue could be examined to find reasons for the difficulty.

Subsequent research, unfortunately, showed that Gay's assumption was not as straightforward as had originally appeared. A study by Zepp (1983) replicated Gay's experiments among the Adjukru in the Ivory Coast, who live only 200 miles from the Kpelle and have a similar culture, but who belong to a distinct language group. Exclusive disjunction is expressed very precisely and simply in Adjukru, but the acquisition of such concepts was found to be very difficult indeed. Thus Gay's hypothesis did not appear to apply to the Adjukru. The study did, however, appear to support the hypothesis that the differences in logic were indeed due to linguistic variables rather than cultural ones, since language was virtually the only apparent difference between the Adjukru and the Kpelle.

Further experiments did not turn up conclusive evidence in support of the strong Whorfian hypothesis. It was found, for example, that the common logical errors in Chinese were almost identical to those made in English (Zepp & Monin, 1987), e.g., reasoning from the converse, failure to apply the contrapositive, etc. Moreover, it was found that Chinese university students could learn new logical principles in English as easily as they could in Chinese (Zepp, 1986b). Finally, it was shown that those logical principles learned in English transferred back to the mother tongue, Chinese (Zepp, 1987). Thus it appeared that the acquisition, use, and transfer of those logical principles were independent of language.

On the other hand, a study by Dawe (1983) gave indications that the Whorfian hypothesis was alive and well. He showed that children's ability to reason logically in their second language (English) correlated more highly with their *first* language competence than with their English language competence. Their knowledge of their mother tongue was somehow affecting their performance in their second language.

The Dawe study appears at first glance to contradict earlier studies in which no Whorfian differences were found. But a closer inspection shows that the paradox can be resolved by examining some of the assumptions on which the earlier studies were based. These assumptions relate to the *way* in which the Whorfian hypothesis was thought to apply.

The earlier studies, which attempt to trace difficulties in mathematics learned in English back to complexities in mother tongue, are based on an assumption that the student, even when studying in English, is somehow doing the real thinking in his mother tongue.

The Dawe study, while apparently supporting this view, actually gives us a hint as to how it might be circumvented. The importance of first language competance suggests that the learning of the first language somehow shapes the mind in an irrevocable way. It suggests that the effect of the first language may lie not so much in individual words and phrases, but in deeper cognitive structures which affect the way we acquire concepts, attack problems, etc. Thus, if Igbo speakers have difficulty with fractions in English, it is not necessarily or directly because their language has difficulty expressing fractions. Rather, their language may have structured their minds in such a way that fractions do not easily fit into their mind set. They may be studying and thinking totally in English, but their *cognitive style* has been fixed, once and for all, in acquiring their first language.

In fact, Gay reported that in interviews after the four-card experiment, subjects were completely at a loss to explain precisely how they had solved the problem. They had acquired the logical concept without being able to verbalize it. It appears that they used a nonverbal strategy to solve the problem. Indeed, the four-card problem can be solved by deaf, dumb, and illiterate persons who have no 'language' at all (Zepp, 1986a).

Differences in cognitive style could explain differences in performance on the four-card test. Perhaps the differences in Gay's experiment were due to the American verbalizing style as opposed to the Kpelle nonverbal style.

It has been claimed that African cultures view learning about an object as a penetration into that object's being rather than an analytical description of the

object. The westerner "learns" by distancing himself from the object, by standing apart from the object and analyzing it 'objectively,' while the African tends to merge with the object.

Such a distinction might partially explain some difficulties in mathematics education. For mathematics is analytical in the extreme: the mathematician distances himself from the object and analytically abstracts what he considers its essence. A member of a culture which places little emphasis on such abstractions may experience difficulty understanding such concepts.

But the real question concerning the Whorfian hypothesis is: To what extent are differences in cognitive style due to differences in language? Almost no research has been done in this area, and it promises to be a fruitful one. The differences between the Kpelle results and the Adjukru results do suggest a possible effect of language on style, for the groups are culturally very similar, and appear to differ in language only. It may also be the case that differences in results could be obtained even between neighboring Kpelle villages with different cognitive styles.

A second question for future research is the role of various language 'competencies,' rather than the generalized term 'competency.' For example, Dawe's work might be replicated in order to ascertain precisely *which* first language skills correlate with ability in mathematical logic. Is, say, reading comprehension a better predictor of logical ability than is oral comprehension?

There is some evidence to suggest that reading is an important shaper of mental structures. Leibnitz, who was nearly as interested in Chinese linguistics as he was in mathematics, claimed that he could tell by looking at a Chinaman's eyes, whether or not he could read. Levi-Strauss (1966), in his book *The savage mind*, claims that the illiterate mind is fundamentally different from the literate one.

In fact, if Whorfian differences are due to the way language structures the mind, it may even be the case that the end product—language competence—is not the key issue at all. Perhaps the method of acquiring language is what really shapes cognitive structures. thus, two people might have equal reading ability, but the method by which they were taught to read could conceivably affect their logical ability.

Language acquisition therefore appears to be a third fruitful field of research activity. Piagetian studies in non-western cultures have shown that the classical developmental stages vary considerably from culture to culture. It could well be that studies in children's language acquisition will reveal differences in their developmental processes, which could in turn lead to different methods of mathematics education from those used in the West.

Finally, if we are to move away from a strict "thinking in language" model of thought, it will be necessary to develop a model for just how thought occurs. It is possible to suggest a model for language and logic based on the perhaps overused analogy of the computer. Computers use an internal machine language. The machine does not think in BASIC or COBOL; rather, it compiles the program written in BASIC into machine language. Perhaps the human mind can be thought of in these terms. A sentence is received by the ear in English, for example, but must be compiled into an internal language for its actual processing. Just as we can communicate with the computer using BASIC or COBOL, we communicate with each other in English or Swahili, but perhaps the actual thinking process is done in some other internal language.

In the above discussion, several areas of research have been suggested. They appear promising and important, because recent research trends indicate that language *can* play a role in mathematics education. However, the nature of that role now appears to be more complex than previously believed, and research can now be done to elucidate the connections between language and mathematics.

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Learning Mathematics in a Foreign Language

Ina Kurth

Many foreign pupils in our country have to learn mathematics and other subjects in the German language whilst they are learning the language itself. German teachers often try to meet the special difficul-

ties of these pupils by letting them work in highly structured learning conditions: no ambiguous questions, no periods of experimentation, no variety of solutions. The teachers rather give them strict and definite rules as how to solve certain problems with mathematical means. They forget that in doing so they very often prevent their pupils from gaining and reflecting upon those experiences that alone would enable them to understand the instructions given by the teacher.

We are looking for possibilities to organize classroom work in such a way that—communication problems between pupil and teacher notwithstanding—the mathematical learning process starts with the description of a situation that touches upon the pupils' everyday experiences and taps their mathematical expertise.

I will try to introduce our conception with an example from our materials. I will focus on two questions:

- What can be done to help the foreign pupils to understand what they are asked to do?
- How can we help them to reflect on what they have done?

For the work with German and foreign pupils in their first year of secondary school, we were looking for different subjects that would give them the opportunity to work with ratios and proportions.

By chance I found Runer Jonsson's story *The Danes* in the Norwegian children's book on Wickie the Viking.

Wickie by no means meets the expectations of his father, the vigorous chieftain of Flake, yet he is the hero of the story, and with wit and imagination repeatedly saves his father from the scrapes he gets himself into.

One day the Vikings are on their way home from a raid with a ship full of booty and are approaching the Danish customs house. "Last year they took 2 out of every 20 bracelets," Snorre sighs. "This year it is even worse! They take 10 out of 25, out of everything you can count." It is up to Wickie to find some way out.

In order to make absolutely sure that the foreign pupils in the class would get the information that was important for the further work, we started—as we did before—with a comic (see comic strip at the end of the article).

On the first page we meet Wickie and find out when and where the story takes place. We also learn that there are three different places where the Vikings could pay their duty on their way home. While the ship is waiting outside the harbor, Wickie goes ashore to find out about customs regulations.

What are the customs collectors doing in Kalundborg? Out of 8 barrels they take 2, out of 12 bottles 3, out of 4 bales of cloth 1, etc.

"Always 1 out of 4," Orhan explains.

We look at the next page: 5 out of 25 in Assens and 3 out of 12 in Helsingör. What is better?

A fierce discussion ensues:

"Helsingör, there they only take 3 pieces and that is less than 5."

"But if they bring 25 and have to give 5 they keep 20 and that is more than 12 minus 3."

It takes some time until they listen to Peter who reminds them: "Not 3, 3 out of 12!"

Then Hacer goes to the blackboard and writes down: 5 out of 25 in Assens, 3 out of 12, that is, 6 out of 24 in Helsingör. Pointing to the numbers she tries to explain that in Helsingör they take more from less than in Assens. "Not good."

And back to Kalundborg, there they take 1 out of 4 and that means 3 out of 12. So they decide to go to Assens. But on their way they are listening to Wickie who has an excellent idea.

During the following weeks, the Danish customs became a familiar framework in order to deal with many different problems: The duty was paid on the cargo according to varying rules. The rules themselves were analyzed.

There was a burglary in the customs house. In order to save himself trouble, the customs officer tried to falsify the data for the cargo to make them correspond to the duty remaining in the store. The Vikings were angry. With the permanently changing rules they always had to calculate anew which was the best rule just now. They demanded and got a standardization of the rules.

It turned out—here, as well as in other cases—the moment the pupils have understood the context as a whole (customs) it is possible to ask questions which—isolated from the context—they would not have been able to grasp.

Obviously not every problem that arises within a given context can be discussed if there is only a low level of linguistic aptitude. This is especially obvious when one tries to analyze the structural component parts of the chosen context, such as the concept of justice underlying the rule of proportional customs duty.

However, the proper choice of context and its appropriate visualization basically allows to work successfully with this method even if there exists only a minimal grasp of the language: We did work in the described manner with Turkish youths who stayed in Germany between two weeks and two months and had their first German lessons at the same time we instructed them.

Given the language barrier between pupil and teacher, the means of representation chosen by both sides are of fundamental importance when the teacher tries to understand the problem-solving attempts of the pupils , when he wants the pupils to understand their mistakes and to work out better problem-solving strategies. But, as the teachers with their highly structured procedure hardly ever learn anything about the pupils' own methods and expertise, the means of representation chosen by the teachers very often are on much too high a level of abstraction.

Remember, the story of Wickie: According to the schoolbooks for this grade one had to introduce fractions in order to describe the proportion of duty. That was to be the result of the analysis. But none of the foreign pupils we worked with ever used a fraction,

other than 1/2, when talking about duty. Some of their German classmates used 1/4 or 1/3, but never anything else, not even 3/4 or 2/3. Their way to describe the situation was: When the Vikings have x pieces on board, they give y pieces to the Danes.

The mathematical thinking of many foreign pupils is dominated by additive structures, and even if they use simple multipliers, as a whole I think that they do not understand the role of division for the quantification of ratios and proportions at all.

Therefore, our Danish custom officers use working sheets of the following format

	Rule	Workspace	
Duty	1		
Cargo	4		50 barrels

which our pupils could fill out in many different ways, e.g.:

Duty	1	2	4	8	12	1/2	12 1/2
Cargo	4	8	16	32	48	2	50
Duty	1	10	2	1/2			12 1/2
Cargo	4	40	8	2			50
Duty	1	0.25					12.5
Cargo	4	1					50
Duty	1						50 : 4 = 12.5
Cargo	4						50

When looking at the way the pupils worked with these sheets, we found similar limitations in their use of the format as we saw in their analysis of the context. They never consider relations in the vertical direction. When looking for a multiplier in order to transform one number into another, they either know it by heart or find it by trial and error. If it is too big or fractional, they either do not answer the question or go back to addition, doubling, and halving.

Thus, after they had solved the original problem, the pupils where asked to look at the completed sheets again, e.g.,

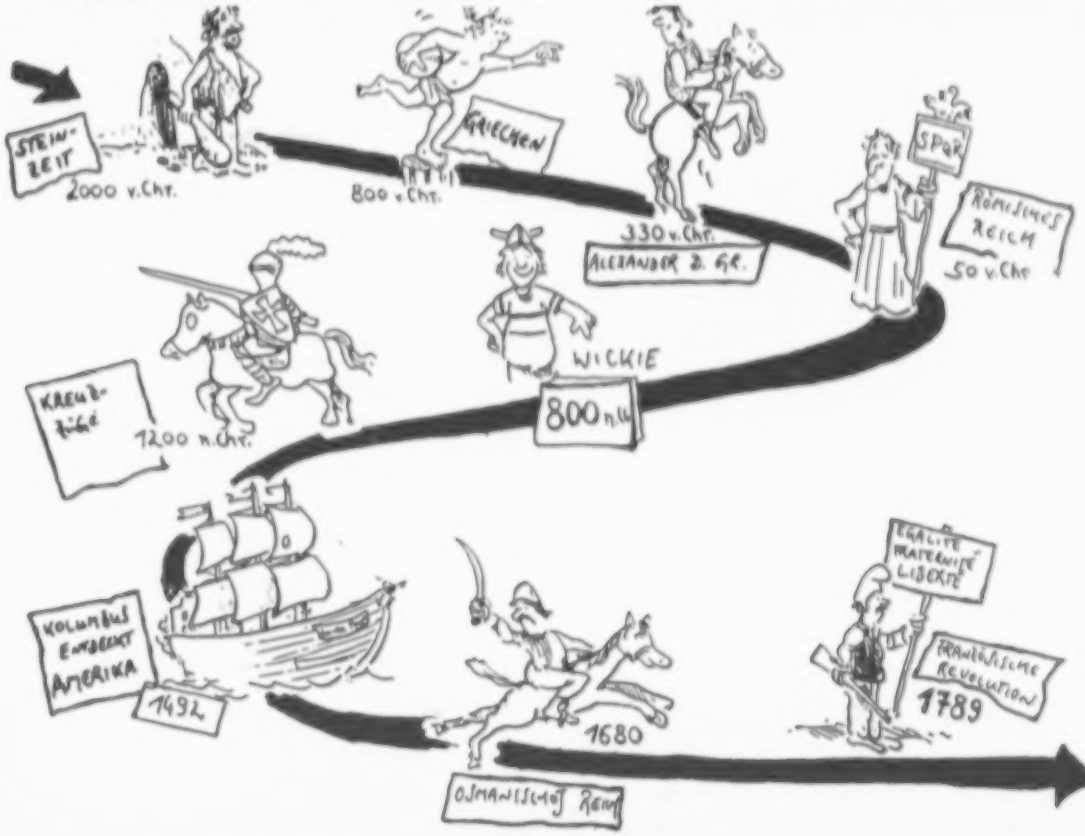
Duty	20	40	10	5	1	76
Cargo	60	120	30	15	3	228

with questions like the following:

- We found out that the duty for a cargo of 228 is 76. Is there any other way to get this result?
- Take your calculator and try to find a multiplier that will transform 60 into 228. Is there another possibility to get the result than trial and error?
- You used many different strategies to calculate the duty for different cargos. Is there a strategy that always works?

In this manner we tried to achieve two things at the same time: We made the pupils use their own strategies in order to solve the given problems. Thus, they could see that they were able to do it by themselves and find out it is worthwhile doing one's own thinking. We also tried to give them a chance to discover the relations they ignored when analyzing the context, but which nevertheless are apparent in their own solutions.

These new insights do not change their way of thinking about the context instantaneously, but maybe it will do so in the long run.

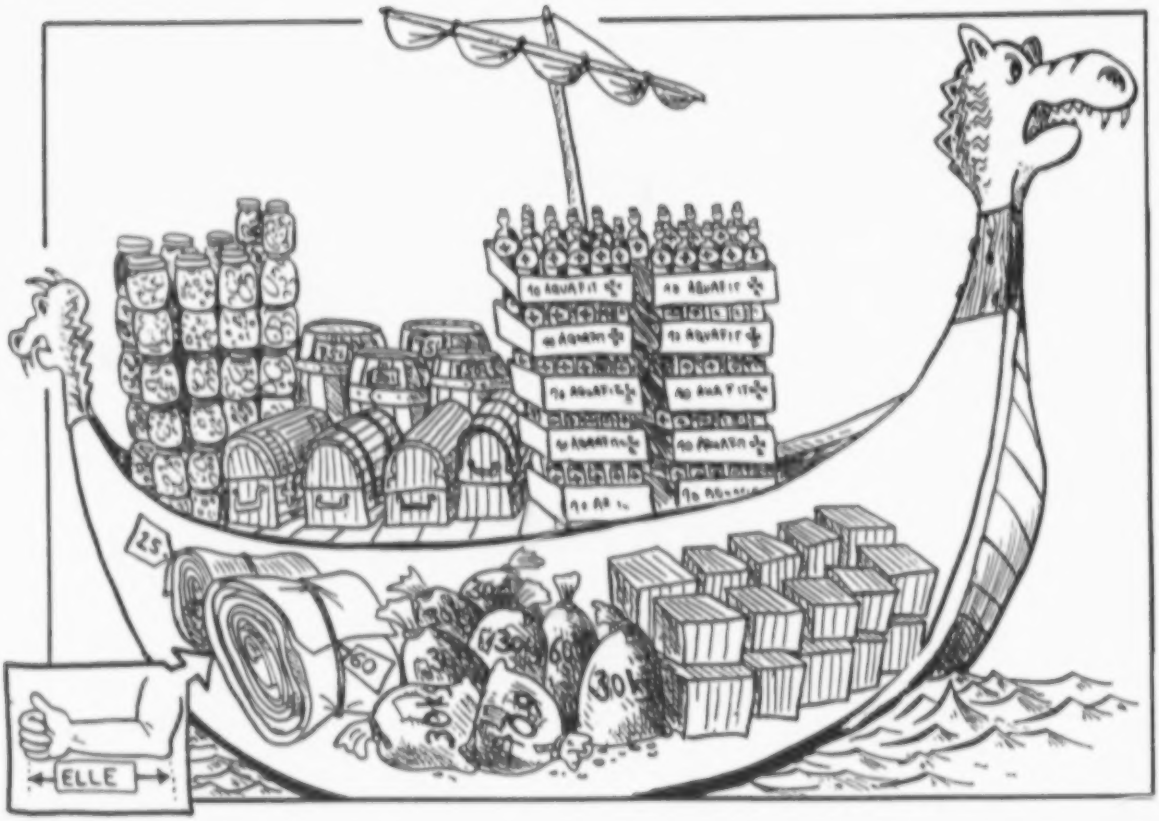


customs houses

① Assens

② Kalundborg

③ Helsingör



ZØLL
ZØLL
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ZØLL
ZØLL
DUANE

How much do they take in Assens and Helsingör?
5 out of 25 in Assens
3 out of 12 in Helsingör

We go to.....
and I know what to do!

Mathematics Education From a Bicultural Point of View

Helen Watson

We are concerned with mathematics learners within the educational institutions of societies where cultural paradigms are in competition. I focus my discussion on mathematics education in the classrooms of contemporary Nigeria, where competing ethnomathematical traditions operate in the society.

Mathematics curriculum and pedagogy in Nigerian schools must acknowledge and use the dual heritage of Nigerian children: the ethnomathematic traditions of Africa, and the ethnomathematic traditions of Western Europe. To deny this dual heritage is to prejudice the cognitive development of Nigeria's bilingual/bicultural children. To use it is to offer the children the chance of enhanced cognitive development (Watson, 1987).

To understand the dimensions of the culture clash which underlies mathematics lessons in the classrooms of Yorubaland in Nigeria, let us consider natural number. The Yoruba language construct of natural number is located in the discourse of *iye*. The discourse achieves for Yoruba speakers what quantification achieves for English speakers—the allocation of objective value. It is important for Yoruba speaking children to be inducted into this discourse, for it is a central discourse in Yoruba life. But for children in Yorubaland this is not sufficient. Yoruba children must also appropriate the discourse of English language objective valuing. For Western technology is a significant social force in Nigeria, and Western technology is accessible only through Western forms of talking about, and valuing over, the stuff in the world. Appropriating English language forms of quantification, and knowing where you stand in two 'knowledge worlds' is an essential prerequisite if Western technology is to be an emancipatory force for Yoruba people.

But more than this is involved. Disparities at the conceptual level, unless confronted, can cripple bilingual/bicultural learners in their progress towards appropriating the abstractions and formalized generalizations that constitute mathematics. As Yoruba speakers, children bring to school a particular understanding of the 'abstract object' natural number, an understanding which encodes particular patterns associated with Yoruba language use and Yoruba material practices of quantification *(kà and wòn)*. In school, in English language mathematics lessons, they meet a different concept: natural number as used in English language quantification. The intuitions of cardinality that are encoded in Yoruba language natural number and in English language natural number are quite disparate. This is important, for understanding of cardinality is central to teachers' strategies in helping children grasp the abstractions of mathematics; it is mostly through their understandings of the workings of natural number that learners gain entry to the world of formal generalizations—mathematics.

We have strong intuitions about 'correct' models of natural number, no doubt related to the fact that number derives from our material practices of tallying. The noted American philosopher Quine intuitively chooses von Neuman's model as 'correct' (Quine, 1960), this account favors intuitions of number as a one-by-one collection of predecessors; the members of an n membered set are paired with the first n numbers of the series of numbers. In this model, 0 is Ø, the empty set. The set which contains the empty set as its sole member is {Ø}, one; the successor of this number is {Ø {Ø}}, or two, and three is the set of all sets smaller than three {Ø, {Ø}, {Ø, {Ø}}}. The successor of any number is generated by adding the successor of 0, that is 1. In this version, a number is the last number of the series reached through one-by-one progression. A numeral names the point at which the progression ceases. This is the view of natural number encoded in the conventional Western mathematics curriculum and associated with Piagetian-based approaches in mathematics pedagogy.

However, an appropriate model for Yoruba language natural number is very different to this, being represented better by Zermelo's account. In this model the number n is a single membered set, the single member of the set is n – 1. Zermelo has 0 as Ø, a set with no members. Then 1 has the empty set as its sole member, {Ø}. The set which contains the unit set, 1, as its sole member is 2, {{Ø}}. Three is the set which has two as its sole member, {{{Ø}}}. In this version each number is totally subsumed by its successor, and any one number has a unified nature. For a Yoruba speaker, this is the model of number which would be taken as the intuitively correct account. Yoruba language numbers carry 'the flavor' of a divided whole, there is no sense of a 'linear stretching' towards the infinite there (Watson, forthcoming).

From a logical point of view, the models agree in overall structure in identifying a number as a particular class of classes. They agree too on relations defined under that set: Each is demonstrably a recursive progression, with a successor function which follows the order of that progression. The ordinary arithmetic operations are defined for these different versions of number in the same way. But, the accounts differ profoundly in the way cardinality is defined. For English language natural number 14 has 14 members. For Yoruba language natural number 14 has one member only. From the point of view of constructing the intuition of natural number, the structures are deeply disparate. And in the primary school mathematics lessons in Nigeria, it is an aspect of learning that is too easily overlooked.

What are the implications of this for curriculum and pedagogy in the Yoruba language primary schools of Nigeria? The complexities of the problem mean that with mathematics Nigerian teachers sometimes lose confidence. With great confusion and consternation they struggle to teach what they have been led to

believe are the "universal truths" of Western mathematics. Their anguish is increased because mathematics as 'the language of science and technology,' is revered in a society struggling to modernize, and school mathematics examinations have become the cutting edge of the social stratification that schools effect in Nigeria.

What does it need for practice in mathematics lessons to change from being injurious to being emancipatory? It needs teachers with the political will to confront 'mental' colonialism: European colonization via the school curriculum. Resolution is necessarily a political matter. It will be achieved by self-reflecting bicultural teachers who understand the place and nature of mathematical concepts in their two language worlds, and who acknowledge and can use the two competing ethnomathematic traditions in their community.

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Mathematics Education and Cultural Continuity

Norma C. Presmeg

There are at least two situations in which cultural discontinuity may be experienced in the context of mathematics education, to the detriment of the learning of mathematics by pupils in such situations. One such situation arises when pupils of one homogeneous culture are taught through a mathematics curriculum which has been designed for a different culture. This is the situation in which mathematics learners in many developing countries find themselves. In this situation, the learner, teacher, and textbook may all be operating in the learner's mother tongue (which may also be the teacher's), and yet difficulties may occur which result in a rote-memorization syndrome. Berry (1985) has illustrated this situation graphically in the case of Mothibi, a six-year-old Motswana boy learning mathematics through the medium of Setswana, in a Botswana village. As Berry suggests, the remedy for this type of cultural dislocation in learning mathematics is to replace the curriculum, which in this case was designed in Britain and carefully translated into Setswana, with a mathematics curriculum which resonates with the language and culture of Botswana.

In another situation, pupils may have difficulties in learning mathematics in classrooms in which they experience cultural dislocation because their home culture and possibly home language do not resonate with those prevailing in the school (Spindler, 1974). This is the multi-cultural classroom situation which is found increasingly around the world.

The two situations pose different problems and therefore suggest different solutions; but in both cases it may be argued that the dislocations experienced by pupils are not inevitable. The remainder of this paper is devoted to the second situation, specifically to some aspects of cultural continuity in the multi-cultural classroom context, a context made complex by the diverse backgrounds and home languages of pupils. Some of the illustrations are drawn from Uthongathi, the first of the multi-cultural New Era Schools Trust (NEST) schools, which came into being in January 1987, on the coast of KwaZulu-Natal, South Africa. The writer and three of her students, Anita Frank, Yanum Naidoo, and Manjul Beharie—all mathematics teachers-in-training—are conducting a pilot research project at Uthongathi to investigate cognitive and affective aspects of learning mathematics in multi-cultural classrooms. Twenty-five pupils (8 black, 8 white, 9 Indian) and their three mathematics teachers (1 Indian, 2 white) are involved in the project. In the still segregated South African social context, schools such as Uthongathi seem to stand out as beacons of hope for the future.

A survey of relevant anthropological literature included studies of the Mistassini Cree (extreme discontinuity), the Sisala of Northern Ghana (continuity), the Ngoni of Malawi (education is "learning how to live") and an urbanizing German village in the Remstal, in which social and economic health were concomitants of rapid social change. In these studies and others, factors which lead to cultural dislocation or to cultural continuity in the school context were identified (Presmeg, 1988). For pupils to experience cultural continuity rather than dislocation in multi-cultural mathematics classes, the following four points would seem to be of importance.

(1) Children need the stability of their cultural heritage, especially during periods of rapid social change.
(2) The mathematics curriculum should incorporate elements of the cultural histories of all the people of the region.
(3) The mathematics curriculum should be experienced as "real" by all children, and should resonate, as far as possible, with diverse home cultures.
(4) The mathematics curriculum should be seen by pupils as relevant to their future lives.

With regard to the first point, need for stability of the cultural heritage, it is noteworthy that KwaZulu Government Schools teach a subject called "Ubuntu-Botho" one period per week in each year of school. As Nzimande wrote,

> Ubuntu is a concept that is usually described as the foundation of all African societies. It is claimed by its proponents that Ubuntu did not only form the basis for the so-called African world-view, but also 'runs through the veins' of all Africans. (1987, p. 4)

It is more than its meaning, "being human," implies. Through Ubuntu in KwaZulu schools, deeply ingrained aspects of Zulu culture and customs are transmitted: for instance, "ukuhlonipha," meaning "respect," is the custom of honoring elders, parents, leaders, particularly men, in the patriarchal Zulu society. The teaching of Ubuntu-Botho has been severely criticized (e.g., by Nzimande, ibid.), but most Zulus are deeply proud of their nationhood and it is possible that some of the positive aspects of Ubuntu-Botho could be valuable in multi-cultural mathematics classes as a basis for a Zulu-world-view component of a mathematics curriculum.

At the same time, the cultural coherence of Zulu nationhood, produces certain side effects which create difficulties for Zulu pupils in multi-cultural schools such as Uthongathi. Of the eight black pupils in the Uthongathi study, only two speak English at home, while all the Indian and white pupils have English as a home language, even if Hindi, Tamil, or Gujarati is also spoken. Although Zulu and Afrikaans are offered as subjects, the medium of instruction at Uthongathi is English, and many of the Zulu pupils, particularly, experience language-related difficulties in their learning of mathematics. These difficulties are evident, especially, in transcripts of interviews with younger Zulu pupils or those who are new to the school. The researchers noticed that although interviewees named pupils of all race groups amongst their friends, their best friends, in a majority of cases, came from their own race group. An Indian girl, Bhavna, articulate as usual, expressed the position lucidly as follows:

> When it comes to speaking and mixing we do that very well; there is no racial barrier there. But you would find in close friends we generally stick to the same race group. That's not done intentionally: in my opinion it is done because we come from similar backgrounds and environment and you enjoy certain things in life—you feel comfortable. (The interviewer was Manjul.)

Perhaps prolonged contact with pupils from different cultural backgrounds at Uthongathi will change this tendency, particularly amongst the boarders. All pupils at the school take a subject called Comparative Religion. A Xhosa pupil, Teboga, whose religion is Anglican, commented,

> Here we compare other religions with our own. We see that some things are similar and others different; for example, the Hindu's Trinity is God the Creator, God the Preserver, and God the Destroyer. (The interviewer was Manjul.)

If the mathematics curriculum reflected the subject "Comparative Mathematics," in essence if not in name, then the second point of importance above would be satisfied. Aspects of the cultural histories of all the people of the region would be incorporated in the mathematics curriculum through the cultural mathematics of all the learners. Paradoxically, it is possible that such a curriculum might simultaneously foster cultural continuity and cultural change, as in creasing mutual understanding leads learners towards each other while retaining their pride in the uniqueness of their cultural roots.

For cultural continuity rather than dislocation in multi-cultural mathematics classes, the third and fourth points suggested that the curriculum should be experienced by the learners as real and as relevant to their future lives. Although the curriculum may embrace cultural mathematics in its content, an increasing emphasis on process and problem solving will be necessary until, by the time they are in high school, learners see mathematics as a powerful way of thinking—a way of knowing (Bishop, 1988)—which in fact transcends differences in cultures.

Working out details of such curricula for the unique needs of learners in each country will involve mathematics educators from diverse cultures in those countries, for decades to come.

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Ethnomathematical Practices

How can the mathematics learning situation be structured as to provide for the acceptance of the child's ethnomathematical knowledge? The panel members offer various examples of ethnomathematics: Salimata Doumbia, Côte d'Ivoire, speaks about the mathematics in some traditional African games; Sergio Roberto Nobre, Brazil, describes the mathematics involved in the most popular, but illegal, lottery in Brazil. Nigel Langdon, Ghana, presents various ethnomathematical activities like mathematics of workmanship, craft, and economy as cultural starting points for the learning of mathematics.

Mathematics in Traditional African Games

Salimata Doumbia

Introduction

As part of the teaching of mathematics, the Institute of Research in Mathematics of Abidjan (IRMA) has set up a workshop "in search of mathematics in the African socio-cultural environment" to study and research in rural settings the mathematical basis of Ivory Coast games.

In a society where one is overrun by "educational games" coming from outside, we have judged it necessary to do such research for two reasons:

(a) The first reason is immediate. It is to recognize the value of traditional African games so that Africa is represented within the development of mankind.

(b) The second reason is a professional one. The best teaching in our opinion must create motivation, and games have this advantage. Thus, we wish to advance the pedagogy of mathematics by integrating the socio-cultural environment of the pupil into the content and the teaching methods of mathematics, thanks to a better understanding of the supports which exist in real life.

The Objectives

The objectives which we have determined are the following:

(a) start an inventory of "mathematizeable" games;
(b) resolve the mathematical problems which are posed in these games;
(c) illustrate these games with mathematical models;
(d) try, as far as possible, to find pedagogical uses for these games in the teaching of mathematics (using the games to teach mathematical ideas).

Method of Working

Inventory of Games

In the first instance it was necessary to collect information. We have chosen the maximum number of documented forms of game, and at least one game in each category as the object of one study. Our sources of information have been surveys, films, and books.

Classification

Once the games have been listed, and the rules of the games have been analyzed, we have proceeded to the classification of the selected games. We have found five categories.

(a) *Verbal Games*
Traditional teaching in Africa is oral. Knowledge was transmitted from generation to generation through proverbs, stories, songs, games, ... In the domain of games one meets games based on speech which we call verbal games. There are two types of verbal games.
Games of memory. A game of memory is a game which calls for mnemonic qualities in the player, who must be able to follow by memory the transformations in the given game. It consists of following a path by memory. This demands a complex organization of the perceptual field and develops in the players combinatorial spirit, speed of mental calculation, as well as visual and tactile memory. The mathematical problem is a problem of enumeration.
Games of counting. A counting game is based on a counting song which the players sing while counting.

(b) *Simple Calculating Games*
These are games which conceal a simple algorithm, the knowledge of which assures victory to the one who knows, which makes him a "magician" or a "sorcerer" in the eyes of those who do not know the algorithm.

(c) *Games on a Checkerboard*
These are games which are played on a board, and are generally complex. They are games of strategy which develop the sense of shrewdness and ingenuity, and the strategy demands the capacity for anticipation, for evaluation, for decision in order better to fulfil the mission by reducing the chances of his adversary. "Thinking what the other would do" is what games of strategy are concerned with.

(d) *Games of Chance*
A game of chance is one in which the result at the end of a single play cannot be determined since a play gives rise to at least two results (true or false, for example).

Study of the Chosen Games

The games have been analyzed according to the following plan:

(a) *the number of players:* more than or equal to 2. This denotes the "social" character of African games;
(b) *the material:* with the exception of "awale," the material is generally fragile, non-transportable, and rudimentary: sand, grain, shells, cowries, sticks, stones, ...;
(c) *the development and rules of the game;*
(d) *the mathematical analysis;*

Results

Our objective here was to make use of motivation in the teaching of mathematics, from pre-school through primary to secondary school. But we realized that the analysis of the most simple game could result in difficult calculations whose development goes beyond the secondary stage (for example: Markov chains, matrix inversion, and graph theory).

Example of Use at Secondary Level

Calculating probabilities with le Nigbe Alladian (Côte d'Ivoire). Two players each throw four cowrie shells. The points scored are as follows (cowries which are sea shells, land either up + or down –, but not with equal likelihood).

+ + + +	5 points
– – – –	10 points
+ + – –	2 points
+ – – –	
+ + + –	0 points

(Cowries were also formerly used as money in Africa.) Experimental study: 1,000 throws of four cowries simultaneously gives p(head) = 2/5.

One defines the random variable x which for one throw of the cowries corresponds to the number of points obtained, and one finds:

	0	with probability 312/625
	2	with probability 216/625
X =	5	with probability 16/625
	10	with probability 81/625

Note that without any knowledge of the calculation of probability, the players have managed, by their attempts, to adopt a clever counting system, in order to balance their chances. The probability of scoring some points is 313/625 (being 50.08%) as against 312/625 (being 49.92%).

Impact

(a) Our research is a modest contribution to the preservation of African cultural heritage. It is to be followed up.
The richness of the universe of African games is such that it is not always possible to find a mathematical theory which will allow the analysis of a particular game. But it is necessary to start somewhere.

(b) There is an impact of a pedagogical kind.
(1) Some experiments are being conducted in secondary, in primary, and in pre-schools in order to develop pedagogical materials, including:
- a brief history of the game;
- the conduct and rules of the game, with possible variants;
- the pedagogic value of the game according to the stage of the pupil.

(2) The analysis of the cowrie games allows us to escape from the classical case of "heads or tails" with a coin.

Thus, the cowrie games and the pea games are used in courses on probability and statistics of the 'lère C et E de la collection IRMA' collections of teaching materials used in the colleges and lycees of the Ivory Coast.

(c) One can simulate games on a computer. The computing section at IRMA has simulated on micro-computer a number of the cowrie games.

The Ethnomathematics of the Most Popular Lottery in Brazil: The "Animal Lottery"

Sergio Roberto Nobre

A blend of dreams, tips, animals, numbers, and luck, "Animal Lottery" is part of Brazilian folklore and is played all over the country—in the slums, the outskirts, the commercial centers, and even in the National Congress and the Government House. The game directly involves 50,000 employments in the state of Rio de Janeiro alone, generating a daily circulation of about US$ 20,000,000.00. All this in spite of being a lottery forbidden by Brazilian law.

There are 25 animals in the list, each one with a respective number.

1	ostrich	14	cat
2	eagle	15	alligator
3	donkey	16	lion
4	butterfly	17	monkey
5	dog	18	pig
6	she-goat	19	peacock
7	sheep	20	turkey
8	camel	21	bull
9	snake	22	tiger
10	rabbit	23	bear
11	horse	24	deer
12	elephant	25	cow
13	cock		

To each number there is a correspondent *group* of animal. For example: *group of the cat* = 14, *group of the turkey* = 20, *group of the donkey* = 3.

To each animal there are also four corresponding *tens*, from *01* to *00*, following the same animal order:

01 – 02 – 03 – 04 – ostrich
05 – 06 – 07 – 08 – eagle
09 – 10 – 11 – 12 – donkey
...
93 – 94 – 95 – 96 – deer
97 – 98 – 99 – 00 – cow

The drawing is done everyday. Either they take advantage of the numbers drawn for the official lotteries, or other drawings are promoted under joint consent among the commanders of "Animal Lottery" in the country. Five prizes are drawn as follows. For example:

1st prize – 8341
2nd prize – 3988
3rd prize – 0402
4th prize – 2715
5th prize – 1219

In the result above, in order to know which animal was drawn, one must observe the *ten* of each prize. The ten of the 1st prize is 41, which corresponds to a ten of the horse (41 – 42 – 43 – 44), so "the 1st prize is the horse." The ten of the 2nd prize is 88, which corresponds to a ten of the tiger (85 – 86 – 87 – 88), thus "the 2nd prize is the tiger." "The 3rd prize is the ostrich," "the 4th prize is the butterfly," "the 5th prize is the dog."

The Mathematics Involved, the Kinds of Games, and the "Folk Knowledge"

In order to check which animal "won," one must look at the ten of the drawn number and, by analogy, find the corresponding animal. In doing this analogy, the person is working with the multiples of the numbers, which is done through mental arithmetic.

Many kinds of games can be done and the amount of money to be won depends on the kind chosen. The prize also depends on how much the gambler bet on it.

Some Kinds of Games

(1) *Group:* The gambler bets on a chosen *animal* and on the corresponding prize (that is, a number which has the *ten* of that animal).
Example: Group 9—1st prize
The gambler bet on the snake and will win if the snake is drawn in the 1st prize. Gain: 1/16—for every "cruzado" (Brazilian currency) he will get 16 cruzados.

(2) *Hundred:* The gambler bets on *a hundred* and on the prize in which it appears.
Example: Hundred: 242—1st prize
The gambler laid is bet on the hundred 242 and will win if it is drawn in the 1st prize. Gain: 1/500.

(3) *Thousand:* The gambler bets on *a thousand* and on the prize in which it appears.
Gain: 1/4,000.

(4) *Inverted Hundred:* The gambler bets on a hundred and its digits can be permuted in order to correspond exactly to the drawn prize.
Gain: 1/83.

(5) *Pass:* The gambler bets on two animals, seeing that the first one must be drawn in the 1st prize and the second must be among the four other prizes.
Gain: 1/80.

There are other kinds of games, but the ones above are enough for us to show, in a few lines, our work.

The Mathematics Used by the Gamblers

"Animal Lottery" is admittedly the most popular of all games of chance practiced in Brazil and, among its followers, there are people from the several cultural and economical classes existing in the country. Most of its gamblers (which corresponds, in terms of percentage, to the majority of the population) are analphabet or semi-analphabet. Nevertheless, they make their bets, choose the most convenient kind of game, check the results and figure out how much they won (if so) very easily. They do calculations which appear to be enviable to many people who have spent years in school, but are not able to do them.

Example

The gambler makes many bets in only one game. He bets on "thousand" and "hundred" from the 1st to the 5th prize.

He will win if the thousand 3785 is drawn in one of the prizes. He himself does the calculations in order to know how much he will win. In this case, he puts Cz$ 100.00 on the T and H (thousand and hundred) for the 1st to the 5th prize.

To figure out his gain, he must know how much was put on in each kind of game for each prize.

100.00	–	thousand and hundred
50.00		on the thousand
50.00		on the hundred
5 prizes	–	10 for each prize of the thousand
		10 for each prize of the hundred

The thousand he had bet on was drawn in one of the prizes! The gambler won the thousand—the thousand pays 4,000 for 1, thus he gained 10 x 4,000 = 40,000.

The hundred was also drawn! The gambler also won the hundred—the hundred pays 500 for 1, thus he gained 10 x 500 = 5,000.

Altogether he made 40,000 + 5,000 = Cz$ 45,000.00. He calculated this many times, without ever having been to school.

The Use of "Animal Lottery" in the Classroom

Taking into account the students' knowledge of the game and their involvement with it through their parents and acquaintances, we introduced the study of "Animal Lottery" in the 2nd degree of high school (which, in Brazil, means the three last years of it). The students were highly motivated, since, after all, we would deal with a subject which is related to their everyday life. We did not need to worry about teaching the students how to play the game. They already knew that, and if not, their friends and relatives would have taught them. Our preoccupation was to emphasize the mathematics involved in it. We can divide in three stages how the work was developed:

The Contact With the Game

Since the game is part of the students' cultural context, there was plenty of motivation, causing the involvement of the whole class. The students already knew the kinds of games, the ways to play them, how much one could gain, etc. All of them knew that, and they even gave the teacher some additional information about the mysticism and the popular beliefs that involve the game.

The Work on the Game and its Relation With Mathematics

Some proposals were worked out in respect to the mathematics of the game. We remarked the following.

We suggested the students who knew about the kinds of games and the gains in each one, *to figure out the probabilities of gain in each kind of game.*

The students, being in the situation of having to solve the problems by themselves, began to come to some results.

Group – probability of gain = 1/25
Hundred – probability of gain = 1/1,000
Thousand – probability of gain = 1/10,000

Until then, everything was getting on well. We had been working on simple situations of probability. The students found the correct results without the teacher's help. From then on, the difficulties began to arise.

In order to calculate the probability of gains in the other kinds of games, it was discussed with the students about the necessity of promoting an improvement in their knowledge of mathematics. Such improvement was done by using the information we already had and, as a result of the students' involvement, we can say there was a meaningful improvement concerning theory of probability and combinatory analysis.

The Conclusions Deduced From the Theoretical Foundation

The conclusions began to be deduced from the moment the students began to compare the *quantity of gain* with the *probability of gain* in each kind of game.

At the beginning the students were excited, searching for a more advantageous kind of game for themselves. Afterwards they concluded that *there is no advantageous kind of game for the gambler.* The greatest profit is always in the hands of the ones who collect the money for the bets and pay the prizes—the owners of the business.

Conclusions

Besides the involvement of the students with the subject seen in class, we must remark that mathematics itself began to be seen as an accessible and easy-to-learn subject. The students were much more motivated to study it from then on.

Another important point is the relation of the subject with the students' social and cultural context: The connection between the mathematical subject and their cultural context made the student begin to understand better the social relations of his own life. In the specific case of "Animal Lottery," the student can come to conclusions about the fact that, despite being forbidden by Brazilian law, it is freely practiced all over the country. This fact is directly related to the economical power of the "owners of the business" (which is obtained through a mathematical "maneuver" in the relation *probability of gain/real gain* in the game) and their connection with the political and police authorities—in a flagrant situation of corruptness.

Having the mathematical elements to understand society in his hands, the student began to see society with other eyes, thus being able to interfere with it and ceasing to be just a spectator.

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Cultural Starting Points

Nigel Langdon

I do not think that it is a coincidence that mathematics educators are concerning themselves with open-ended, investigatory work, and problem-solving activities and, at the same time, exploring multi-cultural issues and ethnomathematics. There *are* other pressures on teachers, pressures arising from mixed-culture urban communities and from people wishing to identify with their root culture. But I also recognize that as some educators are able to free themselves from the constraints of the traditional syllabus, which emphasizes *content* at the expense of *process*. The educators are better able to explore mathematics and ways of learning mathematics. They are also able to explore and to respond to the ways in which people naturally do mathematics. I should also say, they are able to explore situations in which they see mathematics as having a presence. I want to emphasize how important it is for us to continue the exploration of how children (and adults) actually perform mathematical tasks in the community, so that school mathematics and later mathematical processes can be built more solidly, starting with the foundations which people *actually* use.

Research indicates that for many people, there is an assumption that mathematics is only concerned with what is learned in school ... That it is a subject which has little or nothing in common with their language, their methods of working, or their cultural activities and background.

This is accentuated in Ghana, where I am presently working as mathematics advisor. But I think it is true for a very large number of pupils or students in very many countries.

We are exploring the idea that the learning of mathematics will be more successful if it is rooted in

the language, the processes, and the culture of the individual.

Whether your motive for considering multi-cultural issues is:

- to give value to other cultures (providing education and combating racism);
- to use one's own culture to help teach mathematics;
- to negate the effect of colonial education;
- —or political—.

We are suggesting that mathematical activity which has its roots in familiar cultural activity can be an extra dimension and an exciting development in mathematics education.

My experience in Ghana and in the UK, where I have used cultural starting points for different reasons, leads me to believe that, in practice, a multi-cultural response to ethnic minorities in urban societies is very similar to the use of cultural activities which teachers in developing countries employ to counter the adverse effects of colonially inherited education systems.

Both systems are intent on giving value and creditability to activities performed in the community ... and both situations draw out of those activities the mathematics which is inherent in them, as well as providing a missing link between mathematics and the local culture.

In Ghana, we have, so far, discovered that starting points for doing mathematics seem to be more fruitful if they are somewhat familiar to the learner. That is not to say that all starting points must be familiar in order to be successful—something new can be stimulating and highly motivating—but a familiarity in terms of content or process generally makes new knowledge more accessible for the majority of children. As we have explored more problem-solving and investigatory work, there has been a growing realization that many of the valued mathematical skills and processes of doing mathematical tasks in the classroom are already evident in many social or cultural activities. These mathematical aspects may not be explicit. However, they exhibit the basic components of mathematical behavior—the development of which we are all concerned with. For example, we recognize that basket weaving requires and utilizes knowledge about symmetry, rotation, and congruence (and basket making may well be something which children can already do), but that pupils cannot respond to the mathematics therein, thus we have not yet addressed the problem of the transition from ethnomathematics to the more formal mathematics which we value. So, if we wish to make use of ethnomathematics to make school mathematics more accessible and more motivating, then we are looking for effective bridges ... Bridges between culturally based activity and more formal mathematics. It is for this reason that we are looking for cultural starting points—topics and ideas which have roots in the community and which are rich as a teaching source because of the mathematics which is written there. I believe they have been successful within the SMILE project (Inner London Educational Authority: Second on Mathematics Individualized Learning Systems) and now in Ghana because:

- they build upon what people actually do;
- mathematics is seen as already being a part of one's culture;
- mathematics is less threatening because understanding and skill in the subject grows out of more familiar usage.

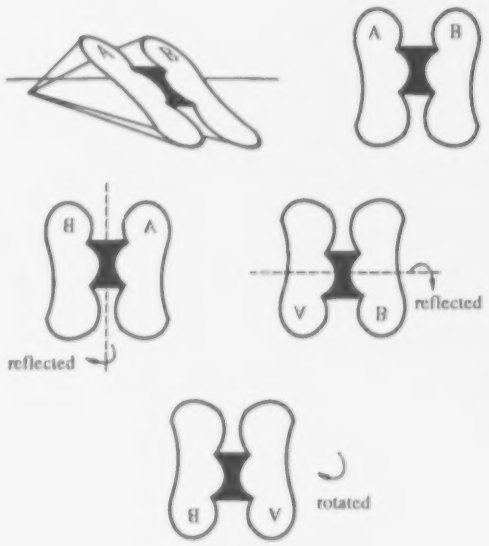
Let me give a few examples of the type of activities which I have termed "cultural starting points."

First of all, as Salimata Doumbia and Sergio Nobre have described, games are sources of enjoyment and of mathematical activity. Much of the thinking which takes place during a game is mathematical thinking. Claudia Zaslavsky (1973) devotes a large section of her book *Africa counts* to exposing the mathematics which is tied up in just one move in a game like Oware. And as Sergio Nobre has illustrated, that students can learn a great deal of mathematics by examining their strategies and the possible outcomes of a move in a game or in a lottery.

Secondly, Paulus Gerdes (1986) gives us a rich analysis of the mathematics that can be explored in basket weaving. Not just the resulting patterns, but also in the creation of the basket by the process of plaiting and folding.

A third example comes from Ghana, where, during a teacher training course, we have explored the designs of Adinkra cloth as a means of teaching about symmetry and rotation.[1] The theorems are not "forced"; the mathematics is easily "unfrozen."

Adrinka design called "Nkonsonkonso"



[1] See Ghana Education Service Teacher Education Division: Modular Programme Mathematics. Module 23.

I do not want to suggest that ethnomathematics or (rather) culturally based starting points are *the* solution to all the problems of poor standards and lack of interest in mathematics. And I think it is important to realize that ethnomathematics is not what it may just seem to be, nor what some people would like us to believe.

Certainly, involvement with culturally based activities has encouraged children (and adults) to mathematicize and has (in the UK and West Africa) been a foundation stone for building up a body of knowledge in mathematics. But I do not think we can claim that ethnomathematics as it is practiced provides "the essence of building up a body of knowledge in a mathematical sense" (D'Ambrosio, 1984, p. 5–6).

My reason for saying this is that we cannot assume that because sand drawings in Angola, or Kente cloth in Ghana or boat building in Brazil are well-known as cultural symbols or cultural activities ... we cannot assume that the knowledge of the object, or of how to make it, is shared by the community.

We cannot bestow upon the whole community, certain skills or attributes which are the property of a small or selected group of artisans.

And, if we recognize that many, perhaps the majority of this group of artisans who create the baskets or the boats are operating in an *instrumental* way, we can see a danger in assuming too much about the existence of ethnomathematics within a culture.

When Salimata Doumbia and Sergio Nobre described using ethnomathematics upon which to build mathematics in the classroom, they were building on particular interests and skills which their students already possessed. And they were able to use that interest and those skills to great advantage.

However, many instances of ethnomathematics do not involve the practitioner in any mathematical thinking other than at a purely instrumental level. I have been trying to identify mathematical thinking in some culturally based activities in Ghana, where I am aware that mathematical considerations may be present. Let me give just three examples.

Firstly, in Ghana, farmers are likely to sow their corn in a square grid pattern

or in an isometric pattern:

Whichever method is used by farmers, the distance between plants is roughly the same, but in the Volta Region there is a predominance of the triangular pattern. When asking farmers why they use the triangular array, I anticipated some answers which would reflect mathematical thinking—but I received none. Farmers told me "We always do it that way here," "My father told me to do it this way," "That's how I saw other people doing it."

Another example is from the market stalls, where small-sized smoked fish is sold in discrete piles, with a generally recognized priced of ten cedis per pile.

At each of several market locations I have used the same questions. The trader and I have agreed upon the price for the total number of piles of fish on display. In the example illustrated, I agreed on 100 cedis for the ten piles. When the trader had grouped all the fish into one pile, I then asked for twice as much, giving the trader 200 cedis. On each occasion of this experiment, the trader has made another ten small piles of fish, which illustrated to me her inability to think mathematically about the situation. Having agreed upon 100 cedis for the larger pile, why did she not make another pile of the same size? Instead, she reverted to making the same small piles, which she recognized as being 10 cedis worth of fish. To what extent was the trader thinking mathematically?

My final example is of basket making for the fishing industry on Ghana's coast. My interest was to find out to what extent the basket makers in Ghana were thinking mathematically.

I asked about different folds. Why like this

and not like this:

Once again, the answers showed little mathematical insight—"It doesn't work if you do it like that," "You have to fold it this way," "This is how to do it."

This leads to the question of whether what *we observe* as mathematical is really mathematical in the mind of the practitioner. It may be that the cultural activity takes the practitioner into the realms of mathematics, but where the cultural process of *doing* excludes mathematical thinking. There may be no intellectual involvement in the activity.

What we may describe as ethnomathematics could simply be instrumental knowledge acquired by rote or by drill.—Quite the opposite sort of skill to that which we would wish to develop in our pupils.

If we accept culturally based activity as valuable, then we must recognize the nature of the activity. To use it effectively may require the teacher to create a bridge between an instrumentally acquired skill and a mathematical behavior which we believe to be valuable.

Amongst contributors to the bank of ideas for cultural starting points, not surprisingly, there is a felt need to justify the activities in traditional mathematical terms. However, the origins of the activity may not be mathematical—or, if they were, they are no longer evident to the artisan performing the activity. Rather, it is we, the mathematicians, who recognize the mathematics, explicit, implicit, or "frozen" in the activity.

When we look at the basket, are we doing anything more than when we look at a pineapple and discover Fibonacci numbers there?

I believe we are doing more, but I am not aware of any research which justifies my belief. I am aware that we cannot pretend that the cultural object is the result of practical mathematics by our pupil. The questions we asked about the basket, or about planting corn, are questions which a mathematician asked—they were not questions which the artisan or the culturally involved pupils were aware of.

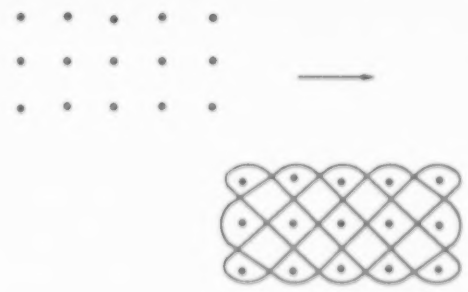
But we want our pupils to be aware of these questions, of this dimension. We wish our pupils to develop mathematical skills from this starting point. We wish that they will recognize mathematical theorems and develop mathematical skills from this starting point.

Our "cultural object" or "activity" is only a vehicle for achieving what we as mathematics educators consider to be a valuable goal. Our experience is that, in addition, pupils acquire a better understanding of mathematics by discovering that it is already a part of their own environment.

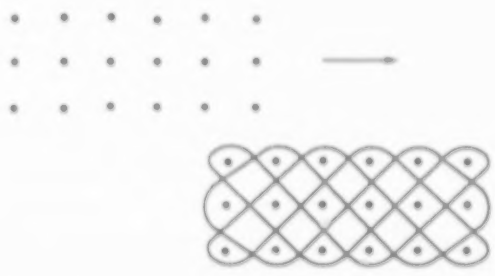
The fact that ethnomathematics is likely to be associated with acquired rote/instrumental skills has required us to use approaches similar to investigations to open up the practice or the knowledge before it can be useful for developing formal mathematics.

For example, in questioning about the Adinkra designs for printing cloth which I illustrated earlier, we asked about whether it would make any difference to the print if the printer picked up the design "Nkonsonkonso" upside-down or the wrong way around? The answer was 'no.' What about the other Adinkra designs? Reflective symmetry? Rotational symmetry? Are there any connections between rotational symmetry and reflective symmetry?

Another example was the use of sand patterns as a starting point which, for most students, led to factors and multiples. They were able to predict the number of loops for a given array.



A 5 x 3 array can be circumscribed in *one* continuous loop. A 6 x 3 array requires *three* loops to complete the picture:



Using such examples from ethnomathematics enabled us to discover that artisans/practitioners are often involved in mathematics without realizing that they are so involved. The starting points therefore had great value in enabling the students to understand that mathematics is already a part of their lives without them having recognized it. And with such awareness they were, perhaps, better at understanding their environment; perhaps better able to make decisions about activities which they were involved with. I say, perhaps, because we have yet to see whether starting points which develop

- a recognition of hidden mathematics in the culture,
- an interest in mathematics,
- a regard for local workmanship and craft,
- an ability in mathematics

will also develop a better understanding in the community in the sense that an informed, aware, mathematically thinking practitioner is likely to be in better control of and have more knowledge about his or her craft.

At this stage I can only report that we have made mathematics both more interesting and more accessible for our students.

Finally, what I have just described places yet more demands upon the teacher—a demand to be aware of cultural starting points and a demand to be able to exploit them to the advantage of the learner. The teacher needs to be aware of the cultural backgrounds of her pupils to understand how the community uses mathematical knowledge in its cultural activities. She

needs to gain experience in utilizing that culture (and what may be instrumentally applied mathematics) to develop mathematical behaviors which are built on understanding rather then rote learning.

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Social Construction of Mathematical Meaning

Mathematics knowledge is both individually and socially constructed, and educators should give a greater priority to the latter than they have typically done. Paul Cobb, USA, outlines an analysis of two selected childrens' construction of arithmetical knowledge during a school year, focusing particularly on the major cognitive restructurings. Erna Yackel, USA, analyzes the social contexts that children mutually construct as they interact to complete mathematics tasks in small groups. Stieg Mellin-Olsen, Norway, describes some of the project work of Norwegian teachers to develop creative uses of mathematics in social contexts within the classroom. Jean-François Perret, Switzerland, reports on empirical investigations about children's understanding of mathematical tasks set to pupils in schools and the discrepancies in meanings exposed between teachers and pupils. Jan Waszkiewicz and Agnieska Wojciechowska, Poland, in his presentation on "The Future Cultural Role of Mathematics and its Impact on Mathematics Education" takes an extreme antiposition to the concept of mathematics as social construction by renewing the platonistic ideal of a universal mathematics independent of any cultural or social development and by relating social context to applications of mathematics only.

Children's Construction of Arithmetical Knowledge in Social Context

Paul Cobb

Recently we completed a year-long classroom teaching experiment in one 2nd grade classroom with 20 seven-year-old children, in which the basic instructional approach for all areas of 2nd grade mathematics including arithmetical computation was small-group problem solving followed by teacher orchestrated whole-class discussions of the children's solutions. The instructional materials developed for the entire 2nd-grade year in the course of the project were designed to be problematic for children at a variety of qualitatively distinct conceptual levels. Detailed models of young children's construction of arithmetical knowledge (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & von Glasersfeld, 1988) were used to guide the development of the arithmetical activities. All mathematics lessons, which were taught by the classroom teacher, were videotaped for later analysis. Additional data sources include photocopies of all the children's written work, three video-recorded clinical interviews conducted with all the children, ethnographic field notes, the teacher's diary of her reflections, and audio-recorded open-ended interviews conducted with the teacher who was also a member of the research and development team.

During the whole-class discussions, the teacher acted with the intention of encouraging the children to verbalize their interpretations and solution attempts. If, as frequently happened, solutions were in conflict, the teacher attempted to frame the situation as a further problem for the children to resolve. However, this aspect of the teacher's agenda ran counter to the children's prior experiences of class discussions. Such discussions had been initiated to steer or funnel (Voigt, 1985) the children towards the officially sanctioned interpretations of solutions (Wood, Cobb, & Yackel, 1988). As a consequence, whole-class discussions were, for the children, situations in which they felt obliged to try and infer what the teacher had in mind rather than to express their own understandings. The teacher therefore had to exert her authority to initiate and guide the mutual construction of obligations and expectations compatible with her agenda for mathematics instruction.

If we view the teacher and children as constituting an intellectual community, the process of renegotiating the social norms that regulated classroom life can be seen as analogous to a change of research tradition. This analogy is plausible in light of Peirce's (1935) claim that the "very origin of the conception of reality shows that this conception involves the notion of a community" (p. 186). This insight that adequate analyses of knowledge construction must take account of the norms embedded in intersubjective communication is as applicable to the mathematics learning of children as it is to progress made by a scientific community. As Knorr-Cetina (1982) put it, scientists are

practical reasoners who "constantly relate their decisions and selections to the expected response of specific members of a community of 'validators'" (p. 7). Similarly, the student's "job is to give a solution to the problem the teacher has given, a solution that will be acceptable with respect to the classroom situation" (Balacheff, 1986, p. 12). For practical reasoners, whether he or she be a research scientist or a 2nd grade mathematics student, established social norms constrain what counts as a problem and as an acceptable solution (Barnes, 1982).

The manner in which the teacher and children renegotiated their obligations and expectations is analogous to the way that scientific communities "hammer out" what it is to be scientific and rational (Rorty, 1980). This hammering out process is reflected in the dual structure of classroom dialogs that occurred at the beginning of the school year. At one level, the teacher and children talked about mathematics, whereas at the other they talked about talking about mathematics (Cobb, Wood, & Yackel, in press). In contrast to her non-evaluative stance with regard to the children's mathematical interpretations and solutions, the teacher was relatively directive when she and the children talked about talking about mathematics. However, she did not attempt to modify the social norms by specifying context-free rules for the children to follow. Instead, she capitalized on unanticipated events that occurred as the children did and talked about mathematics and framed them as paradigm cases within which to talk about her expectations for the children. It is because of their concrete, highly contextual nature that we talk of the renegotiation and mutual construction of classroom social norms. Such norms, whether they regulate activity in a scientific laboratory or a mathematics classroom, derive from the collaborative cognitive activity of individuals. Despite the teacher's intentions, the norms would not have come into existence unless the children figured out the possible significance of the paradigm cases and thus created obligations for their own activity. The children's attempts to fulfill these obligations led to further incidents that the teacher and, sometimes, the children framed as paradigm cases, and these gave rise to additional opportunities for the children to modify or elaborate their obligations. Thus, as in a scientific laboratory, the evolving tradition in the classroom informed both the participants' practice and was modified by further practice. This notion of an evolving tradition or, as we like to call it, a communal story about the nature of mathematical activity, is of course compatible with the view that what counts as science and rationality cannot be specified in terms of ahistorical criteria (Feyerabend, 1975; Kuhn, 1970). Instead, scientific knowledge and the encompassing tradition are constructed together. These two levels are analogous to the dual structure of the dialogs in the 2nd grade classroom.

By participating in the teacher-initiated and guided negotiation process, the children reconstructed their tacit obligations and expectations during mathematics instruction. In effect, a new social reality was negotiated and institutionalized, and thereafter was taken for granted by the teacher and children. At a more global level, we can conceptualize this tacit knowledge in terms of the children's beliefs about their own role, the teacher's role, and the nature of mathematical activity (Cobb, Yackel, & Wood, in press). The profound influence of the children's reconstruction of their beliefs on their construction of mathematical knowledge is readily apparent. We are, in effect, talking about the children's learning of what is traditionally called the hidden curriculum. What counts as a problem and as a solution have changed beyond recognition, as have the self-evident ways that one resolves misunderstandings and learns from the teacher and one's peers.

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The Negotiation of Social Context for Small-Group Problem Solving in Mathematics

Erna Yackel

This paper discusses insights gained about the negotiation of norms for cooperative small-group problem solving in a research and development project designed to study young children's mathematical learning. A classroom teaching experiment, conducted for an entire school year in a 2nd grade mathematics classroom, forms the basis for this paper, which focuses on how the norms were negotiated and the relationship of these norms to children's mathematical beliefs, and for two companion papers by my colleagues Terry Wood and Paul Cobb, one on whole-class interactions and the other on children's construction of arithmetical knowledge in social context.

The instructional materials developed for use in the project classroom are compatible with constructivist learning theory in general (Confrey, 1987; von Glasersfeld, 1984) and models of early number learning in particular (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & von Glasersfeld, 1988). A typical hour-long class session was divided approximately equally between small-group problem solving and subsequent class discussion in which children talked about their solution methods. Children were paired for small-group work. All class sessions were taught by the regular classroom teacher. Video-recordings of all class sessions, ethnographic field notes, and copies of the children's written work form the data source for the analysis.

Negotiation of Norms for Small-Group Problem Solving

When children work in small groups to complete activities in the mathematics classroom, the interactions between the children and between the teacher and the children are important in establishing the context within which learning takes place. These interactions are constrained by the social norms of the classroom and conversely serve to construct the social norms that constrain the individual's activities. Thus, we say the classroom norms are mutually negotiated in the course of mathematical activity. From the psychological perspective, the norms come into being through the expectations that the teacher and children have for each other and the largely implicit obligations that they have for themselves in specific situations (Voigt, 1985). Following Voigt, we analyze the teacher's and the children's expectations and obligations in concrete situations as they teach and learn mathematics to account for the construction of classroom norms (Cobb, Wood, & Yackel, in press).

The project teacher initiated and guided the mutual construction of a variety of social norms including that children cooperate to solve problems, that persistence on a personally challenging problem is more important than completing a large number of activities, and that partners should reach consensus as they work on the activities. In addition, there were norms for individual activity including that children figure out solutions that are meaningful to them, that they explain their solution methods to their partners, and that they try to make sense of their partner's problem-solving attempts.

The strategy the teacher used to initiate and guide the mutual construction of these obligations and expectations was to use concrete situations that occurred in the course of mathematical activity as paradigm cases to initiate a class discussion of obligations and expectations with respect to these particular cases. Consider as an example the following episode in which three children worked together for the first time. The teacher observed that one child was completing all the activities and the other two did not understand his solutions and had failed to construct solutions of their own. She reminded them that she expected them to work together to complete the activities and began the subsequent whole-class discussion of the children's solutions with the following dialog:

Teacher: I'd like to have John, Jennie, and Adam explain what happened to your group today.
Adam: Well, I went ahead and did almost all the problems and forgot to tell them [John and Jennie] how I did the answers. I had done one and a half pages and then Mrs. M came along and she asked us if I was telling them how I did the answers and I said that we weren't.

By calling this specific situation to the attention of the entire class, the teacher attempted to convey to the children in concrete terms what she expected of them as they worked together in groups.

At times there was tension when partners each attempted to act in accordance with one obligation but not another. For example, if one child strove to figure something out for himself while his partner simultaneously explained her solution method, then it was impossible for him to fulfill his obligation. The teacher therefore had to initiate the renegotiation of obligations to optimize mathematical learning opportunities for the children. The teacher might say, for

example, "Just a minute. Let him try to figure it out for himself first. The you can explain your method."

Children's Mathematical Beliefs

Our discussion of obligations and expectations directly addresses the children's evolving beliefs about their own and the teacher's role as they learn mathematics and about the activity of doing mathematics. For example, the obligations nurture the belief that mathematical activity should be explainable, justifiable, and rationally grounded. Further, the children had the opportunity to view mathematics as an activity under their control rather than as disembodied, objectified, subject matter content. The children also came to realize that mathematical problems can have multiple solutions. Further, the children's acceptance of the obligation to think through their problems for themselves as they worked in small groups, as indicated by their persistence, evidenced the belief that it sometimes takes hours rather than minutes to solve mathematical problems. For a detailed discussion of emotional acts, beliefs, and social context in mathematical learning see Cobb, Yackel, and Wood (1989).

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Creative Uses of Mathematics in Social Contexts

Stieg Mellin-Olsen

I am going to tell you about the experiences I have when I provoke teachers of Norwegian primary and secondary schools. I provoke them by enforcing them to do a project with their classes, which is based on the students' interests of knowledge, thus supporting these interests.

First I want to give you a description of the theoretical background of my praxis.

Activity theory as it was developed by A. N. Leont'ev and his collaborators is rooted in Friedrich Engels' *Dialectics of nature*. By this dialectic, Engels analyzes how Man, during history, has created Nature while, at the same time, Man is determined by Nature. Transferred to Activity theory, these dialectics can be expressed as follows: Man creates Society at the same time as Society determines Man. Knowledge functions as *thinking tools:* In order to create Society, Man needs knowledge as tools (Leont'ev, 1978; Vygotskii, 1976).

This dialectic can be expressed in several ways:
Man both reproduces and produces culture (Willis, 1978).
Man both lives *in* society and *with* society (Freire, 1972).
Man assimilates activities of society and accommodates to these activities (Piaget).

One didactical implication I draw from all this, is that over time students cannot fully appreciate the functionality of mathematical knowledge if they are not permitted to use them in social creative ways.

More generally: The individual who is only permitted to reproduce society by means of the school knowledge passed to her, thus not using the knowledge for the purpose of participation and creation of society, gradually develops an unhealthy relationship to school learning. I have developed this in detail elsewhere (Mellin-Olsen, 1987).

In everyday language: The student who is only permitted to use mathematics in connection with tasks given to her by the teacher, and eventually by the textbook, is not on a prosperous way to knowledge.

A dialectic approach to understand teaching and learning includes the view that the learner apprehends the unity of creation-recreation possibilities of knowledge. This implies that the learner eventually can accept reproduction of knowledge, because she can see such reproduction in perspective of its productive uses.

Again in everyday language: The hypothesis is that students of mathematics accept to a far extent what we can call traditional teaching methods, if they are also permitted to use mathematics in creative ways.

This way of reasoning leads to a theory of learning based on the student's motivation to learn rather on the methodization of the teaching process; the moti-

vation of the student being the basic drive for the learning process.

You are now probably waiting for the interpretation of my notion about "creative uses" of mathematics.

In order to get just signs of such uses, I find that I have to *provoke* teachers. I can do that since I am a teacher educator.

In our half year post-graduate course in teacher training I ask teachers of primary and secondary schools to do a project with their students. The teachers are asked to help their students to document important issues of their lives; "important" issues that are related to their knowledge interests in their social environment. The documentation tools are chosen from mathematics.

As a part of the post-graduate course, the teachers are to report back the experiences of their projects.

I can divide the reports into three groups according to how the teachers conceptualize "important issues in the students' lives."

(1) Reports describing projects which are related to adult interests rather than the interests of children and adolescents. An example of such a project is one about the cost of school books and the necessity to take care of them.

(2) Reports describing how the teachers gave their students an overview of possible mathematical tools suitable for documentation. Following this, the teachers had provoked their students to find topics themselves which they could investigate.

(3) Reports describing how the teachers introduce a theme to be explored, and how discussions and investigations of these topics develop among the students.

Examples of such topics:

- Norwegian school children achieve a gold medal if they walk 100 km on skis during the winter. Graphic tools were introduced in the class to control the distance obtained in relation to the 100 km.
- Road safety. Investigation of factors concerning safety on the roads leading to the school.
- Equal opportunities for men and women. Investigation of sex differences in relation to wages, job opportunities, classroom behavior, etc.

On the whole, the teachers who introduced a theme rather than letting the students choose achieved the most. Let us see how.

(1) The projects where the students had to choose their own themes consisted mostly of counting a phenomenon. There was counting of favorite food, of drinking situations (alcohol), of hours of watching television, etc. No discussions among the students about the results were reported.

(2) The projects where the students were guided into a topic to be studied led to discussions among the students. These discussions were related to both the properties of the various thinking tools (what is the advantage of analyzing *this* relation by means of *that* mathematical tool?) and they were related to the development of the project itself. In the projects about road safety and sex differences, the students took over the management of the project after some introductory work.
They thus grasped control over the goal of the project, relieving their teachers from considerable work on organization and planning.

(3) In this second group of projects the students analyzed and investigated relations, or relational knowledge. They would ask "what is road safety?" and "what are the basic circumstances which road safety is related to?"
"What is the rate of women out of work in various age groups, and why is it that the 40–45 group has the highest rate?"
It was interesting to see how even the youngest students managed discussions about relations, thus being on the narrow road to theoretical thinking (Otte, 1988).
  - A theory about which factors make a road dangerous to children.
  - A theory about which factors keep women out of jobs.

In the groups of teachers whose students were free to find their own themes, no investigations of relational knowledge were reported.

(4) I can argue that the three projects of the third group led to creative action in the social environment of the students.
Road safety: Location of dangerous spots, documentation about these to the authorities, follow-up strategies if nothing was done. The class took the initiative and responsibility for the project.
Sex differences: New insight. New attitudes and new praxis in relation to the other sex. The class took the initiative and responsibility for the project.
Ski medal: Control of strategies for obtaining the distance.

Summary

I try to examine and develop mathematics teaching and learning through motivation. *Motivation* is related to creative—or productive—uses of mathematics related to the lives of the students.

In short, the teachers who succeed with their projects observe new energy for learning in their students. This happens is particular when students take control over the projects.

According to our experiences, the teacher has to guide her students to a suitable theme for a project if motivation is to be gained. It is not sufficient—still according to our experiences—to ask the students to find their own themes according to their interests. In this case, the students apparently do not believe in the didactical situation set up for them.

They start counting a phenomenon without relating the phenomenon to other issues. In fact, several of these students turn the project against themselves. In our case, they documented how many hours they watch television, on how many occasions they consume alcohol or drugs. It is as if they cannot do any-

thing wrong in relation to a new sort of didactical situation, if they turn the project against themselves.

Experience thus shows that the teacher has a task to find project themes for the students, especially when the students have few experiences with creative uses of mathematics. This implies that the mathematics teacher has a more vital role than usually recognized—one who conveys her students into her society.

When the students are convinced that the project really can be theirs, unexpected capacities and energy for learning might be released.

So far, it is difficult to be more detailed about which teacher will succeed, and which will not. So far my key words are: solidarity with students, sensitivity of what issues are important in their lives, and generosity. I mention generosity as I am not convinced that it is trivial for all teachers to hand over her strongest thinking tools to their students, at the same time as they demonstrate the most vital uses of them.

Signs of teachers' success might be students discussing the project, leading to:

- students taking over project by redefining its goals and defining new tasks;
- relational thinking eventually leading to theoretical thinking;
- students taking advantage of knowledge from their culture for the benefit of the project, which in turn implies that the teacher can learn something about the lives of their students.

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The Meaning of Mathematical Tasks for Pupils and Teachers

Jean-François Perret

In a series of empirical investigations we have studied how children behave when they learn mathematics in school. We have paid special attention to their understanding of the numerical system (Perret, 1985). We were then struck by the discrepancy between the meaning that the children gave to the school tasks and the meaning that the teachers attributed to these tasks.

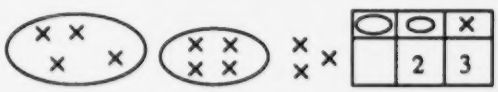
We will report on this and then say how these results raise questions on the nature of the so-called "mathematical tasks" set to children in school.

First, let me present some of our investigation results. In the studies of children's mathematical learning, the most widespread approach is the evaluative approach. It consists of assigning tasks or problems to the pupils in order to examine under which conditions the pupils *provide the correct solutions*. In our current investigation we have not adopted such an evaluative approach but a quite *different* one paying close attention to the *pupils' strategies* when they deal with a task: of course we are interested in the nature and quality of their performances, but we are even more interested in describing and understanding on which *skills and knowledge* pupils rely when they try to solve school tasks in mathematics. This more psychological and more qualitative approach is the one we have used to study the elementary school child's understanding of the numerical system.

We have analyzed pupils' written responses to different mathematical tests. We have observed pupils doing group work. And we have conducted interviews with pupils and interviews with teachers. In all of these investigations we have repeatedly found that for each mathematical task in school, children develop very specific *know-hows* that are, to a large extent, bound to the task, with only a narrow local validity. The "know-how" that the children develop and rely upon is very *task specific*. Let me give you an example:

The Swiss curriculum for the elementary school contains a series of activities to explore various numerical bases. The aim of these exercises is to invite the pupil to a wider understanding of the foundations of our numerical system and in particular to comprehend better the importance of *position* in the writing of numbers. In this scope, children are presented with different activities that are supposed to invite them to compute and to write numbers in different bases.

For instance, one of these activities consists of *grouping* objects of a collection and then in writing the *code* of the collection in a given numerical basis.



Another example: Here the child is asked to group the objects of the collection by *tens*. As the collection has 28 elements grouped by tens, the code they have to write is | 2 | 8 |.

Seven and 8-year-old children do this *easily* and their performances can be evaluated as *very good*. But we have ventured into looking at the meaning that this code bears in the pupils' eyes. And we have observed that most of these pupils *did not consider such a code as being a written form of the number 28!* For them, this code was exclusively a way to note down that they had done what they had been asked to do.

This code was just a way to write down what they had done at the moment. The meaning of this code, for them, is dependant on the context of its elaboration.

Often, this type of behavior tends to be interpreted as lacking generalization, as difficulties to transfer learning from one task to another. But we believe that the difficulty lies in the fact that the pupils' solving strategies are very task specific and that the know-how that they develop at this occasion is in fact not appropriate for generalization.

On these coding tasks there is a large discrepancy between the teacher's expectation and the pupils'. For them it is just a notation of an action. For the teacher the aim is to offer the child an opportunity to discover the positional system.

In many topics of school math we can observe such misunderstandings between teacher and pupils about the nature and aims of their joint activity in the classroom.

Another example: One of the activities of classification required the pupils to find a procedure in order to fill in the blanks of a table, in which part of the classification was already done. Teachers could legitimately believe that pupils who succeeded in this task had understood the logic underlying this classification. In fact, this was *not* the case: the same pupils were incapable of constructing the entire table if it was presented empty.

Having presented to you some of these results, I would now, in the second part of my speech, like to turn to some of the solutions that have been proposed to deal with these misunderstandings. I suggest that they are not sufficient.

To try to reduce this discrepancy between the teacher's and the pupils' representations of the tasks and of mathematical activities, many researchers have suggested to give to the pupils *more global and meaningful activities*, to set *goals* that are more explicit and common to pupils and teachers. For example:

- *Games* (with dice, numerical games, magic squares, mastermind, tangrams, and other games requiring strategies or construction);
- Another suggestion was to introduce *adequate didactic material* (Dienes' Blocks, Cuisenaire's Sticks, etc.);
- or to imitate daily *life situations;*
- or to do math in connection with other subjects of the curriculum;
- and so on.

Many of these suggestions are stimulating indeed. In particular they are interesting because they invite the children to be active in finding adequate solutions by themselves. There is not the above-mentioned discrepancy between the teacher's aim and the pupils'.

But new problems arise this time concerning the *meaning of the task*. For instance, many children refuse to consider that they learn when they play! *In most of these tasks they also do not understand what they have to retain as mathematical knowledge!*

In conclusion, there is a series of questions that these investigations raise:

- What are our *aims* when we assign tasks to pupils?
- How do we choose the *problems?*
- On what kind of objects do we want them to operate?
- To what extent can they understand the *meaning* of the tasks that they are doing?
- What are the pupils' *representations* of what mathematics is about?
- What connections can they make between *school math* and *reality?*

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The Future Cultural Role of Mathematics and its Impact on Math Education

Jan Waszkiewicz and Agnieszka Wojciechowska

In all considerations of this session we were trying to understand what the influence of social environment on math and its understanding is. But our task is not only to be mathematicians and math educators, but also to be human beings and educators of being humane. So we now want to change the perspective and ask the opposite question of mathematical construction of social meaning. A person needs a coherent picture of the world to be humane—our duty is to use mathematics to help build this picture.

It seems that one of the basic human rights is the right to live in the safe, secure, and hence self-evident universe. The picture of the universe, which is given by modern science, does not meet this basic need: the chaotic universe full of black holes and other unintelligible phenomena, starting with the Big Bang and going to the inevitable end, and even more imperiled, the lone Earth with its crazy and weak human beings almost hopelessly trapped on it ... this is part of the picture given by modern science and imprinted into young minds by the schools and the mass media. The clear, evident, and optimistic picture constructed over centuries is nowadays destroyed, and not only some general civilizational trends, but also mathematics and math education have done their part in this business.

One of the reasons for such a turn can be called globalization of the Earth. With this we mean the change of the vision of the world in the individual minds, as well as the picture of the world accepted by the culture, consisting of seeing the Earth really and deeply as the one, complex wholeness of unique value for mankind. "Only one Earth"—this is the way of

expressing this new vision in the title of a well-known book. But the main symptom of the change is the changing scale of problems (or our vision of them). The problems which one generation ago were local, nowadays are global, i.e., important (or at least interesting) for people on the whole Earth. Problems of environment, resources, and interconnected economical processes are the most evident examples of this tendency. The same can be said about the policies of both the great powers and of the do-it-yourself scale. Two examples from our biographies—the student movement of 1968 and "Solidarity," show this evidently.

The fundamental character and far-reaching consequences of globalization are visible if its topological nature is recognized.

Throughout centuries, Earth was seen as the surface, which was the topological equivalent to a bounded planar area. One blank on the spherical map (meaning the area is non-existing for us, not belonging to our world in any reasonable sense), no matter how small, makes an appropriate transformation possible. In consequence, Earth's area could be seen as inscribed into a circle with its own village, own city, or the capital of its own state (or empire) in the center of it. This picture changes rapidly the moment the global character of the Earth is taken into account (not only superficially, but as the essence of the human world). It seems that the Apollo mission can be seen as a turning point in this process—when the picture of the lone Earth as visible from its closest neighbor was transmitted to all mankind. The metaphor "spaceship Earth" expresses this new consciousness very well.

The new topology of human imagination has far-reaching consequences, two of which follow immediately from what was said above. The first is new geocentrism—centering on the Earth and its problems. The second is polycentrism which is irremovable from this new picture of the world.

In this situation the problem of the new picture of the world arises. It is still more difficult, since it involves geometrical problems stated by the modern cosmology. Moreover, this new picture should be somehow common to different cultures of the unified world.

The next consequence is the need for adequate means of information exchange, data processing, and control on a global scale. (Indeed it is the stimulator of integration rather then its result.) Forecasts of so-called information society should be mentioned here. Let us, moreover, point out that the better understanding of the wholeness of the world opens new vistas at the global engineering, not as naive and self-confident as it was during the time of TVA or most sound cosmic and military projects of the last decades. This gives rise to the new socio-cultural phenomenon which, being by its very nature a branch of technology, widens essentially the scope if its goals, means, and methods. In previous papers we called this phenomenon super-technologies.

Also, the traditional science transforms. Looking for the new coherent picture of the world in all its dimensions gives to it this fundamental character which is alien to its positivistic model. This gives us the new demarcation line between traditional areas of the culture: science and technology. This new border also goes inside mathematics. In fact, we have to deal with two different approaches to mathematics, and two kinds of social (and cultural) demands for it. When seen from the point of view of the super-technologies as well as of needs of information society, mathematics is the set of methods and models which are useful in two main areas, both connected with decision making: data processing and problem solving. Mathematical models and methods help here to organize data, to simplify (and understand) the situation, to find and optimize the solution.

Models and methods used in such a situation are mostly ad hoc ones. The theory in the good old style is sometimes less important then practical plausibility tested experimentally, e.g., by computer simulation. Moreover, as is shown by the solution of the problem of four colors (and many others similar in this respect), the very notion of the proof should be reexamined now. The logical consistency being the pride of mathematics throughout the centuries is only of secondary importance.

Such functions and approaches are consciously accepted in the growing number of methodological works on applied mathematics. This field already has methodological self-consciousness, which is one of the determinants of autonomy of any science.

Let us now turn our attention to the second, pure part of mathematics. As was pointed out above, its future role is to help elaborate the new picture of the world. As is already seen, efforts to create this picture will go very deeply. They can involve all main categories of rational reasoning. The fundamental character of incoming changes opens new opportunities for mathematics, assuming it is able to be used appropriately to this task. And it means coming back to old Platonic and Pythagorean visions of the world and the role of mathematics in it.

This point is crucial for us, so let us discuss it further. It is obvious that the return to the past ideas will not be simple. With this, we only mean the return to some general attitude towards mathematics. It means convincing oneself—whether teacher, student, or researcher—that mathematics is relevant to reality. General applicability of mathematics is nothing but a reflection of the fact that the world is mathematical in its deepest core. There is a social world in which everybody is living. There is a biological world underlying it. There is a physical world more fundamental than the former two. But there is also a mathematical world somewhere in the deepest layers of reality. This world can be approached by intuition, it can be visualized—as Emmer is doing it. But basically it can be grasped by rational analysis.

Such an approach again roots mathematics and school math in the old tradition of looking for

archetype—the first principle of the cosmos (i.e., universal order or harmony), and roots it in values such as truth, beauty, and rationality as well. Such mathematics will be able to confront the demands of the future.

Of course such an approach can have many instances, but it is possible and it works, at least on the secondary school level, where we taught according to it a few years ago.

But what to do with this applicable, computer-based part of mathematics, which we were discussing so extensively at the Congress? Here we can give the following analogy. The physicists are supposed to understand well the processes and facts underlying driving a car. But sometimes an almost illiterate driver does it better and teaches it better.

Let us leave computers to run by computer people and let us concentrate on what we can do better.

Mathematics Education in the Global Village

In the five hearings of this timeslot questions were put, on behalf of society, by the chair/questioner to a panel. The questioners focused on the accountability of the international mathematics education community in relation to the particular hearing theme. The new format of a hearing aimed at concentrating on questions instead of statements, but was not successfully applied throughout. The hearings were to be a summary and conclusion of the special program of the day in concentrating the panels on very specific questions. Selected speakers of the special program were questioned in these hearings that contributed to special themes and topics in order to get their discursive or conflicting answers.

Hearing 1
Which and Whose Interests are Served by Mathematics Education?

The following questions were raised at the hearing by the two questioners, **Christine Keitel** (Technical University of Berlin (West)) and **Yves Chevallard** (Université Marseille, France): What is the relationship between social needs (e.g., relative to society as a collective body) and individual interests in changing social settings? Who defines qualifications provided by mathematics education? What are the conditions of a democratic debate negotiating demands on mathematics education? What about the relevance of the distinction between explicit, live mathematics used and created by few professionals in science and technology and implicit (dead, "frozen," crystallized) mathematics in the finished products and technical constructs or services? Is the fact, that mathematics, by its social use, has become more implicit and invisible although more widespread as technology, reflected in explicit mathematics teaching? What could be meant by democratization through education, and how can mathematics education contribute to building up democratic competencies, but avoiding 'cultural imperialism'? How could mathematics education help to promote mutual accountability and give full scope to democratic imagination to establish new forms of social contracts, communications, and discourse? Within the hearing the panelists brought out several particular points.

John Fauvel (Open University, United Kingdom) explained the role of the mathematical practitioner in the Renaissance and compared it with social practice of mathematics today. By referring to the new technology he argued that teaching should emphasize judgment and wisdom rather than particular skills in order that people should not feel scared of this technology.

Kathryn Crawford (Sydney University, Australia) agreed that what was needed was not only a greater confidence to deal with a highly mathematized society but also a better understanding of mathematical structures in relation to everyday life, social organization, and collective welfare in order to be fit for democratic control. There must be a common core of a much greater expertise in the sense of opportunity in school for information, interpretation, and making judgment about, instead of so much emphasis traditionally being placed on computational and procedural skills: Knowing what to do instead of knowing how to do it. Also technology brought into the Third World mostly disempowers people and continues in their being exploited.

For **Ole Skovsmose** (Aalborg University, Denmark) the link between interests and mathematics education was the idea of democratic competence in society; and in today's technological society he sees a need to increase people's competence in understanding and reflecting mathematical models, the intentions behind their construction and use, in addition to applied mathematical knowledge.

Murad Jurdak (American University, Lebanon) felt that unlike language teaching, mathematics teaching did not develop in relation to the learners' interests. Rather the interests of the state and of mathematics experts appeared to be of paramount influence.

Claudia Zaslavsky (USA) outlined how in the USA mathematics education was geared to the interests of an elite, through the control of the curriculum by national testing procedures, resulting in feelings of inadequacy amongst large numbers of peoples. Speakers from the floor contributed to the hearing by raising the important point of general power imbalance between 'exploited' and industrialized countries, the dominance of interest groups through language, the patronizing stance of Euro-American educators, and the plight of the oppressed black people in the Republic of South Africa. There was a general wish that the issues which this hearing addressed had to be complemented by the question: Which and whose interests *should* be served by mathematics education? and to be debated much more widely within the congress.

Hearing 2
How Does Mathematics Education Relate to Destructive Technological Developments?

In his introduction, questioner **Jens Naumann** (Max Planck Institute for Human Development and Education, Berlin (West)) reminded the panelists that in the Enlightenment it was considered a great social advance to liberate reasoning, theorizing, and research from everyday life and from the moral and ethical limitations guiding social life. The hope and promise behind that was the optimistic idea that this liberation would eventually lead to the discovery of more rationality, more efficient and better ways of social organization; science, and technology were seen as equivalent to

progress and to the search for better human conditions. This thinking was the proper hidden religion and driving force of the bourgeois capitalism as well as modern socialism. The core of the scientific disciplines were natural sciences and mathematics associated with engineering disciplines; whereas the humanities and social sciences in this thinking never reached the state of real sciences, as they remained too fuzzy, too much involved with and too close to religion, aesthetics, moral, and ethical concerns and limitations.

He noted that there are only two professional disciplines which institutionalized with some success some kind of ethical code for their members that are organized and administered by their professions themselves: the professions of law and medicine. Nothing similar and comparable exists so far for any other discipline. Today, it seems, at least in the advanced capitalist countries, that the time of scientific optimism is over. This is witnessed by the critical debates about the dangers of advanced technology, the enormous and unforeseen ecological crises and the potentially destabilizing effects of military research. With this background he asked the panelists if there had been any qualitative or quantitative changes in the recent decades scientific and technological development calling for a much more prudent and cautious approach to the use and applications of scientific advances, and if so, what are the implications for mathematics education.

Jan Waszkiewicz (Wroclaw University, Poland) voiced the opinion that, in the concrete case of Poland, mathematics and mathematics education do not contribute to any destruction. On the contrary, they make the citizens critical.

In **Hugh Burkhardt**'s (Nottingham University, United Kingdom) view it is not the scientist who is in the end responsible for the use of his/her discoveries. The social control belongs to all citizens. Mathematics education should educate all citizens to be able to question proposals and decisions made by militarians and politicians as was done in the star-war debate.

Jens Høyrup (Roskilde University, Denmark) argued that the level of understanding of the social implications of the work of mathematicians and scientists has deteriorated as they have become only elements in a segmented hierarchical system-like bureaucracy.

Bernhelm Booß-Bavnbek (Roskilde University, Denmark) completed the reasoning by pointing out that as mathematicians are integrated in such a system they cannot individually control or assess their work. It is necessary to change the relation between human beings and their products, and mathematics education should contribute to this. It is not the scientific attitude in itself that should be blamed for destructive developments, but the 'military technical pollution' of scientific thinking. There are, today, unprecedented risks of technological applications in the civil and military field on the bases of models and simulations void of theoretical comprehension and insensitive to the limits of validity of existing empirical knowledge. Mathematics education should acknowledge this situation.

Hearing 3
Do Mathematics Educators Know What They are Doing?

Questioner **Jeremy Kilpatrick** (University of Georgia, USA) asked the panelists for their view on the question and discussion started from this task.

Nicolas Balacheff (University of Grenoble, France) raised the issues of who are the mathematics educators—mathematics teachers, teacher trainers, researchers; what kind of knowledge do mathematics educators need and how can research provide this 'professional' knowledge. He argued that professional knowledge in mathematics education for a long time has been considered only as practical experience and practical knowledge mostly related to traditions, schools of ideas, ideologies and opinions about the principles of teaching design, pedagogical methods and fashions. The major step forward came with the recognition within the community of mathematics educators that the problems of mathematics education need scientifically based research, empirical investigations with specific theoretical and methodological tools as the source of evident, valid professional knowledge. He claimed that in many societies research is not recognized and accepted as part of the professional activity and responsibility of mathematics educators and that they have to strive for acknowledgment and support by politicians and decision-makers.

Marilyn Nickson (Essex Institute, United Kingdom) argued that it is from the teachers that everything stems. Mathematics educators in general can only be accountable in terms of what mathematics teachers do in the classroom. So she particularly emphasized the new opportunities in research fields like action research and ethnographic research which can help mathematics educators improve their contact with classrooms.

João Ponte (University of Lisboa, Portugal) presented the case that a mathematics educator is anyone professionally concerned with the intersection of mathematics and education. He therefore looks for a more professional engagement with relevant activities, such as curriculum development, assessment, teacher training, and research. ICMI has for him a strong role to play in improving society generally.

Hans-Georg Steiner (University of Bielefeld, Federal Republic of Germany), argued for non-simplistic approaches. He showed that much depended on what theoretical perspectives are adopted as well as on the context in which the question was raised. All the panelists agreed that whatever micro-situation a mathematics educator worked in, there must be acceptance of some responsibility for the macro-situation as well.

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Hearing 4
What are the Challenges for ICMI in the Next Decade?

Questioner **David Wheeler** (Concordia University, Montreal, Canada) offered the panelists the main question and three more specific ones based on the idea of ICMI as a *scientific, international institution*. What should ICMI's *scientific* priorities be in the 1990s? How could ICMI improve the effectiveness of *international* cooperation in mathematics education? How should ICMI as an *institution* adapt to the challenges of the 1990s?

Aderemi Kuku (University of Ibadan, Nigeria) argued that ICMI should continue to do what it does now, but also that it should concentrate on implementing the recommendations in its study series, that it should develop more themes such as the influence of first language on mathematics learning and increasing the role of women in mathematics, and that it should seek more funding for its important work.

Stieg Mellin-Olsen (Bergen College of Education, Norway) argued for much more open debate within ICMI. He deplored the subdividing of groups and issues into 'sections' which prevented debate, and he challenged the commission to get the debates out of the backrooms and corridors into the plenary context.

Gilah Leder (Monash University, Melbourne, Australia) agreed that in relation to its scientific work ICMI should promote more debate, but she also wished for more action and fewer words concerning international cooperation. She recommended ICMI to be more open to alternative approaches to mathematics education issues.

Bienvenido Nebres (Ateneo de Manila University, Manila, Philippines) felt that a major challenge for ICMI is how to keep discussion and involvement alive between the four yearly congresses. He saw more regional conferences and other study groups (like PME, the International Group for the Psychology of Mathematics Education) as ways to achieve this. He would also like to see ICMI as an international body making more contact with the key decision-makers in individual countries, who determine the kind of support mathematics education receives.

Ed Jacobsen (UNESCO, Paris, France) proposed an ICMI study on how to provide *more* mathematics education with *less* resources, a challenge facing members from many countries. He wanted ICMI to improve its communicating role both internally and externally, through examining the status and role of national representatives, through using the bulletin in better ways and through increasing the international participation rate at the congresses.

Hearing 5
What can we Expect From Ethnomathematics?

In his introduction, questioner **Ubiratan D'Ambrosio** (Campinas State University, Brazil) clarified the meaning of ethnomathematics. Ethnomathematics has to do with the social-cultural roots of the dialogue of Mankind with Nature, trying to use his or her power of thinking (counting, measuring, sorting, classifying, etc.). Out of this, throughout the ages, Mankind developed a codified, institutionalized mathematics. Confronted at school with this codified, institutionalized mathematics, both children and adults feel insecure. The school tries to impose certain modes of thought different from the social and cultural mathematical experience of the pupils. If we want mathematics education to contribute to the liberation of Mankind from fear, then ethnomathematics is a must. But many have doubts: "Children will go to school without learning mathematics, if you always keep talking about ethnomathematics. Where is ethnomathematics leading to?" Then questioner Ubiratan D'Ambrosio asked the panelists why each of them came to ethnomathematics.

Mary Harris (University of London, United Kingdom) contrasted the low achievement of girls in mathematics at schools with their capacity to do complicated needlework, sewing, etc. The problem lies in who defines what mathematics is, in who defines which are the standards. In a male-dominated society, women's mathematics is not acknowledged. She wants women to become more interested in mathematics and to profit from it. Her project 'Maths in work' aims to make closer links between the mathematics that goes on in school and the mathematics that goes on at the workplace. By producing learning materials that reproduce some of the activities of the workplace, she tries to identify and make the mathematics explicit within them. She looked at textiles as mathematical objects. After a brief analysis of the mathematics of a t-shirt from Turkey and the designs on a skirt from Ghana, she introduced an important question: All over the world most persons who produce clothes are women. The largest single group of people that hates learning mathematics are women, too. Given that 100% of the people at this conference wear clothes most of the time and are mathematicians most of the time, *why* is it that the problems of the designs of their clothes are not considered to be mathematical? This, she hopes, ethnomathematics can probably answer.

Patrick Scott (University of New Mexico, USA) described the objectives and activities of the International Study Group on Ethnomathematics (ISGME), of which he is the secretary. He explained that there are two major models in ethnomathematics: the *D'Ambrosio-Gerdes* model (ethnomathematics in Third World countries to bridge the gap between home culture and 'modern' science) and the *Zaslavsky-Ascher* model (ethnomathematics in First World countries to bring the world into the classroom and to

have an appreciation of the math of other cultures). On the basis of his experience in the USA and in Guatamala, he pointed out that there is a risk with the use of both these models in schools: the risk that people who introduce the ethnomathematics will not really understand all of the cultural aspects and will just present caricatures of ethnomathematics that will neither support the learning of mathematics nor the cultural understanding.

Munir Fasheh (Birzeit University, occupied Palestine) attacked the Western mathematics as the worst fundamentalism there is as it leaves no choice to believe in it or not. It justifies itself by arguments of universalism and objectivity. He criticized also the fundamentalism of the dominant epistemological model, the type of logic which believes in true and false statements only. For Fasheh every social reality is so rich that telling students from the 1st grade to the 12th grade that mathematics rests on certain principles is dehumanizing. He compares the monopoly of Aristotelian logic with that of Coca-cola: Instead of drinking the pure water in their environment, people drink Coca-cola, thus reinforcing their economic dependency. The real need is for water, and most people have their own springs for mathematics. Nobody can survive for more than a few hours without relating to the world in a mathematical way. And then comes somebody from outside, from one of the centers of learning in the world, to tell what real mathematics is, that the mathematics used is not real mathematics. The dominant type of mathematics learned and taught in school is useless and meaningless to most of the students in the world, especially in the Third World. His quest is not to assert a Palestinian mathematics, an Islamic mathematics, or another mathematics, but to assert a *meaningful* mathematics.

Paulus Gerdes (Eduardo-Mondlane-University, Maputo, Mozambique) explained how racial and colonial ideology negated the capacity of Africans to do mathematics. He underlined the building up of confidence in the people. To destroy this confidence of the people in their own culture was probably the most efficient and active tool of colonialism. To win (cultural) self-confidence, it is important to show the students that mathematics is not alien to their home culture, that mathematics does not come from outside Africa, that mathematics exists in Mozambican society, and that it is possible to build on that traditional knowledge. In order to defend the country against continuing aggression, in order to improve the quality of life, more mathematics is needed than only ethnomathematics. In school ethnomathematics may serve as a starting point: Starting with ethnomathematics and other cultural elements with a high mathematical potential, one may accelerate the access to all knowledge that is meaningful in a given situation. One takes from worldculture, from worldmathematics, what is necessary for the improvement of the quality of life. Here worldmathematics (or mathematics) may be briefly defined as the union of all ethnomathematics.

Ubiratan D'Ambrosio emphasized that ethnomathematics is essentially ethical: to bring in moral issues—the concern about the quality of life of the people—fits very well with a kind of activity that comes out of the community as in the case of ethnomathematics.